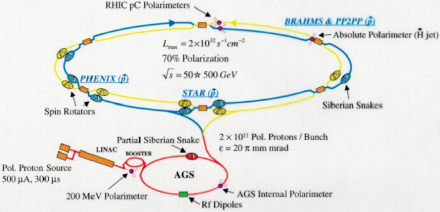
Polarized proton collisions in RHIC



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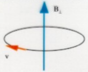
Spin Dynamics

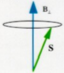
Particle trajectory governed by Lorentz force:



$$
\begin{aligned}
\frac{d(\gamma m\vec{v})}{dt} &= e\left\{\vec{E}+\vec{v}\times\vec{B}\right\} \\
\frac{d\vec{v}}{dt} &= -\frac{e}{m\gamma}\left\{\vec{B}\times\vec{v}\right\} \qquad \left(for\ \vec{E}=0\right) \\
&= -\left[\frac{e\vec{B}_\perp}{m\gamma}\right]\times\vec{v} \\
&= -\vec{\Omega}_{rev}\times\vec{v}
\end{aligned}
$$

(assuming γ changing slowly)

In a frame rotating with the particle velocity vector, the equation for spin precession is:



$$
\begin{aligned}
\frac{d\vec{S}}{dt} &= -\frac{e}{m\gamma}\left\{G\gamma\vec{B}-(\gamma-1)G\frac{(\vec{v}\cdot\vec{B})\vec{v}}{v^2}+\gamma\left(G-\frac{1}{\gamma^2-1}\right)\frac{\vec{E}\times\vec{v}}{c^2}\right\}\times\vec{S} \\
&= -\frac{e}{m\gamma}\left\{G\gamma\vec{B}_\perp+G\vec{B}_\parallel\right\}\times\vec{S} \qquad \left(for\ \vec{E}=0\right) \\
&= -\left[\frac{e\vec{B}_\perp}{m\gamma}G\gamma\right]\times\vec{S} \qquad \left(for\ \vec{E}=0,\ \vec{B}_\parallel=0\right) \\
&= -G\gamma\vec{\Omega}_{rev}\times\vec{S}
\end{aligned}
$$

Here, the fields are those in the lab frame, whereas the spin vector is in the particle's rest frame.

For a pure vertical guide field in a circular accelerator, the spin precesses $G\gamma$ times per revolution. Thus, the "spin tune" is $\nu_s = G\gamma$.

Depolarizing Spin Resonances

Field perturbations cause undesired precession, and when resonance conditions hold can generate depolarization...

- Imperfection Resonances
  - arise from sampling of error fields, fields due to closed orbit errors, etc.
  - $G\gamma = integer$
- Intrinsic Resonances
  - arise from sampling of focusing fields due to finite beam emittance
  - $G\gamma \pm \nu_y = integer$,

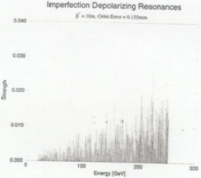
    ν_y = vertical betatron tune

    ν_x = horizontal betatron tune

    $G\gamma \pm \nu_y \pm \nu_x = integer$

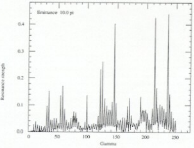
These resonance conditions can be avoided through the use of "Siberian Snakes."

$\varepsilon_{imp} \propto$ rms orbit distortion
Need orbit as flat as possible.



Intrinsic depolarizing resonance

$\varepsilon_{int} \propto \sqrt{\text{emittance}}$



<u>How snake works</u>

Transport of spin in magnetic/electric field is a Rotation $R(\vec{n}, \varphi)$ with 3 parameters $SO(3)$ or $SU(2)$

$SO(3)$ is a group $\rightarrow$ $R_n \cdots R_2 \cdot R_1 = R_{eff}$

↳ piece wise component in the ring

$\rightarrow$ Effective spin rotation for one turn $R(\vec{n}_0, \varphi_0)$

or $R(\vec{n}_0, \frac{\varphi_0}{2\pi})$

$\vec{n}_0$: stable spin direction; $\frac{\varphi_0}{2\pi}$: spin tune

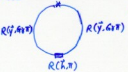
If No horizontal fields: $\vec{n}_0 = (0, 1, 0)$ [vertical], $\nu_{sp} = G\gamma$

Snake: $R(\vec{h}, \pi)$: effectively reverses rotation from vertical field

$$R(\vec{h}, \pi) \cdot R(\vec{y}, \varphi) = R(\vec{y}, -\varphi)\, R(\vec{h}, \pi) \qquad [\sigma_x \sigma_y = -\sigma_y \sigma_x]$$

$$= R^{-1}(\vec{y}, \varphi) \cdot R(\vec{h} \cdot \pi)$$

Example One Snake



one turn rotation:

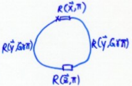
$$R(\vec{y}, G\gamma\pi) \cdot R(\vec{h}, \pi)\, R(\vec{y}, \pi G\gamma)$$

$$= R(\vec{y}, G\gamma\pi) \cdot R^{-1}(\vec{y}, G\gamma\pi)\, R(\vec{h}, \pi)$$

$$= R(\vec{h}, \pi) \longrightarrow \text{spin tune } 1/2$$

↑ stable spin direction,

Two snakes



one turn rotation:

$$\underbrace{R(\vec{y}, G\gamma\pi)\, R(\vec{z}, \pi)\, R(\vec{y}, G\gamma\pi)}\, R(\vec{x}, \pi)$$

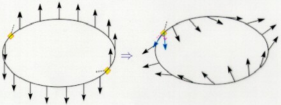
$$= R(\vec{z}, \pi) \cdot R(\vec{x}, \pi)$$

$$= R(\vec{y}, \pi) \longrightarrow \text{spin tune } 1/2$$

↑ stable spin direction

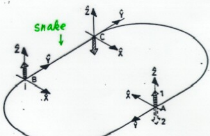
Longitudinal Polarization in RHIC

- Inject vertically polarized protons with both snakes on.
  $E \sim 24.3$ GeV ($G\gamma \sim 46.5$)
- Accelerate beams to 100.48 GeV ($G\gamma = 192$)
- Turn off one snake in each ring: polarization $\Rightarrow$ horizontal plane.
  (Long. Pol. at IR's.)



Siberian Snake

(Ya. S. Derbenev, A.M. Kondratenko; Russia)



(a)

$\pi\nu_s$
↓

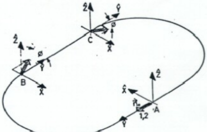
$A \to B \quad 2k\pi + \phi$

$B \to C \quad \pi - 2\phi$

$C \to A \quad 2k\pi + \phi$

$(4k+1)\pi$

(b)



(c)

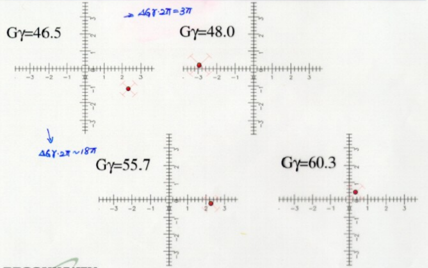
$A \to B \quad 2k\pi + \phi$

$B \to C \quad -2\phi$

$C \to A \quad 2k\pi + \phi$

$4k\pi$

Acceleration with single snake



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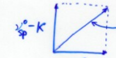
Resonance Crossing

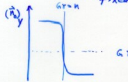
Oscillating horizontal field

amplitude $= \varepsilon$, freq. $= K f_{rev}$



$$\nu_{sp}^2 = (\nu_{sp}^0 - K)^2 + \varepsilon^2$$
$$= (G\gamma - K)^2 + \varepsilon^2$$

ε fixed in rotating frame



For a partial snake

$$R(\vec{n}_0, \varphi_0) = R(\vec{y}, 2\pi \cdot G\gamma) R(\vec{z}, \delta)$$

$$\cos \varphi_0/2 = \cos \pi G\gamma \cos \frac{\delta}{2}$$

$$\varepsilon = \frac{\delta}{2\pi}$$

If $\delta = 0$, $G\gamma = n$.

$R(\vec{n}_0, \varphi_0) = 1$

Any pol. direction is stable!

AGS 5% partial snake. $9°$ $\delta = \frac{\pi}{20}$, $\varepsilon = 0.025$

$K =$ integer : · imperfection resonances
· partial snake

$K \neq$ integer: · intrinsic resonances
· external RF field : spin flipper

Froissart-Stora Formula

Let α characterize the rate at which the resonance is crossed:

$$\boxed{\alpha \equiv \frac{d(G\gamma)}{d\theta} = \frac{G}{2\pi f_{rev}} \frac{d\gamma}{dt}}$$

Consider a particle whose initial spin component along the stable spin direction is P_i. After crossing the resonance, its spin component P_f along the resulting stable spin direction is given by the Froissart-Stora formula:

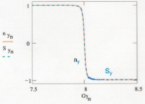
$$\boxed{\frac{P_f}{P_i} = 2e^{-\pi\frac{|\epsilon|^2}{2\alpha}} - 1}$$

Comment on intrinsic resonances and spin flippers:

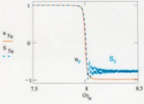
The above discussion describes the case of a spin imperfection resonance. Intrinsic resonances, or artificially induced resonances from "spin flippers," are essentially the same, except that the horizontal field error is not constant. After a transformation into a reference frame that rotates at the frequency of the horizontal field, these resonances can be understood in the same way as imperfection resonances.

Some examples of crossing a resonance of strength $\varepsilon = 0.015$ ($\varepsilon^2 = 2.25\text{x}10^{-4}$) at various speeds...

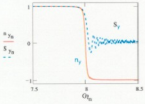
$\alpha = 0.53\text{x}10^{-4}$



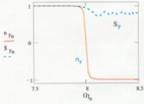
$\alpha = 1.6\text{x}10^{-4}$



$\alpha = 5.3\text{x}10^{-4}$



$\alpha = 16\text{x}10^{-4}$



Remarks:

- Passing through the resonance, the stable spin direction changes sign -- "spin flip."
- So long as the resonance is passed adiabatically, the particles will follow the stable spin direction and polarization of the beam will be preserved.
- If the resonance is passed very quickly, then the stable spin direction can change sign quickly enough that the spin simply begins precessing about the new direction; again polarization would be preserved.
- In intermediate cases, where either the crossing is neither quick nor adiabatic, the spin precession of the particles will not follow the change of the stable spin axis, and, because there is an inherent spread in precession frequency of the particles, beam depolarization will result. The resulting depolarization can be estimated using the Froissart-Stora formula.

Snake

B field starts and ends with $B_y \neq 0$

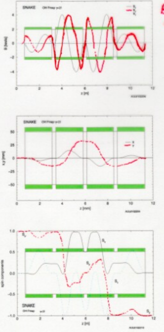


Figure 4.1: Field, orbit, and spin tracking through the four helical magnets of a Siberian Snake at $\gamma = 25$. The spin tracking shows the reversal of the vertical polarization.

spin rotator

B field starts and ends with $B_x \neq 0$

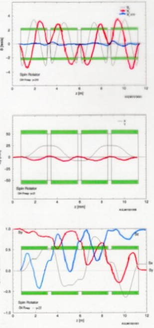
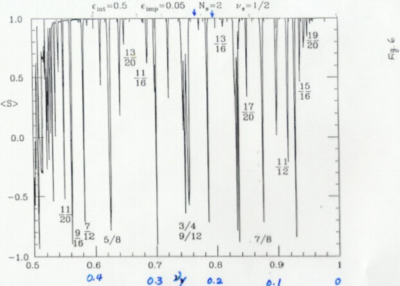


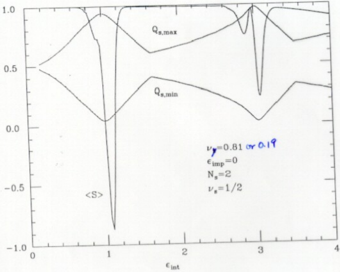
Figure 4.3: Field, orbit, and spin tracking through the four helical magnets of a Spin Rotator at $\gamma = 25$. In this example, the spin tracking shows how the polarization is brought from vertical to horizontal.

Snake resonance



When ϵ_{int} very strong, we are dealing with overlapping resonances (int. and imp.)
this leads to additional resonance conditions: $\nu_s \pm k\nu_y$ = integer. snake resonance.
snake resonance order.

$\Rightarrow$ need to control betatron tune very well along ramp. $\Delta\nu_y < 0.02$.



1.0
0.5
0.0
−0.5
−1.0
0
1
2
3
4
ϵ_{int}
$Q_{s,max}$
$Q_{s,min}$
<S>
ν_y=0.81 or 0.19
ϵ_{imp}=0
N_s=2
ν_s=1/2

Fig. 2.

Snake Configuration Requirements

N_s snakes with snake axes angles $(\phi_1, \phi_2, \cdots \phi_{N_s})$ distributed in an accelerator, $\theta_{i,i+1}$ be the orbit angle between ith, (i+1)th snakes.

1. To ensure ν_s independent of beam energy.

$$\theta_{odd} = \theta_{even} = \pi, \quad \theta_{odd} = \sum_{k=odd}^{N_s} \theta_{k,k+1}, \quad \theta_{even} = \sum_{k=even}^{N_s} \theta_{k,k+1}$$

If the odd orbital angle deviates from π. RHIC: Two snakes π apart

$$\nu_s = \frac{1}{2} + G\gamma\left(1 - \frac{\theta_{odd}}{\pi}\right)$$

or $$\Delta\nu_s = G\gamma\left(1 - \frac{\theta_{odd}}{\pi}\right)$$

2. $$\nu_s = \frac{1}{\pi}\sum_{k=1}^{N_s}(-1)^k \phi_k = j + \tfrac{1}{2}, \quad j = \text{integer}.$$ RHIC $|\phi_1 - \phi_2| = \frac{\pi}{2}$

Snake can be built with solenoid (Low energy), dipole, or helical dipole (reduce orbit excursion)

RHIC: choose $\phi_1 = 45°$, $\phi_2 = -45°$

$\Rightarrow$ Two magnetic devices can be mechanically the same, operating at the same strengths, but opposite polarities.

Each snake: two knobs. B_1 B_2 to determine ϕ (axis) and μ (rotation angle).

For two identical snakes:

$$\cos\frac{2\pi\nu_s}{N_s} = -\cos(\phi_2 - \phi_1) + \sin^2\left(\frac{\Delta\phi}{2}\right)\left[\cos(\phi_2 - \phi_1) + \cos\frac{2G\gamma\pi}{N_s}\right]$$

$\Delta\phi$ is the rotation angle error.

If $\phi_2 - \phi_1 = \frac{\pi}{2}$, $G\gamma = \text{int}$. ν_s shift is small.

$\Delta\phi \neq 0$, or $\phi_2 - \phi_1 \neq \frac{\pi}{2}$, ν_s shifts, $\hat{n}$ away from vertical.

For snake to work perfectly, need to control $\Delta\phi$ small, and $\phi_2 - \phi_1 \sim \frac{\pi}{2}$. Spin flipper can help.

Spin flipper (AC dipole)

For RHIC, $\nu_s = \frac{1}{\pi} |\phi_1 - \phi_2|$, ϕ_1, ϕ_2 : axis angles of two snakes.

$(\phi_1, \phi_2) = (\frac{\pi}{4}, -\frac{\pi}{4})$, $\Rightarrow \nu_s = 1/2$.

An AC dipole with horizontal oriented oscillating dipole field induces a spin resonance at

$$\nu_m = \frac{\text{AC dipole freq.}}{f_{rev.}}$$

with strength $\epsilon = \frac{1+G\gamma}{4\pi} \frac{B_m L}{B\rho}$

Spin flipping:

tune the snake axes so that the spin tune is away from 1/2.

adiabatically turn on AC dipole with freq away from ν_s.

ramp AC dipole adiabatically through spin tune.

$\Rightarrow$ an adiabatic spin flip is induced.

$$\frac{P_f}{P_i} = 2e^{-\frac{\pi|\epsilon|^2}{2\alpha}} - 1$$

$$\alpha = \frac{d}{d\theta}[G\gamma - (kP \pm \nu)]$$

Summary on Polarized Proton Acceleration

1. Why there are spin resonances?
2. How does snake work?
3. What's the difference between snake and spin rotator?
4. How does spin flipper work?
5. What are important parameters for polarization preservation?