Sivers function and its parameterizations

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Outline

- Parton distributions
- $k_\perp$ dependent parton distributions
- Sivers function
- Interpretation of the Sivers function
- Semi-inclusive deep-inelastic scattering (SIDIS)
- Extraction from HERMES data
- Gauge-Link
- Sivers asymmetry in Drell-Yan (DY)
- Predictions for various experiments
- Summary and Outlook

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Parton distributions

Parton model

Model to describe the distribution of the nucleon's constituents (partons). Mathematical description via quark-quark-correlator

$$\Phi_{ij}(x) = \int \frac{d\xi^-}{4\pi} e^{ixP^+\xi^-} \left\langle P \left| \bar{\Psi}_j(0) \mathcal{W}(0,\xi) \Psi_i(\xi^-) \right| P \right\rangle$$

collinear parton model

All partons move in the same direction as the nucleon
$\Rightarrow$ no transverse parton momentum

parton distributions

1. $f_1^q(x)$ or $q(x)$, e.g. given by $\mathrm{Tr}(\Phi\gamma^+)$
2. $g_1^q(x)$ helicity distribution
3. $h_1^q(x)$ transversity

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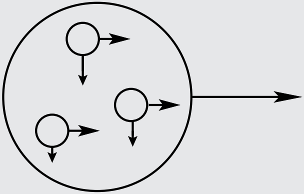
$k_\perp$ - dependent parton distributions

Parton model with $k_\perp$

- Partons can also move into transverse direction (compared to the target motion)
- $\Rightarrow$ more parton distributions, depending on x and also on $k_\perp$



- Nucleon momentum: $P^\mu = \left(P^+, P^-, \vec{0}\right)$
- Parton momentum: $k^\mu = \left(k^+, k^-, \vec{k}_\perp\right)$
- By integration over the transverse momentum one gets back to the three forward parton distributions.

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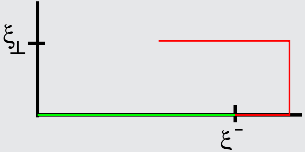
Quark-Quark correlator

Correlator

$$\Phi_{ij}\left(x,\vec{k}_\perp\right) = \int \frac{d\xi^- d^2\vec{\xi_\perp}}{(2\pi)^3} e^{i\left(xP^+\xi^- - \vec{k_\perp}\cdot\vec{\xi_\perp}\right)} \left\langle P \left| \bar{\Psi}_j\,(0)\,\mathcal{W}\left(0,\tilde{\xi}\right)\Psi_i\left(\tilde{\xi}\right)\right| P\right\rangle$$

- $\tilde{\xi} = \left(0, \xi^-, \vec{\xi_\perp}\right)$
- $\mathcal{W} = \mathcal{P}exp\left(ig\int d\alpha^\mu A_\mu(\alpha)\right)$



- A_μ : Gluon field
- Light-cone gauge $A^+ = 0$
- $\mathcal{W} = 1$ in light-cone gauge, but only if there is no transverse momentum

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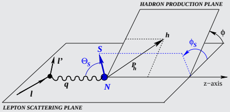
Sivers function

- introduced to explain the large Single-Spin asymmetries in e.g. $p^{\uparrow}p \longrightarrow \pi X$ @ FNAL (1990's)
- Measurement of non-zero Sivers Asymmetry also in SIDIS @ HERMES



- $\propto (S_T \times p_T)\, P_N$
- the Gauge-Link is necessary for the existence of the Sivers effect (e.g. calculation of the Sivers effect in a diquark model)
- the Sivers effect is quantified in terms of the Sivers function $f_{1T}^{\perp}$
- Probability to find an unpolarized quark in a transversely polarized target

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Sum Rules for Sivers function

Burkardt Sum rule

[M.Burkardt, Phys.Rev. **D**69 (2004) 091501]

$$\sum_{a=q,\bar{q},g} \int_0^1 dx \int d^2\vec{k}_\perp \vec{k}_\perp^2 f_{1T}^{\perp a}\left(x, \vec{k}_\perp^2\right) = 0$$

$\langle k_\perp \rangle$ vanishes when summed oder all partons.

Has been checked in model calculations.
[K. Goeke, S. Meissner, A. Metz, M. Schlegel, Phys.Lett. B637 (2006) 241]

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Connection to GPD's

Connection to $E_q(x, \xi, \Delta)$

[M.Burkardt, Phys.Lett. **B** 595 (2004) 245-249]

- Transform $E_q(x, \xi, \Delta)$ to impact parameter space $\mathcal{E}_q(x, b_T)$
- rewrite Sivers function (from scalar diquark spectator model)

- From model calculations
  $E_q \propto (1-x)^5 \quad \Rightarrow \quad f_{1T}^{\perp} \propto (1-x)^5$
- Newer calculations predict $f_{1T}^{\perp} \propto (1-x)^4$
  [S.J. Brodsky, F.Yuan, hep-ph/0610236]

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Connection to quark angular momentum

In the calculation of the Sivers function the same light-cone wave functions enter as in the calculation of the anomalous magnetic moment
[S.J. Brodsky, S. Gardner, hep-ph/0608219]
But the SSA matrix elements can only be related to those contributing to κ and are not identical

$$\kappa_p = 1.79$$
$$\kappa_n = -1.91$$
$$\kappa^{u/p} > 0$$
$$\kappa^{d/p} < 0$$

u- and d-quark Sivers function should have opposite sign!

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Measuring the Sivers function

- The Sivers function can be measured via a Single-Spin asymmetry (SSA).
- Assume a model for the Sivers function and fit parameters
- Semi-inclusive deep-inelastic scattering off a lepton on a transversely polarized target
- Drell-Yan with one transversely polarized nucleon
- Can also be measured with hadronic final state $p^{\uparrow}p \longrightarrow jet_1\, jet_2\, X\,,jet\, H\, X$

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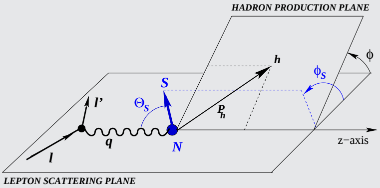
Semi-inclusive deep-inelastic scattering (SIDIS)

$$A_{UT} \propto \underbrace{\sin(\phi_H + \phi_S)\, h_1 H_1^{\perp}}_{Collins Effect} + \underbrace{\sin(\phi_H - \phi_S)\, f_{1T}^{\perp} D_1}_{Sivers Effect} + \ldots$$

$$\sigma = \sigma_{partonic} \times \text{pdf} \times \text{FF} \times \text{Soft factor}$$



$$q = l - l' \qquad Q^2 = -q^2$$

$$x = \frac{Q^2}{2P \cdot q} \qquad z = \frac{P \cdot P_H}{P \cdot q}$$

$$A_{UT}^{\sin(\phi_H - \phi_S)} \propto \frac{f_{1T}^{\perp}(x)\, D_1(z)}{f_1(x)\, D_1(z)}$$

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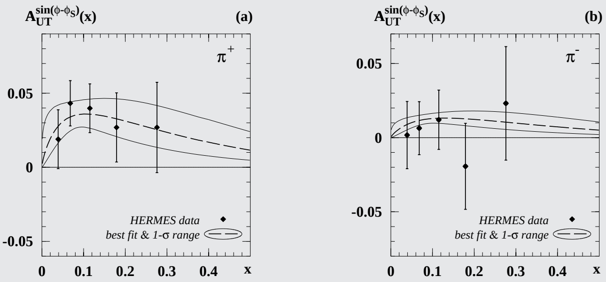
Hermes data



HERMES Collaboration, Phys.Rev.Lett. 94 (2005) 012002

- we fit the first moment of the Sivers function $f_{1T}^{\perp(1)}(x) = \int d\vec{k}_T \frac{\vec{k}_\perp^2}{2M_N^2} f_{1T}^\perp(x, \vec{k}_T^2)$
- k_T-dependence is assumed to be Gaussian

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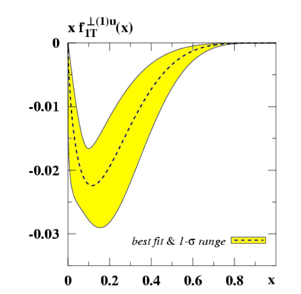
Fit to HERMES data

Reduce number of parameters

- $f_{1T}^{\perp,s,\bar{q},g} = 0$
- Large - $N_C \Rightarrow f_{1T}^{\perp,u} = -f_{1T}^{\perp,d}$ [Pobylitsa, hep-ph/0212027]

$\Rightarrow$ only need to fit $f_{1T}^{\perp,u}$



Phys.Rev. D73 (2006) 014021

- Ansatz for Sivers function

$$xf_{1T}^{\perp(1)} = Ax^B(1-x)^5 \quad .$$

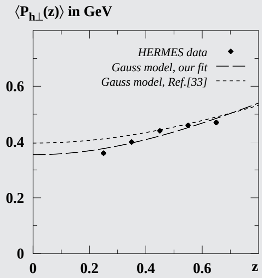
  (also checked with $(1-x)^4$)
- Gaussian $k_\perp$ and $p_\perp$ dependence

Determining the gauss width

$$f_1\left(x,\vec{k}_\perp\right) = f_1(x)exp\cdot\left(-\frac{\vec{k}_\perp^2}{k_{unp}^2}\right)$$

$$f_{1T}^\perp\left(x,\vec{k}_\perp\right) = f_{1T}^\perp(x)exp\cdot\left(-\frac{\vec{k}_\perp^2}{k_{siv}^2}\right)$$

$$D_1\left(z,\vec{p}_\perp\right) = D_1(z)exp\cdot\left(-\frac{\vec{p}_\perp^2}{p_{D1}^2}\right)$$



$$\langle P_{hT}(z)\rangle_{Siv} \stackrel{Gauss}{=} \frac{2}{\sqrt{\pi}}\sqrt{z^2k_{unp}^2 + p_{D1}^2}$$

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Determining the Gauss width

- $\Rightarrow$ 3 Parameters to fit from external conditions
- p^2_{D1}, k^2_{unp} from HERMES $P_{h\perp}$-data

$$p^2_{D1} = 0.16\text{GeV}^2 \qquad k^2_{unp} = 0.33\text{GeV}^2$$

- use positivity constraint
$$\frac{|p_\perp|}{M} f_{1T}^\perp\left(x, \vec{k}_\perp^2\right) \leq f_1\left(x, \vec{k}_\perp^2\right) \Rightarrow k^2_{siv} \leq \frac{k^2_{unp}}{1 + \frac{2M_N^2 k^2_{unp}}{e k^2_{siv}}\left(\frac{f_{1T}^\perp(x)}{f_1(x)}\right)^2}$$
- $\Rightarrow \qquad 0 < k^2_{siv} < k^2_{unp}$
- k^2_{siv} -range gives only 10% inaccuracy
- choose average $k^2_{siv} = 0.15$ GeV ^{2}

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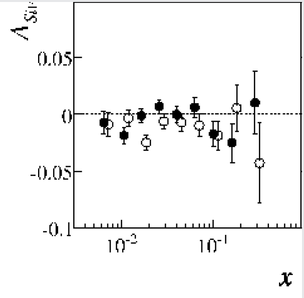
Comparison to COMPASS data

COMPASS does not measure any significant Sivers asymmetry!



COMPASS uses a Deuterium target $\Rightarrow$ number of u- and d-quarks is equal. Large-N_C Ansatz implies this result naturally, because

$$f_{1T}^{\perp,u} = -f_{1T}^{\perp,d}$$

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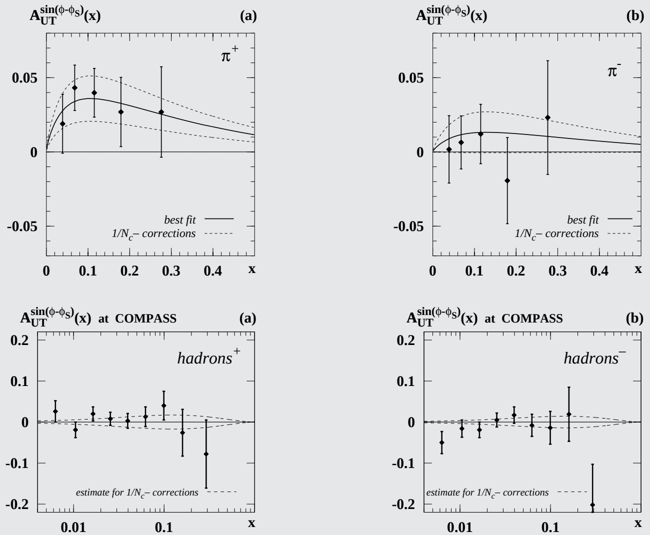
Validity of Large-N_C Ansatz



Comparison to other groups

hep[-ph/0511017]

Phys. Rev. D 71 074006 J.C.Collins, A.V.Efremov, K.Goeke, S.Menzel, A.Metz, P.Schweitzer :

$$f_{1T}^{\perp(1),u} = Ax^b(1-x)^5 \frac{\pi}{\langle k_\perp^2\rangle} e^{-\frac{\vec{k}_\perp^2}{\langle k_\perp^2\rangle}}$$

Phys. Rev. D 70 117504 M.Anselmino, M.Boglione, U.D'Alesio, A.Kotzinian, F.Murgia, A.Prokudin :

$$\Delta^N f_{q/p^\uparrow} = 2N_q(x) f_{q/p}(x) g\left(\vec{k}_\perp^2\right) h\left(\vec{k}_\perp^2\right)$$

$$N_q(x) = N_q x^{a_q}(1-x)^{b_q} \frac{(a_q+b_q)^{(a_q+b_q)}}{{a_q}^{a_q}\, {b_q}^{b_q}}$$

$g\left(\vec{k}_\perp^2\right), h\left(\vec{k}_\perp^2\right)$ Gaussian or other function

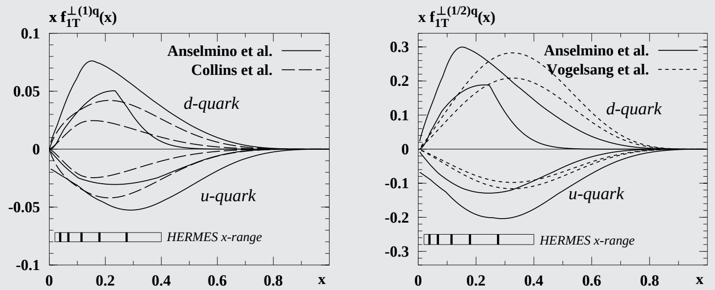
Phys. Rev. D 72 054028 W.Vogelsang, F.Yuan:

$$F_{siv}(x) = f_{1T}^{\perp(1/2)} = \frac{1}{2}\int d^2\vec{k}_\perp \frac{|\vec{k}_\perp|}{M_N} f_{1T}^{\perp}\left(x, \vec{k}_\perp^2\right)$$

$\vec{p}_\perp$ of D_1^q is neglected

Comparison to other groups

[hep-ph/0511017]



- all 3 describe the data equally well
- all 3 describe only $f_{1T}^{\perp u,d}$

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Gauge-Link

- ensures the color gauge-invariance of the quark operator
- generated by so-called final state interactions (FSI)
- the scattered parton can exchange gluons with the target remnant
- also called Wilson line

Definition

$$\mathcal{W} = \mathcal{P}exp\left(ig\int d\alpha^{\mu}A_{\mu}(\alpha)\right)$$

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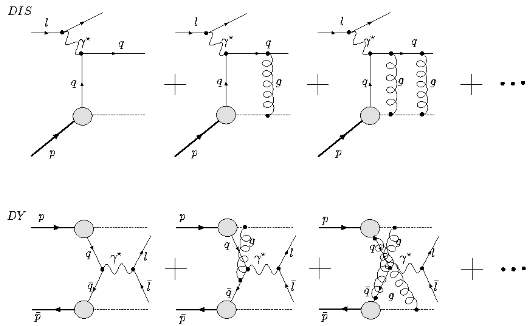
Gauge-Link in SIDIS and in Drell-Yan



Gauge-Link

Gauge-Link in SIDIS and in DY run the other way round
Collins, Phys. Lett **B** 536,43 (2002)

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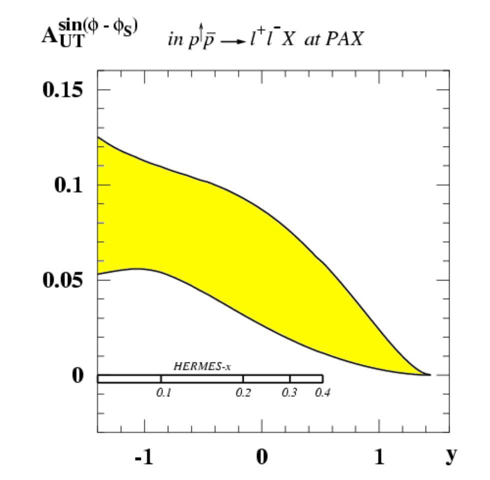
DY@PAX



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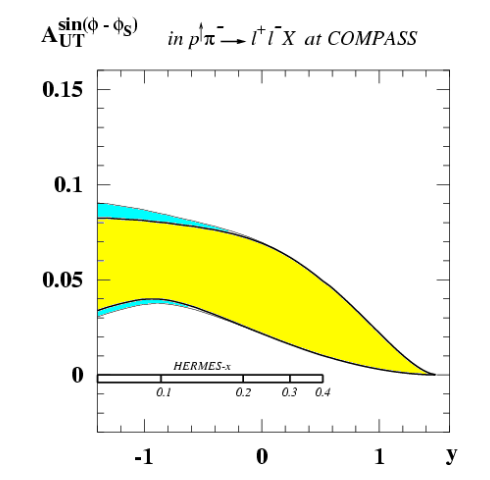
DY@COMPASS



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DY@RHIC $\longrightarrow$ antiquark

$\bar{q}$ can give significant contribution

PAX: Collision of protons and antiprotons
COMPASS: Collision of protons an π^-
$\Rightarrow$ Sivers antiquark distributions hardly visible
RHIC: Proton-proton collisions
sensitive to $f_{1T,DY}^{\perp,u}(x_1) f_1^{\bar{u}}(x_2)$ and $f_1^u(x_1) f_{1T,DY}^{\perp,\bar{u}}(x_2)$ on equal footing
$\Rightarrow$ Antiquarks should give significant contribution!

Models for Antiquark distributions

1. $f_{1T}^{\perp\bar{q}} = 25\,\%\, f_{1T}^{\perp q}$
2. $\frac{f_{1T}^{\perp,q}}{f_{1T}^{\perp,\bar{q}}} = \frac{f_1^u + f_1^d}{f_1^{\bar{u}} + f_1^{\bar{d}}}$

Results

Measurement can give first boundaries on $\bar{q}$ distributions

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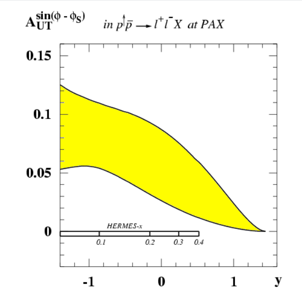
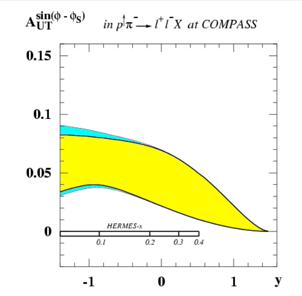
Antiquarks at PAX and COMPASS





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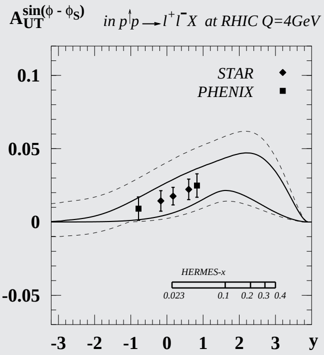
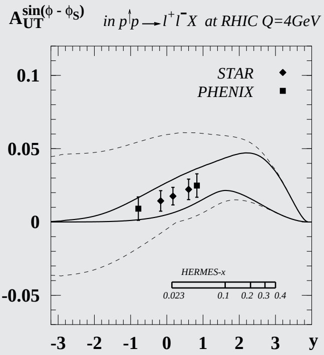
DY@RHIC some $Q = 4$ GeV

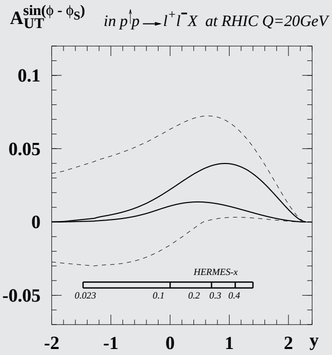
J.C. Collins, A.V. Efremov, K. Goeke, M. Grosse Perdekamp, S. Menzel, B. Meredith, A. Metz, P.Schweitzer
[Phys.Rev. **D**73 (2006) 094023]

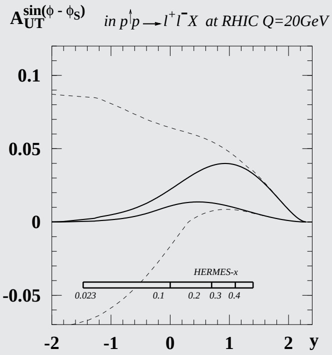




DY@RHIC $Q = 20$ GeV

J.C. Collins, A.V. Efremov, K. Goeke, M. Grosse Perdekamp, S. Menzel, B. Meredith, A. Metz, P.Schweitzer
[Phys.Rev. **D**73 (2006) 094023]





Summary

- Definition and descriptions of the Sivers function
  - Properties
  - Interpretation
- Parameterizations of the Sivers function
  - Gauss-Ansatz and Large N_C
  - Comparison of different parameterizations
- Future measurements of the Sivers function
  - Testing QCD (change of sign in SIDIS $\longrightarrow$ DY)
  - Sivers antiquarks (RHIC)

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Outlook

- Get more accurate data of A_{UT}
  - Decide which parameterization is right
  - Relax Large-N_C ansatz
- Future experimental data
  - A_{UT}^{SIDIS}:
    - COMPASS (proton target)
    - HERMES (weighted data, more statistics)
  - hadronic final states
    - RHIC ($p^{\uparrow}p \longrightarrow jet_1\, jet_2\, X;\quad jet\, H\, X;\quad etc.$)
    - RHIC ($p^{\uparrow}p$)
  - A_{UT}^{DY}:
    - PAX ($p^{\uparrow}\bar{p}$)
    - COMPASS ($p^{\uparrow}\pi^{-}$)
    - RHIC ($p^{\uparrow}p$, Antiquarks)

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