Are GPDs and TMDs related?

Marc Schlegel,
Theory Center, Jefferson Lab

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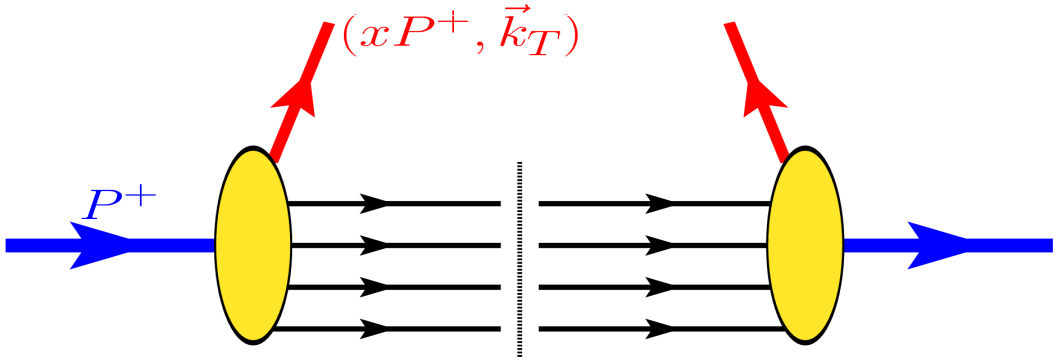
k_T-dep. Parton Distributions (TMDs)

What are Transverse Momentum Dependent Parton Distributions?

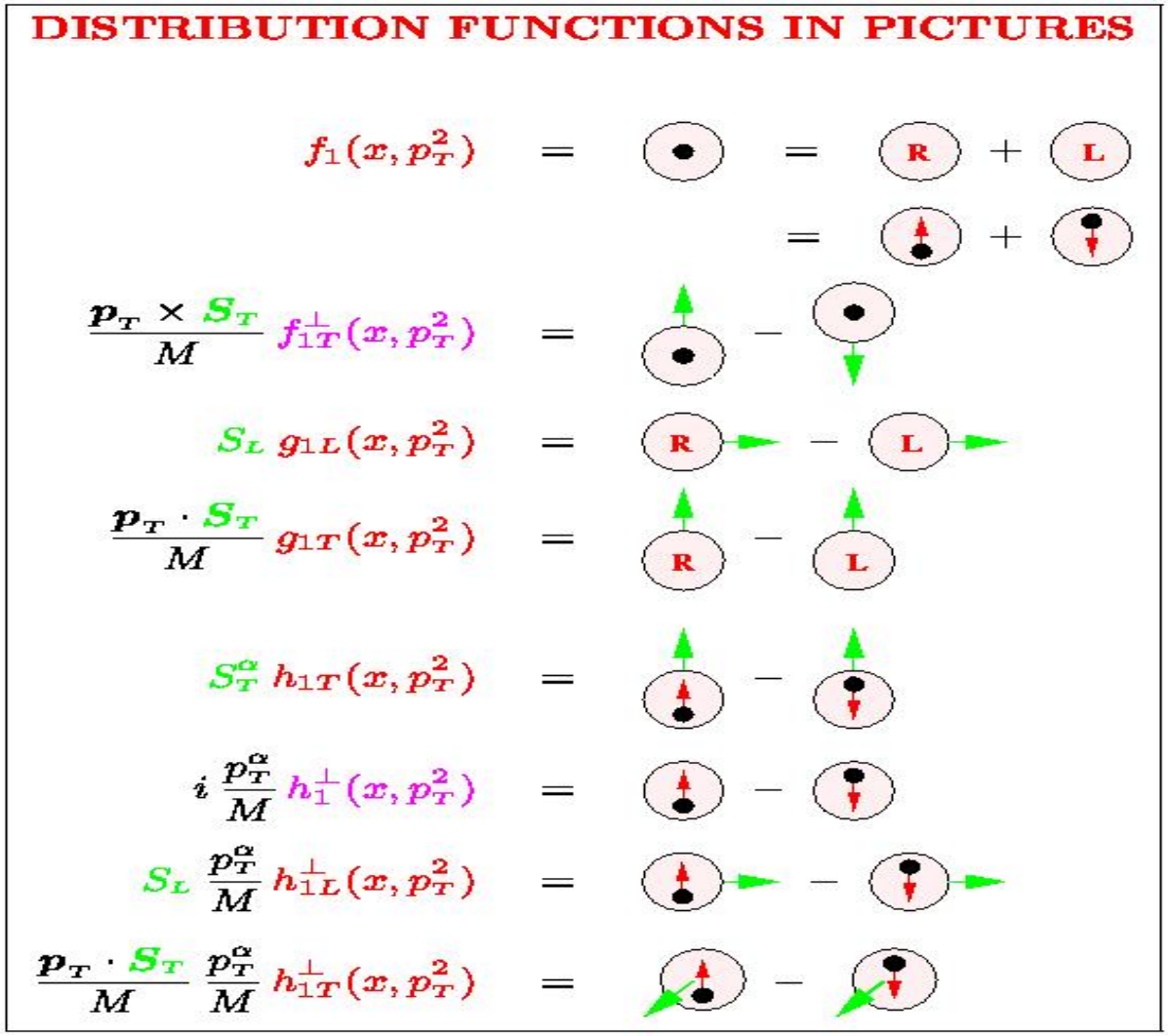


$$\Phi_{ij}(x, \vec{k}_T; S) = \int \frac{dz^- d^2 z_T}{2(2\pi)^3} \, e^{ik\cdot z} \langle P, S | \bar{\psi}_j(0) \mathcal{W}_{\text{SIDIS/DY}}[0, z] \, \psi_i(z) | P, S \rangle \Big|_{z^+=0}$$

- Integration over k_T → ordinary collinear parton distributions f_1, g_1, h_1
- Project out polarizations: 8 leading twist TMDs

unpolarized:	$(f_1 \, , \, f_{1T}^{\perp})(x, \vec{k}_T^2)$
longitudinally:	$(g_{1L} \, , \, g_{1T})$
transversely [chirally-odd]:	$(h_1 \, , \, h_{1T}^{\perp} \, , \, h_{1L}^{\perp} \, , \, h_1^{\perp})$

TMDs in pictures



"unpolarized" → f_1

"Sivers"

"helicity" → g_1

"transversity" → h_1

"Boer-Mulders"

"pretzelosity"

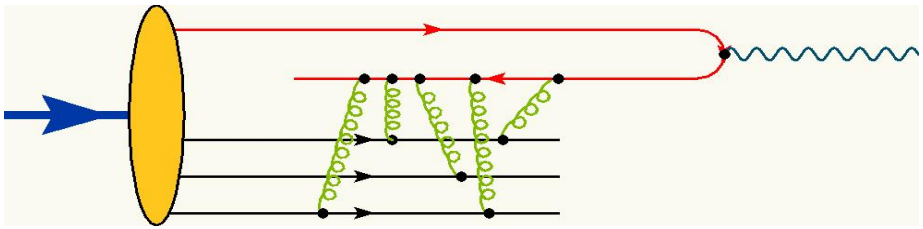
T-odd TMDs

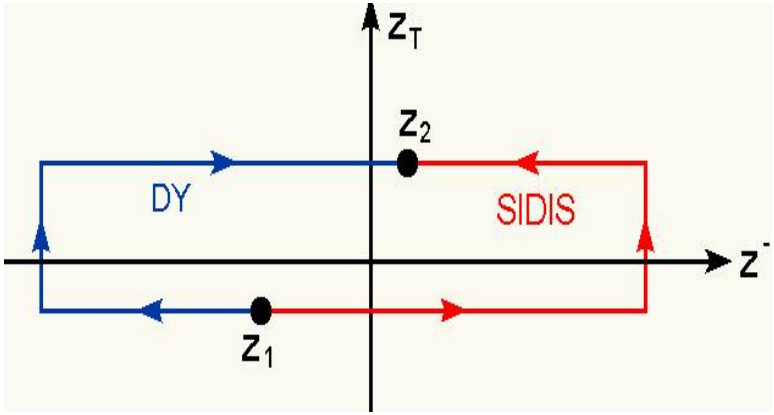
- *Neglect* gauge link operator:

Time-reversal forbids → Sivers function $f_{1T}^{\perp}$

Time-reversal forbids → Boer-Mulders function $h_1^{\perp}$

- If T-odd TMDs $\neq 0$: Gauge Link not neglegible, physical effect:





$$\mathcal{W}[z_1; z_2] = \mathcal{P}\mathrm{e}^{-ig\int_{z_1}^{z_2} ds\cdot A(s)}$$

Initial / Final state interactions

Time reversal → switches sign:

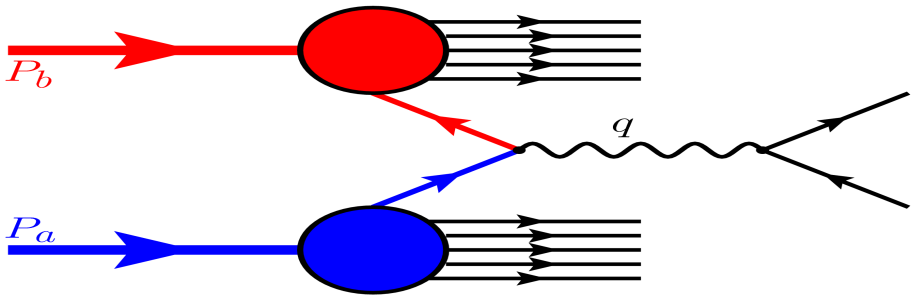
$$f_{1T}^{\perp}\Big|_{DIS} = -f_{1T}^{\perp}\Big|_{DY}$$

$$h_1^{\perp}\Big|_{DIS} = -h_1^{\perp}\Big|_{DY}$$

TMDs in physical processes

Theoretically "clean": semi-inclusive processes with simple final states

Direct acces to TMDs: Drell-Yan dilepton pair production



TMDs appear in the kinematical regime: $q_T^2 \ll Q^2 \equiv q^2$

Simple parton model:

$$f_A \otimes \bar{f}_B = \sum_q \int d^2k_{aT} d^2k_{bT}\, \delta^{(2)}(\vec{q}_T - \vec{k}_{aT} - \vec{k}_{bT})\, w(\vec{k}_{aT}, \vec{k}_{bT}) f_A(x_a, \vec{k}_{aT}^2)\, \bar{f}_B(x_b, \vec{k}_{bT}^2)$$

Drell-Yan factorization theorems: [Collins, Soper, Sterman; Ji, Ma, Yuan; Collins, Metz]

$$\frac{d\sigma}{d^4q\, d\Omega} \propto H \otimes f_A \otimes \bar{f}_B \otimes S$$

- Soft factor due to soft gluon radiation
- Wilson lines slightly modified

Structure functions

[Arnold, Metz, M.S., arXiv:0809.2262]

- Spin observables can be measured at RHIC, J-PARC, JINR, IHEP, FermiLab, COMPASS (CERN), GSI (FAIR)
- Collection of spin observables ⇒ decomposition into structure functions

$$\frac{d\sigma}{d^4q d\Omega} \propto L_{\mu\nu} W^{\mu\nu}$$

$$W^{\mu\nu}(P_a, S_a; P_b, S_b; q) = \int \frac{d^4x}{(2\pi)^4} \, \mathrm{e}^{iq\cdot x} \langle a, b | J^\mu(0) J^\nu(x) | a, b \rangle$$

- Constraints by current conservation, hermiticity and parity

$$q_\mu W^{\mu\nu} = W^{\mu\nu} q_\nu = 0$$

$$W^{\mu\nu} = [W^{\nu\mu}]^*$$

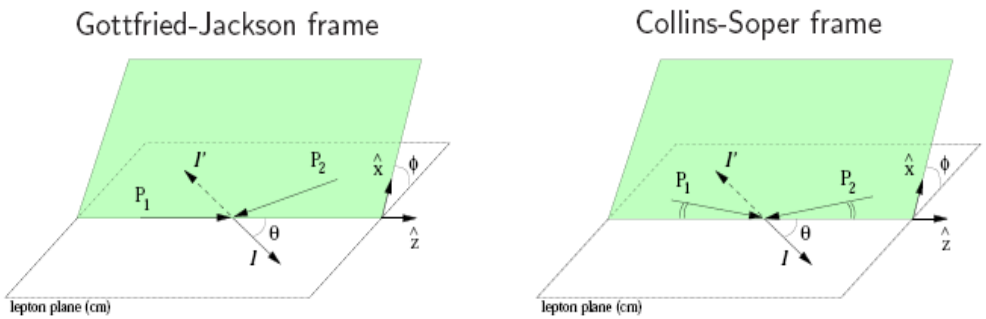
$$W^{\mu\nu}(P_a, S_a; P_b, S_b; q) = W_{\mu\nu}(\bar{P}_a, -\bar{S}_a; \bar{P}_b, -\bar{S}_b; \bar{q})$$

- Leads to 4 + 8 +8 + 28 = 48 structure functions $W(x_a, x_b, q^2)$
- 24 leading twist structure functions in the *parton model*, but only 8 TMDs
- Each TMD appear at least twice: Drell-Yan self-consistent. Cross-checks possible

Prominent Observables in Drell-Yan

- Dilepton rest frame:

simple azimuthal dependencies



$$\frac{d\sigma}{d^4q d\Omega} \propto (1+\cos^2\Theta)F_{UU}^1 + (1-\cos^2\Theta)F_{UU}^2 + \sin 2\Theta \cos\phi F_{UU}^{\cos\phi} + \sin^2\Theta \cos 2\phi F_{UU}^{\cos 2\phi} + \ldots$$

Double transverse polarization: $F_{TT}^{\cos 2\phi - \phi_a - \phi_b} \propto \cos(2\phi - \phi_a - \phi_b) h_1 \otimes \bar{h}_1$

Single transverse polarization: $F_{TU}^{\sin(2\phi - \phi_a)} \propto \sin(2\phi - \phi_a) h_1 \otimes \bar{h}_1^\perp$

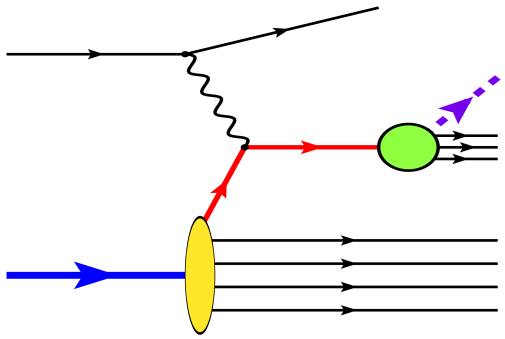
Sivers asymmetry: $F_{TU}^1 \propto f_{1T}^\perp \otimes \bar{f}_1$

Unpolarized azimuthal asymmetry: $F_{UU}^{\cos 2\phi} \propto \cos(2\phi) h_1^\perp \otimes \bar{h}_1^\perp$

Semi-inclusive DIS

Experimentally easier: SIDIS (measured at HERMES, COMPASS, JLAB)

- TMDs accessible: $|\vec{P}_{h\perp}| \ll Q$



Structure functions:
[Bacchetta, Diehl, Goeke, Metz, Mulders,M.S., 2006]

$$\frac{d\sigma}{dx\,dy\,dz\,d^2\vec{P}_{h\perp}\,d\phi_s} \propto F_{UU} + (2-y)\cos\phi F_{UU}^{\cos\phi} + (1-y)\cos(2\phi) F_{UU}^{\cos(2\phi)} + \ldots$$

- Parton Model: 8 leading twist spin observables, 8 TMDs
  → complete set of observables

- Factorization theorems for structure functions:

$$F(x, z, \vec{P}_{h\perp}^2) \propto H \otimes f \otimes D \otimes S$$

- Relatively low Q → higher twist effects more important
  → e.g. longitudinal single-spin asymmetries @ HERMES and JLAB

Prominent Observables in SIDIS

1.) Sivers asymmetry:

$$\sigma_{UT}^{\rm SIDIS} \propto \sin(\phi_h - \phi_S) f_{1T}^{\perp} \otimes D_1$$

2.) Collins asymmetry:

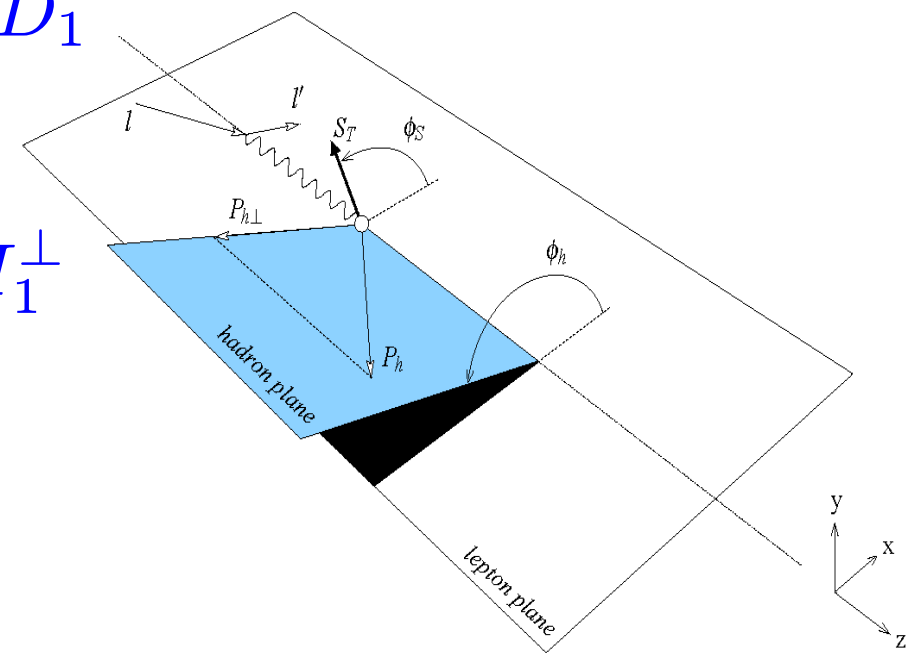
$$\sigma_{UT}^{\rm SIDIS} \propto \sin(\phi_h + \phi_S) h_1 \otimes H_1^{\perp}$$

3.) Boer-Mulders:

$$\sigma_{UU}^{\rm SIDIS} \propto \cos(2\phi) h_1^{\perp} \otimes H_1^{\perp}$$

4.) "pretzelosity":

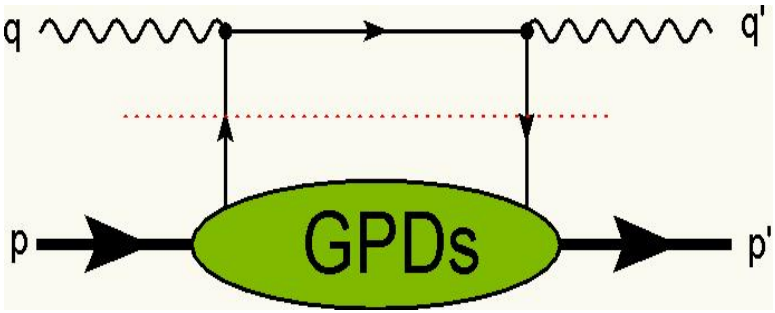
$$\sigma_{UT}^{\rm SIDIS} \propto \sin(3\phi_h - \phi_S) h_{1T}^{\perp} \otimes H_1^{\perp}$$



Collins-function important → BELLE data

Generalized Parton Distributions

- Exclusive processes (DVCS, meson production, ...):



$$P = \frac{1}{2}(p + p')$$

$$\Delta = p' - p$$

Skewness-parameter:

$$\Delta^+ = -2\xi P^+$$

$$t = -\frac{4\xi^2 M^2 + \vec{\Delta}_T^2}{1 - \xi^2}$$

- GPDs → "off-diagonal" matrix elements of quark-quark operator:

$$F_{ij}(x, \xi, \vec{\Delta}_T) = \int \frac{dz^-}{2(2\pi)} \, e^{ixP^+ z^-} \langle p' | \bar{\psi}_j(-\frac{z^-}{2}) \left[-\frac{z^-}{2} ; \frac{z^-}{2} \right] \psi_i(\frac{z^-}{2}) | p \rangle$$

- Project out quark polarizations: 8 leading twist GPDs

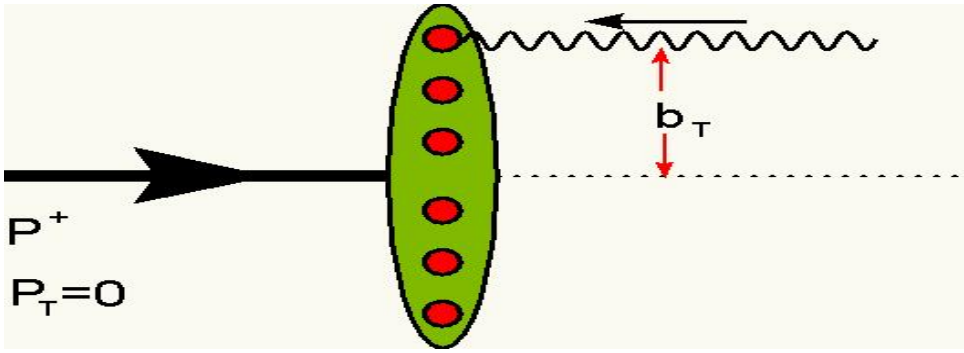
unpolarized: $(H \, , \, E)(x, \xi, t)$

longitudinally: $(\tilde{H} \, , \, \tilde{E})(x, \xi, t)$

transverse [chirally-odd]: $(H_T \, , \, E_T \, , \, \tilde{H}_T \, , \, \tilde{E}_T)(x, \xi, t)$

Impact Parameter Space

- Impact Parameter Space: (ξ =0) [M. Burkardt, PRD62, 071503]



- Impact parameter b_T and transv. momentum transfer Δ_T → FT

$$\mathcal{F}_{ij}(x,\vec{b}_T) = \int \frac{d^2\Delta_T}{(2\pi)^2}\, \mathrm{e}^{-i\vec{\Delta}_T\cdot\vec{b}_T}\, F_{ij}(x,0,\vec{\Delta}_T)$$

- Impact parameter space → "diagonal" matrix element $z_{1/2} = \mp\frac{z^-}{2}n_- + b_T$

$$\mathcal{F}_{ij}(x,\vec{b}_T) = \int \frac{dz^-}{2(2\pi)}\, \mathrm{e}^{ixP^+z^-}\, \langle P^+;\vec{0}_T|\bar{\psi}_j(z_1)\,[z_1;z_2]\,\psi_i(z_2)|P^+;\vec{0}_T\rangle$$

- (ξ =0): probability density of partons in transverse plane.

Relations between GPDs and TMDs

Non-trivial relations for "T-odd" parton distributions:

M. Burkardt [Nucl.Phys. A735, 185], [PRD66, 114005]

Average transverse momentum of unpolarized partons in a transversely polarized nucleon:

$$\langle k_T^i \rangle_T(x) = \int d^2k_T\, k_T^i\, \frac{1}{2}\Big[\mathrm{Tr}[\gamma^+\Phi(\vec{S}_T)] - \mathrm{Tr}[\gamma^+\Phi](-\vec{S}_T)\Big] \propto f_{1T}^{\perp,(1)}(x)$$

Manipulation of Gauge Links + Impact parameter representation

$$\langle k_T^i \rangle(x) = \int d^2b_T \int \frac{dz^-}{2(2\pi)}\, \mathrm{e}^{ixP^+z^-} \langle P^+;\vec{0}_T;S_T|\bar{\psi}(z_1)\gamma^+[z_1;z_2]I^i(z_2)\psi(z_2)|P^+;\vec{0}_T;S_T\rangle$$

$$z_{1/2} = \mp\frac{z^-}{2}n_- + b_T$$

Impact parameter representation for GPD E

$$I^i(z^-) = \int dy^-\,[z^-;y^-]gF^{+i}(y^-)[y^-;z^-]$$

coll. "soft gluon pole" matrix element

Non-trivial Relations

- *Conjecture:* factorization of final state interactions and spatial distortion:

$$\langle k_T^i \rangle = -M \epsilon_T^{ij} S_T^j \, f_{1T}^{\perp,(1)}(x) \simeq \int d^2 b_T \, \mathcal{I}^i(x, \vec{b}_T) \, \frac{\vec{b}_T \times \vec{S}_T}{M} \, \frac{\partial}{\partial b_T^2} \mathcal{E}(x, \vec{b}_T^2)$$

$\mathcal{I}^i(x, \vec{b}_T^2)$: Lensing Function = net transverse momentum

- Av. transv. momentum of transv. pol. partons in an unpol. hadron:

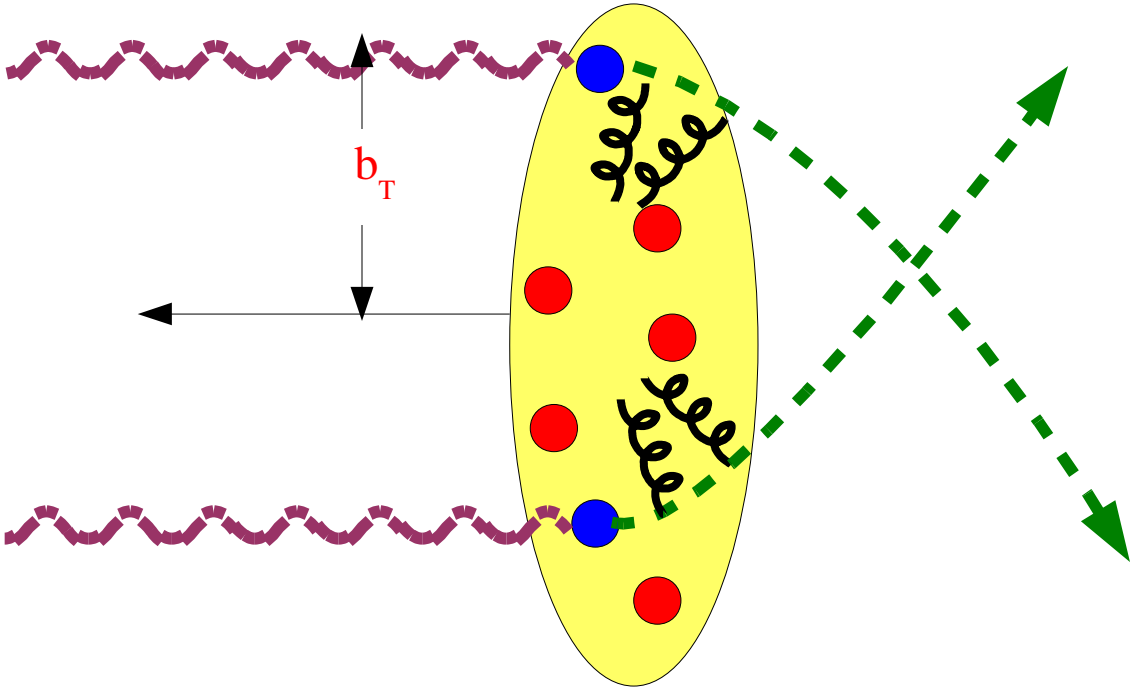
$$\Rightarrow \quad -2M^2 \, h_1^{\perp,(1)}(x) \simeq \int d^2 b_T \, \vec{b_T} \cdot \vec{\mathcal{I}}(x, \vec{b}_T) \, \frac{\partial}{\partial b_T^2} \left(\mathcal{E}_T + 2\tilde{\mathcal{H}}_T \right)(x, \vec{b}_T^2)$$

- Quantify distortion effect → flavor anormalous magnetic moment κ^q

  → Prediction of signs for u- and d-quark T-odd TMDs.

Physical picture of the Relation

Intuitive picture of the Final State Interactions:



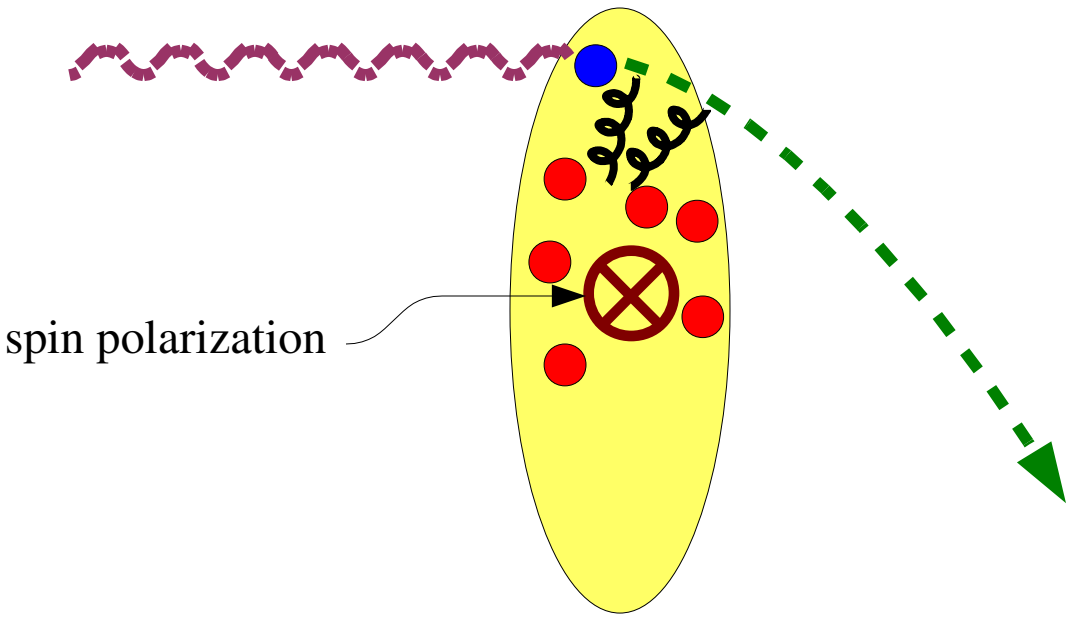
Final State interactions are assumed to be attractive

→ Lensing!

Physical picture of the Relation

Intuitive picture of the Sivers asymmetry:

Spatial distortion in the transverse plane due to polarization!



Mechanism leads to non-zero Sivers asymmetry!

Predictions

Intuitive picture seems to work "numerically":

Distortion effect given by flavor dipole moment:

$$d^{q,i} = \int dx \int d^2 b_T \, b_T^i \, \frac{\vec{b}_T \times \vec{S}_T}{M} \mathcal{E}'(x, \vec{b}_T^2) = -\frac{\epsilon_T^{ij} S_T^j}{2M} \int dx E^q(x,0,0) = -\frac{\epsilon_T^{ij} S_T^j}{2M} \kappa^q$$

with flavor an. magn. moment $\kappa^{u/p} \simeq 1.7$ $\kappa^{d/p} \simeq -2.0$

$$f_{1T}^{\perp,(1)}(x) \propto \int d^2 b_T \, \mathcal{I}(x, \vec{b}_T) \frac{\vec{b}_T \times \vec{S}_T}{M} \mathcal{E}'(x, \vec{b}_T^2)$$

Predicts opposite signs of u- and d- Sivers functions.

- in agreement with large-N_c prediction [Pobylitsa, 2003]
  model calculations in spectator models, MIT-bag model, etc.

Predictions

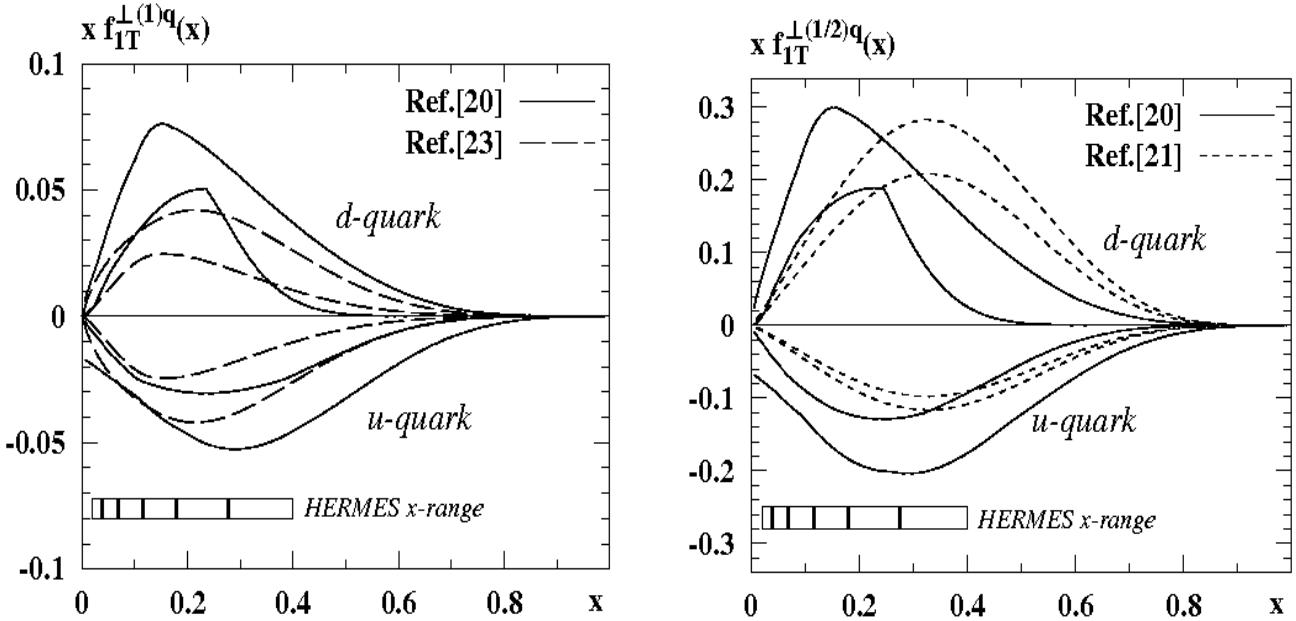
Intuitive picture also predicts the absolute sign

if:

Final state interactions are attractive, $\mathcal{I}(x, \vec{b}_T) < 0$

$$f_{1T}^{\perp,u} < 0 \quad f_{1T}^{\perp,d} > 0$$

Confirmed by HERMES, COMPASS data:



Fits taken from:

[20] Anselmino et al., PRD72 (05)

[21] Vogelsang, Yuan, PRD72 (05)

[23] Collins et al., hep-ph/0510342

Chiral-odd Relation

- Av. transv. momentum of transv. pol. partons in an unpol. hadron:

$$\langle k_T^i \rangle^j(x) \;=\; \int d^2k_T\, k_T^i\, \frac{1}{2}\Big(\Phi^{[i\sigma^{i+}\gamma^5]}(S) + \Phi^{[i\sigma^{i+}\gamma^5]}(-S)\Big)$$

$$\Rightarrow \quad -2M^2\, h_1^{\perp,(1)}(x) \simeq \int d^2b_T\, \vec{b}_T \cdot \vec{\mathcal{I}}(x,\vec{b}_T)\, \frac{\partial}{\partial b_T^2}\Big(\mathcal{E}_T + 2\tilde{\mathcal{H}}_T\Big)(x,\vec{b}_T^2)$$

- Spatial distortion in transv. plane of transv. pol. quarks quantified by

$$\kappa_T \;=\; \int dx\, \Big(E_T + 2\tilde{H}_T\Big)(x,0,0)$$

- Lattice QCD, const. quark model: $\kappa_T^u > 0$ and $\kappa_T^d > 0$

$\Rightarrow$ Boer-Mulders function negative for u- and d-quarks!

[in agreement with large-N_c, models.]

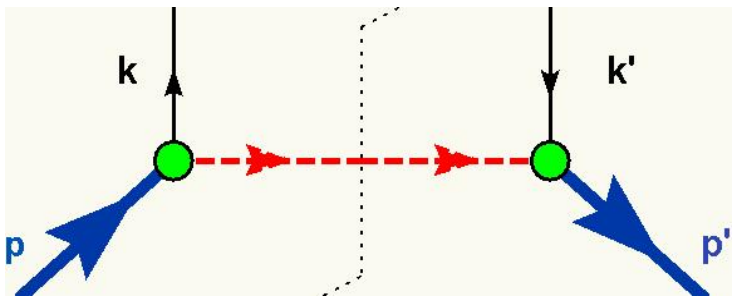
Relations in Spectator Models

Explicit checks of relations in a diquark spectator model:

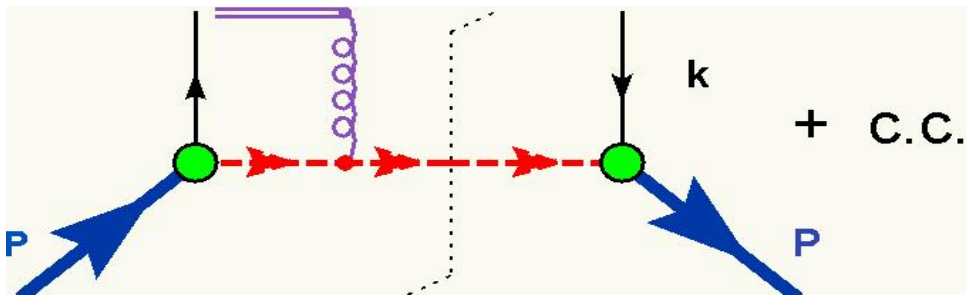
[Burkardt, Hwang, PRD69, 074032], [Meissner, Metz, Goeke, PRD76, 034002]

Lowest order calculations:

GPDs:



(T-odd) TMDs:



Non-trivial relations are *exactly* fulfilled!

$$-M\epsilon^{ij}S_T^j f_{1T}^{\perp,(1)} = \int d^2b_T\, \mathcal{I}^i \frac{\vec{b}_T\times\vec{S}_T}{M}\mathcal{E}'$$

$$-2M\, h_1^{\perp,(1)} = \int d^2b_T\, \frac{\vec{b}_T\cdot\vec{\mathcal{I}}}{M}\left(\mathcal{E}_T + 2\tilde{\mathcal{H}}_T\right)'$$

Relations in Spectator Models

In the diquark-spectator model:

- Relations between *arbitrary* moments:

$$f_{1T}^{\perp,(n)}(x) \propto E^{(n)}(x)\,,\ 0 \leq \mathrm{n} \leq 1$$

- TMD: $f^{(n)}(x) \sim \int d^2k_T\,(\vec{k}_T^2)^n f(x,\vec{k}_T^2)$
- GPD: $E^{(n)}(x) \sim \int d^2\Delta_T\,(\vec{\Delta}_T^2)^{n-1} E(x,0,-\frac{\vec{\Delta}_T^2}{(1-x)^2})$

- Relation between GPDs and *T-even* TMDs:

$$h_{1T}^{\perp,(n)}(x) \sim \tilde{H}_T^{(n)}(x)$$

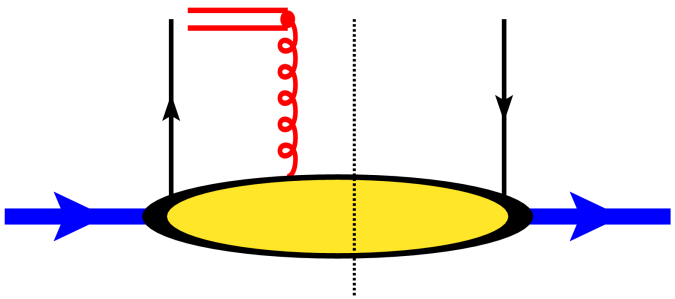
→ No FSI / Lensing function needed!

valid also in a light-cone constituent quark model
(Pasquini et al.,arXiv:0806.2298)

- Relations for gluon-GPDs and gluon-TMDs.
- Relations are likely to be broken for higher order diagrams.

Beyond the One-Loop Approximation

So far: Most phenomenological approaches to T-odd TMDs
→ Final state interactions modeled by a one-gluon exchange



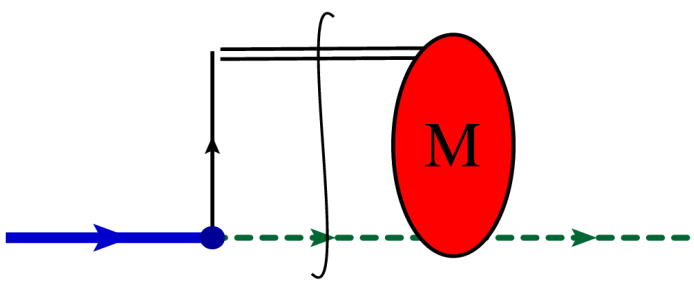
e.g. Diquark-model, MIT-Bag model etc.

Sivers-effect ~5%, $f_{1T}^{\perp,(1)u,d} \simeq \mp 0.05$

$\alpha_s \simeq 0.2 - 0.3$ "strength of FSI"

Can we do better? Can we learn about the quality of the relations?

[L. Gamberg, M.S., in preparation]



- Still work within spectator framework, but *non-perturbative model of FSI*.
- In order to separate out GPDs, "cut" the diagram → "natural" picture of FSI.

$$f_{1T}^{\perp,(1)u}(x) = -\frac{1}{2(1-x)M^2} \int \frac{d^2 p_T}{(2\pi)^2}\, p_T^y I^y(x, |\vec{p}_T|) E^u\left(x, 0, -\frac{\vec{p}_T^{\,2}}{(1-x)^2}\right)$$

Relativistic Eikonal models

Lensing function given in terms of quark-diquark scattering amplitude M:

$$I^y(x,\vec{p}_T) = p_T^y \Im[M(x,|\vec{p}|_T)] + \int \frac{d^2 q_T}{(2\pi)^2}(2q_T^y - p_T^y)\Im[M(x,|\vec{q}_T|)]\Re[M(x,|\vec{q}_T - \vec{p}_T|)]$$

- Calculate the amplitude M in a relativistice eikonal model:
  [1970's: Fried, Quiros, Levy, Sucher, Zuber, etc....]

Functional methods in "quenched" QED

Generating functional: $Z[J,\bar{\eta},\eta,j^\dagger,j] = \frac{1}{N}\int \mathcal{D}[A,\bar{q},q,\phi^\dagger,\phi] e^{iS+\int JA+\bar{\eta}q+\bar{q}\eta+j^\dagger\phi+\phi^\dagger j}$

Exact 4-point function for quark-diquark scattering:

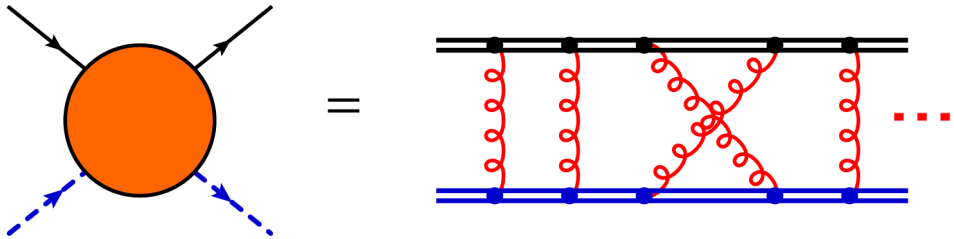
$$T = -e^{-iL_{12}}\left[\left(e^{-\frac{i}{2}L_{11}}\mathcal{G}^{-1}(x_2,x_1|\bar{A}_1)\underbrace{e^{\frac{1}{2}\mathrm{Tr}\ln\mathcal{G}(\bar{A}_1)}}_{\text{neglect}}\right)\times\left(e^{-\frac{i}{2}L_{22}}\mathcal{K}^{-1}(y_2,y_1|\bar{A}_2)\underbrace{e^{-\frac{1}{2}\mathrm{Tr}\ln\mathcal{K}(\bar{A}_2)}}_{\text{neglect}}\right)\right]\Bigg|_{\bar{A}_1=\bar{A}_2=0}$$

Linkage operator: $L_{ij} = -\int d^4z_1 d^4z_2 \frac{\delta}{\delta\bar{A}_i(z_1)}\mathcal{D}^{-1}(Z_1 - z_2)\frac{\delta}{\delta\bar{A}_j(z_2)}$

Eikonal approximation: $L_{ii}\mathcal{G}^{-1} = 0$

Relativistic Eikonal models (II)

- Generalized Ladder approximation:



- Eikonal Propagator:

Idea: highly energetic particle looses spin information $\gamma^\mu \leftrightarrow v^\mu$

$$\mathcal{G}^{-1}(x,y|\bar{A}) = \int_0^\infty ds\, \mathrm{e}^{-is(m_q - i0)} \delta^{(4)}(x-y-sv) \mathrm{e}^{ie_q \int_0^s d\alpha\, v\cdot A(y+\alpha v)}$$

Quark: Eikonalization comes "for free" $v^\mu = n_-^\mu, \;\; n_-^2 = 0$

Diquark: similar Eikonalization, here an assumption $v^\mu = n_+^\mu, \;\; n_+^2 = 0, \;\; n_- \cdot n_+ = 1$

- Eikonal Amplitude and Coulomb Phase:

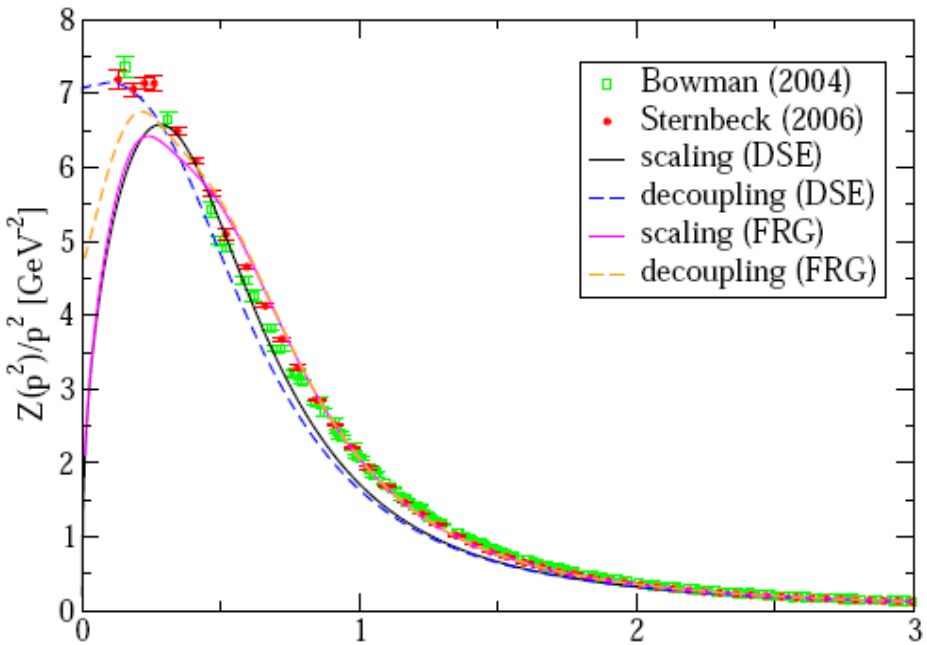
$$M^{\mathrm{eik}}(|\vec{q}_T|) \propto \int_0^\infty dz\, z J_0(z|\vec{q}_T|)(\mathrm{e}^{i\chi(z)} - 1)$$

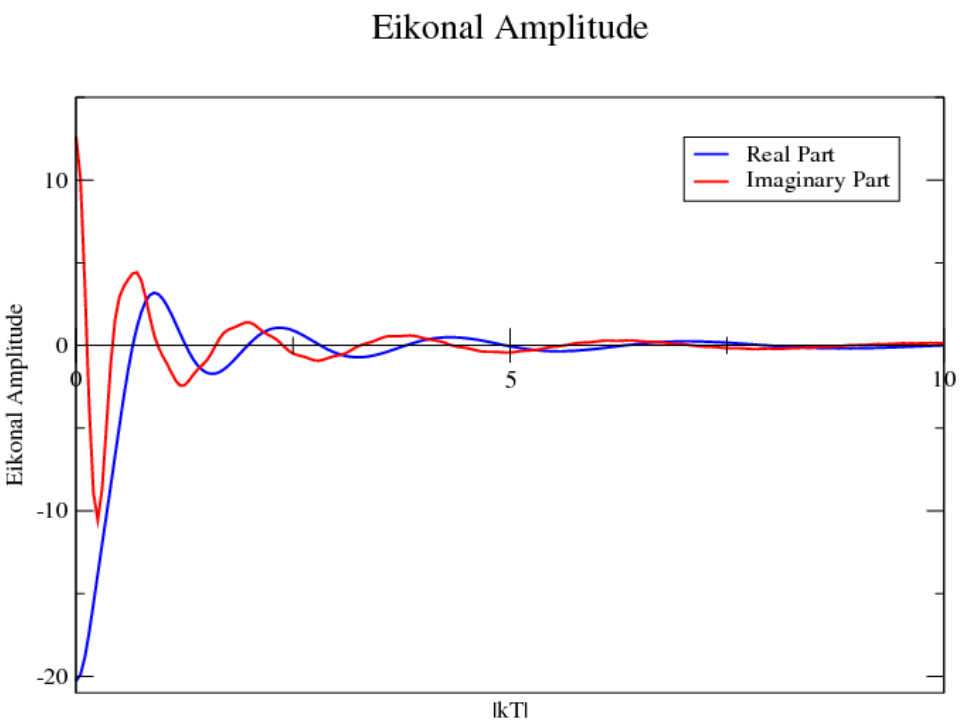
$$\chi(z) = -2C_F \alpha_s \int_0^\infty dr\, r J_0(rz) n_-^\mu n_+^\nu \mathcal{D}_{\mu\nu}^{-1}(-r^2)$$

Eikonal Phase and Amplitude

Scattering of two highly energetic particles with low energy transfer
→ soft gluon exchanges
→ replace the perturbative gluon propagator by a non-perturbative

- Non-perturbative gluon propagator → Lattice QCD + Dyson-Schwinger-Eqs.
- Also from DSE: $\alpha_s(\mu \to 0) \simeq 3$



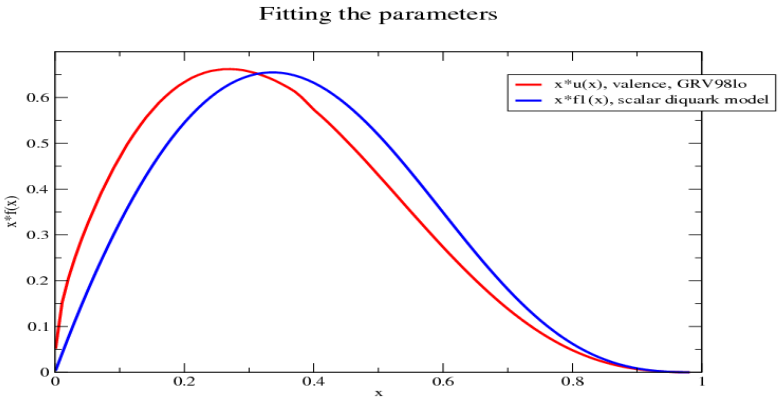


[Plot from Fischer, Maas, Pawlowski, arXiv:0810.1987]

GPD E(x,0,0)

Fixing of the model parameters:

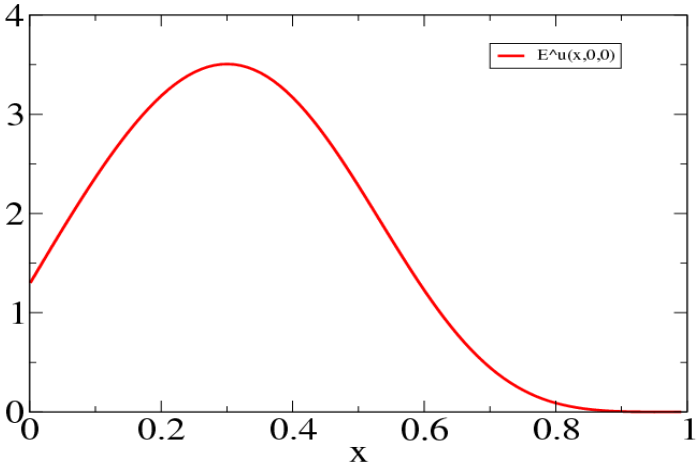
- Comparison of the unpolarized PDF f_1 in the model to GRV



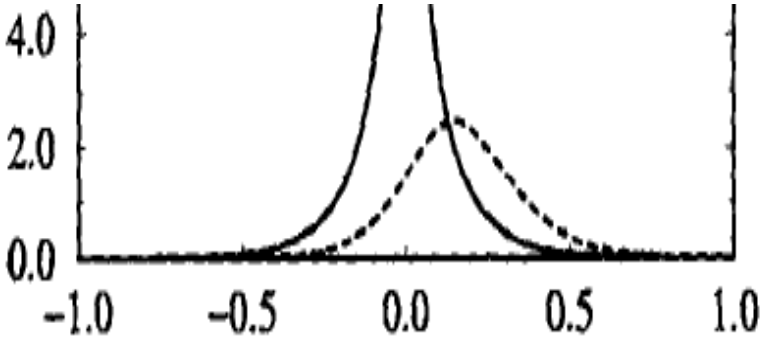
Constraint from the contribution of u-quarks to the an. magn. moment

$$\int_0^1 dx\, E(x,0,0) = \kappa^u \simeq 1.7$$

E(x,0,0) in the scalar diquark model:

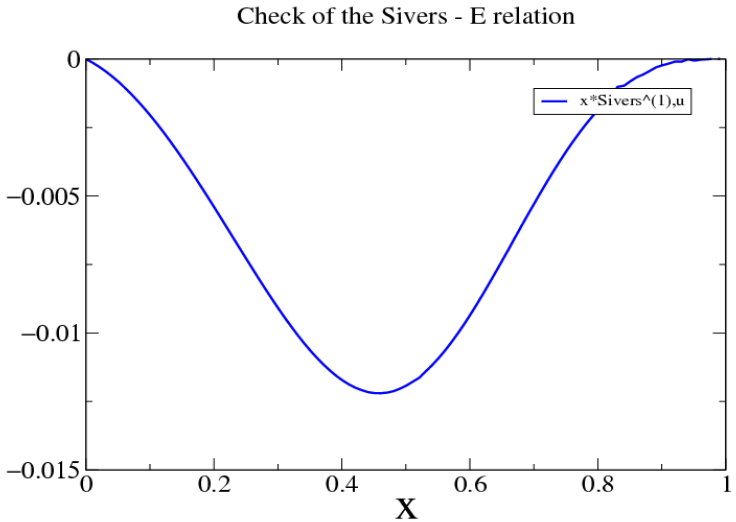


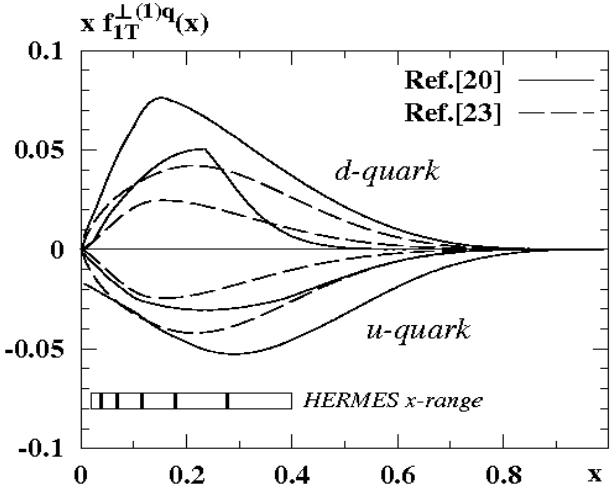
E(x,0,0) in the chiral quark soliton model:



(Preliminary) Results

Lensing function and GPD E → input for the relation:





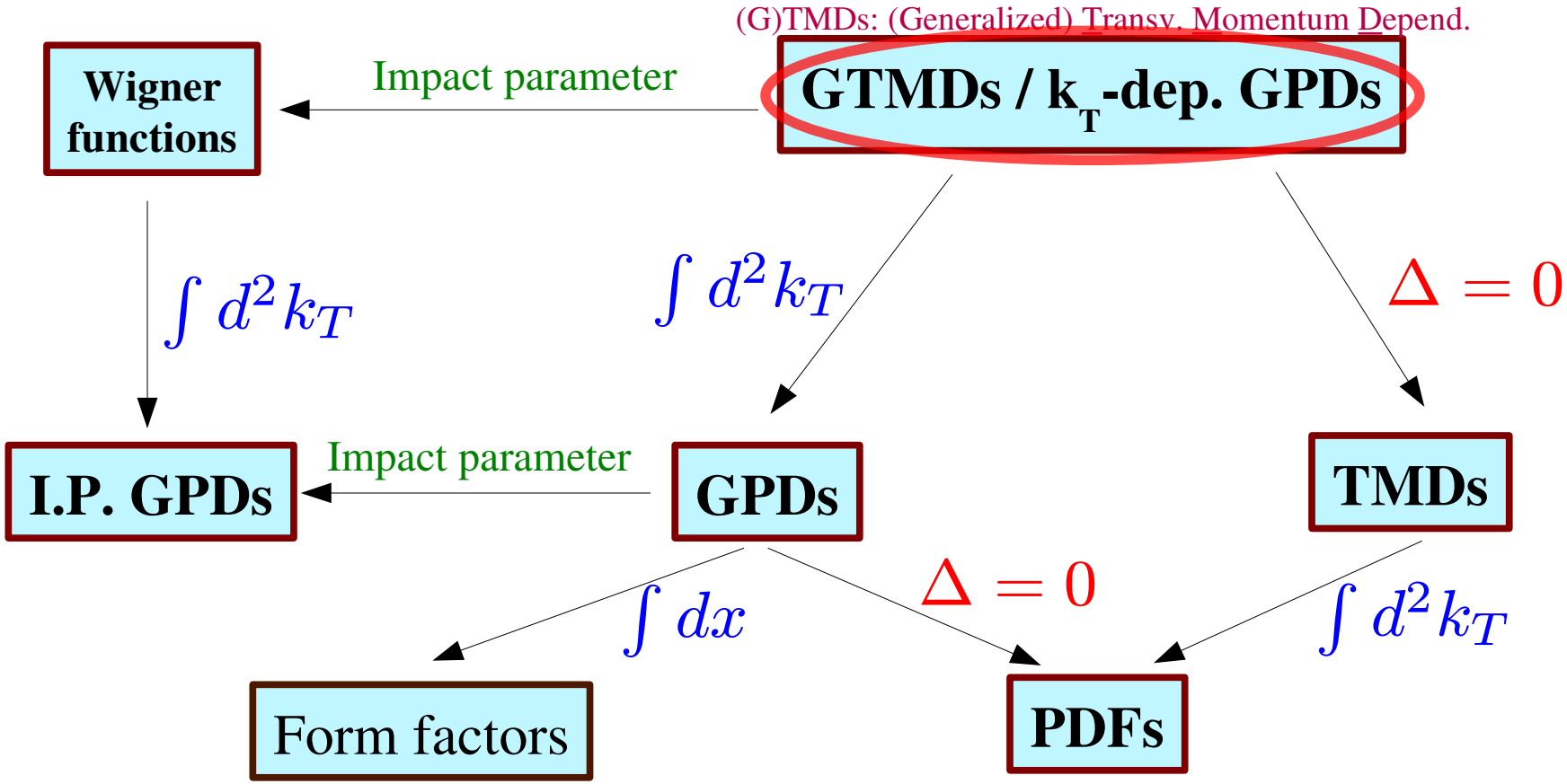
- Only ~30% of the effect covered by the relation (in this model!)
- Lensing Function behaves differently compared to the one-gluon exch.

Outlook: Possible "improvements" of the model:

- Implementation of non-perturbative scalar-glue/ fermion-glue vertices.
- Inclusion of axial-vector diquarks → prediction for d-quarks
- Try other nucleon-quark-diquark vertices.

Mother functions

Relations between functions:



Which GPDs and TMDs have the same mother functions?

Classification of Generalized TMDs

Define "off-diagonal" k_T-dependent matrix element:

[Meissner, Metz, M.S., Goeke, JHEP08(2008)038]

$$W_{ij}(x,\xi;\vec{k}_T;\vec{\Delta}_T) = \int \frac{dz^- d^2 z_T}{(2\pi)^3}\, e^{ik\cdot z} \langle p',\lambda'|\,\bar{\psi}_j(-\tfrac{z}{2})\,\mathcal{W}[-\tfrac{z}{2},\tfrac{z}{2}]\,\psi_i(\tfrac{z}{2})\,|p,\lambda\rangle\Big|_{z^+=0}$$

For spin-0: Classification 4 leading twist GTMDs

chiral-odd: $$\mathrm{Tr}[W\, i\sigma^{i+}\gamma_5] = -\frac{2i}{M}\left(\epsilon_T^{ij} k_T^j\, H_1^k + \epsilon_T^{ij}\Delta_T^j\, H_1^\Delta\right)$$

- GTMDs are complex-valued functions)

$$X(\eta) = \Re[X] + i\eta\Im[X]$$

Limits: TMD $$h_1^\perp(x,\vec{k}_T^2) = \eta\Im[H_1^k](x,0,0,0,\vec{k}_T^2)$$

GPD $$H_1^\pi(x,\xi,t) = \int d^2k_T \left[\frac{\vec{\Delta}_T\cdot\vec{k}_T}{\vec{\Delta}_T^2}\Re[H_1^k] + \Re[H_1^\Delta]\right]$$

No exact, model-independent relation!

Approx. relation?

Limiting cases: Spin 1/2

Example: Sivers function:

GTMDs: [Meissner, Metz, M.S., in preparation]

$$\mathrm{Tr}\left[\tilde{W}_{\lambda',\lambda}\gamma^+\right] = \frac{1}{P^+}\bar{u}(p',\lambda')\left[\frac{P^+}{M}\mathcal{F}_1 + \frac{i\sigma^{i+}k_T^i}{M}\mathcal{F}_1^\perp + \frac{i\sigma^{i+}\Delta_T^i}{M}\mathcal{F}_1^\Delta + \frac{P^+}{M}\frac{i\sigma^{ij}k_T^i\Delta_T^j}{M^2}\mathcal{F}_1^{\perp\Delta}\right]u(p,\lambda)$$

TMD-limit: $\Delta = 0$

$$f_{1T}^\perp(x,\vec{k}_T^2) = \Im[\mathcal{F}_1^\perp](x,0;\vec{k}_T^2,0,0)$$

GPD-limit: $\int d^2k_T\,,\ \xi = 0$

$$(E+H)(x,0,\vec{\Delta}_T^2) = 2\int d^2k_T\left[\Re[\mathcal{F}_1^\Delta] + \frac{\vec{k}_T\cdot\vec{\Delta}_T}{\vec{\Delta}_T^2}\Re[\mathcal{F}_1^\perp]\right]$$

No model-independent *exact* relation! Approximate relations?

- Relation $h_{1T}^\perp \leftrightarrow \tilde{H}_T$ *not* supported by GTMD-analysis!

Summary

- TMDs play a major role in Drell-Yan and SIDIS.

- Non-trivial relations between GPDs and (T-odd) TMDs were suggested on the basis of a separation between:

  distortion of parton distribution in the *transverse plane*

  + *Final state interactions*

- Relations seems to work "numerically"

  ⇒ predicts signs of u- and d-quark *Sivers*- and *BM*-function.

- Relations are exact in lowest-order spectator models.

  Appr. 30% of the relation between Sivers and E are recovered in eikonal models.