Pion & Jet Production @ RHIC and the Extraction of Δg

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Outline

- **brief tour of the theoretical framework**
- **single-inclusive pion and jet production:** pQCD expectations; resummations
- **towards a global analysis:** Mellin technique
- **Δg revisited:** impact of the new data

I. Brief tour of the theoretical framework

the pQCD approach to hard scattering

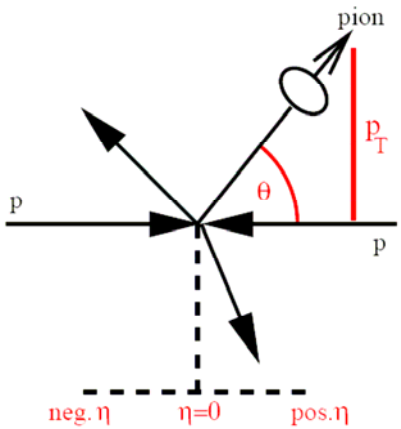
if hardness of probe is large enough ($\alpha_s(Q) \ll 1$), perturbative QCD can be used to make *quantitative predictions* (exploiting asymptotic freedom of QCD)

Gross, Wilczek; Politzer

starting point: factorization theorem & universality of pdfs

Libby, Sterman; Ellis et al.; Amati et al.; Collins et al.; ...

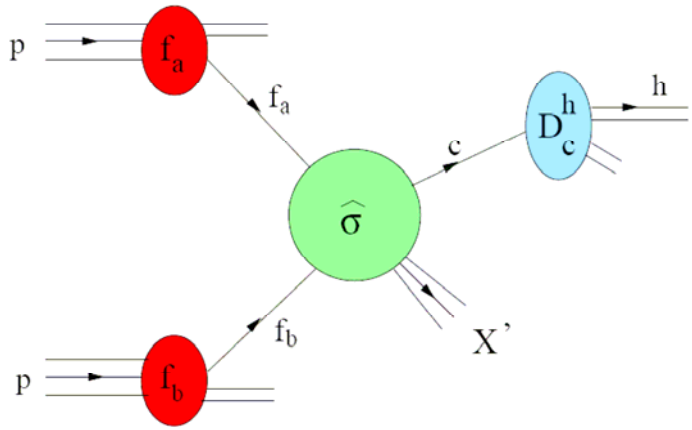
example : (un)polarized inclusive high-p_T pion production



factorization

⟶

theorem



the pQCD approach (factorized cross section)

in more "mathematical" terms:

long-distance

from exp.; μ-dep.: $d\sigma/d\mu = 0$ (pQCD)

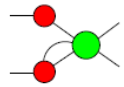
$$\frac{d\Delta\sigma^{\vec{p}\vec{p}\to\pi X}}{dp_T d\eta} = \sum_{abc} \int dx_a\, dx_b\, dz_c\, \Delta f_a(x_a,\mu_f)\, \Delta f_b(x_b,\mu_f)\, D_c^{\pi}(z_c,\mu_f')$$
$$\times \frac{d\Delta\hat{\sigma}^{ab\to cX'}}{dp_T d\eta}(x_a P_a, x_b P_b, P^{\pi}/z_c, \mu_f, \mu_f', \mu_r) + \mathcal{O}(\frac{\lambda}{p_T})^n$$

short-distance

calculable in pQCD: power series in α_s

power corrections

neglected



"features":

- separation between short- and long-dist. not unique (*fact. scheme*)
- theory calculation depends on *unphysical fact./renorm. scales*
- factorized "picture" good up to power corrections

the pQCD approach (scale dependence)

measured cross section *must not* depend on theoretical scales: $\mu_f \frac{d\Delta\sigma}{d\mu_f} = 0$

$\leftrightarrow d\Delta\hat{\sigma}(\mu_f \to \mu_f + d\mu_f, \mu'_f)$ canceled by scale behavior of $\Delta f_{a,b}(x_{a,b}, \mu_f)$

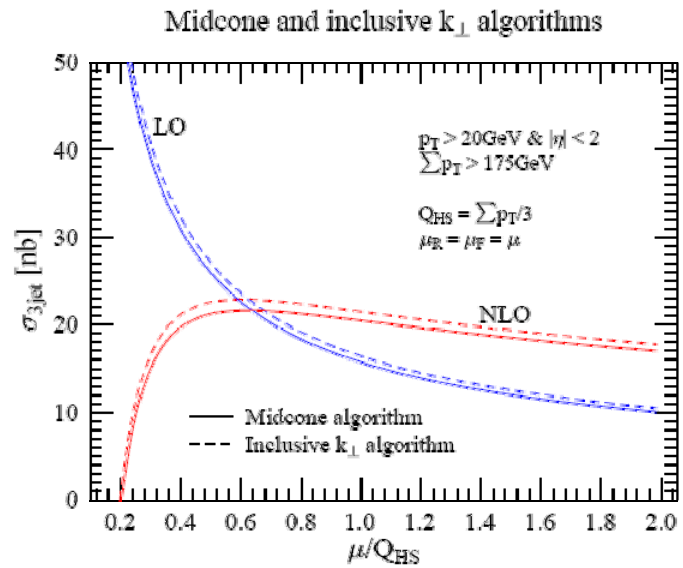
catch: . . . we work with a perturbative expansion in α_s:

$$d\Delta\hat{\sigma} = \underbrace{d\Delta\hat{\sigma}^{(0)}}_{\text{LO}} + \underbrace{\alpha_s\, d\Delta\hat{\sigma}^{(1)}}_{\text{NLO}} + \underbrace{\alpha_s^2\, d\Delta\hat{\sigma}^{(2)}}_{\text{NNLO}} + \ldots$$

μ_f cancellation only happens to all orders

LO	:	no cancellation whatsoever
NLO	:	cancellation starts to work
NNLO	:	better and better
. . .		

$\rightarrow$ higher order calculations mandatory



typical example: 3-jets @ TeVatron (Z. Nagy)

going beyond the LO is in every aspect

a major enterprise

$$d\Delta\hat{\sigma} = \overset{\text{LO}}{d\Delta\hat{\sigma}^{(0)}} + \overset{\text{NLO}}{\alpha_s\, d\Delta\hat{\sigma}^{(1)}} + \overset{\text{NNLO}}{\alpha_s^2\, d\Delta\hat{\sigma}^{(2)}} + \ldots$$

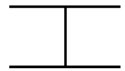
number of partons

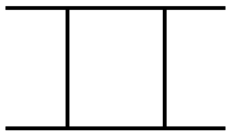
complexity of calculation

... pushes computer algebra programs to their limits

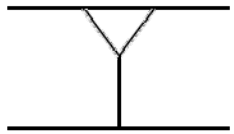
NLO corrections in a nutshell (to-do list)

at $\mathcal{O}(\alpha_s^3)$ (NLO) one has to consider:

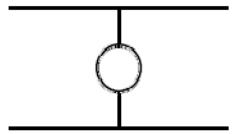
- one-loop (virtual) corrections to all LO $2 \to 2$  processes



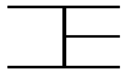
"box"



"vertex"



"selfenergy"

- all conceivable $2 \to 3$  parton-parton scattering processes

this includes additional gluon emission to existing LO processes

$$qq' \to qq'g,\ q\bar{q} \to ggg,\ gg \to ggg, \text{ etc.}$$

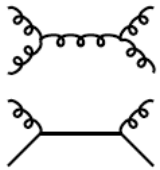
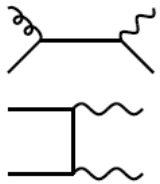
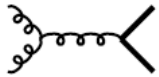
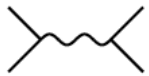
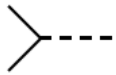
as well as genuine NLO processes not possible at $\mathcal{O}(\alpha_s^2)$

$$qg \to qq'\bar{q}',\ qg \to qq\bar{q}, \text{ etc.}$$

Cross sections relevant for RHIC-spin

Δg

$\Delta \bar{q}$

Reaction	Dom. partonic process	probes	LO Feynman diagram
$\vec{p}\vec{p} \to \pi + X$ [61, 62]	$\vec{g}\vec{g} \to gg$ $\vec{q}\vec{g} \to qg$	Δg	
$\vec{p}\vec{p} \to \text{jet(s)} + X$ [71, 72]	$\vec{g}\vec{g} \to gg$ $\vec{q}\vec{g} \to qg$	Δg	(as above)
$\vec{p}\vec{p} \to \gamma + X$ $\vec{p}\vec{p} \to \gamma + \text{jet} + X$ $\vec{p}\vec{p} \to \gamma\gamma + X$ [67, 73, 74, 75, 76]	$\vec{q}\vec{g} \to \gamma q$ $\vec{q}\vec{g} \to \gamma q$ $\vec{q}\vec{\bar{q}} \to \gamma\gamma$	Δg Δg $\Delta q, \Delta\bar{q}$	
$\vec{p}\vec{p} \to DX, BX$ [77]	$\vec{g}\vec{g} \to c\bar{c}, b\bar{b}$	Δg	
$\vec{p}\vec{p} \to \mu^+\mu^- X$ (Drell-Yan) [78, 79, 80]	$\vec{q}\vec{\bar{q}} \to \gamma^* \to \mu^+\mu^-$	$\Delta q, \Delta\bar{q}$	
$\vec{p}\vec{p} \to (Z^0, W^\pm)X$ $p\vec{p} \to (Z^0, W^\pm)X$ [78]	$\vec{q}\vec{\bar{q}} \to Z^0$, $\vec{q}'\vec{\bar{q}} \to W^\pm$ $\vec{q}'\bar{q} \to W^\pm$, $q'\vec{\bar{q}} \to W^\pm$	$\Delta q, \Delta\bar{q}$	

Jäger,Schäfer, MS, Vogelsang; de Florian

Jäger,MS,Vogelsang; Signer et al.

Gordon,Vogelsang; Contogouris et al.; Gordon, Coriano

Bojak, MS

Weber; Gehrmann; Kamal; Smith et al.

NLO corrections known in all cases

II. Single-incl. pion & jet production

pQCD expectations & resummations

Unpolarized "benchmark" results

important to have an adequate *and* proven theoretical framework

→ **relevance of unpol. "benchmark" results**

- demonstrate applicability of pQCD methods
  (or map out the regions where they fail !!)

- measurements at 200 GeV (and 500 GeV) never done before !!
  → energy dependence of cross sections at a given p_T :

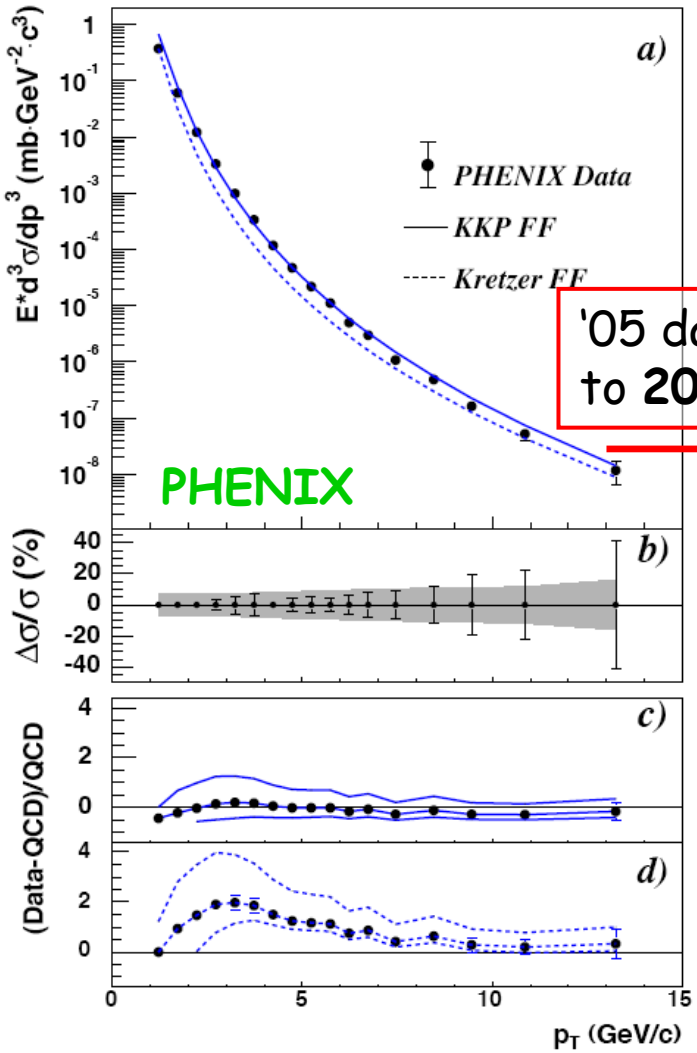
  (fixed target) → RHIC → SPS → TeVatron → LHC

- valuable source of information about unpol. pdfs and frag. fcts

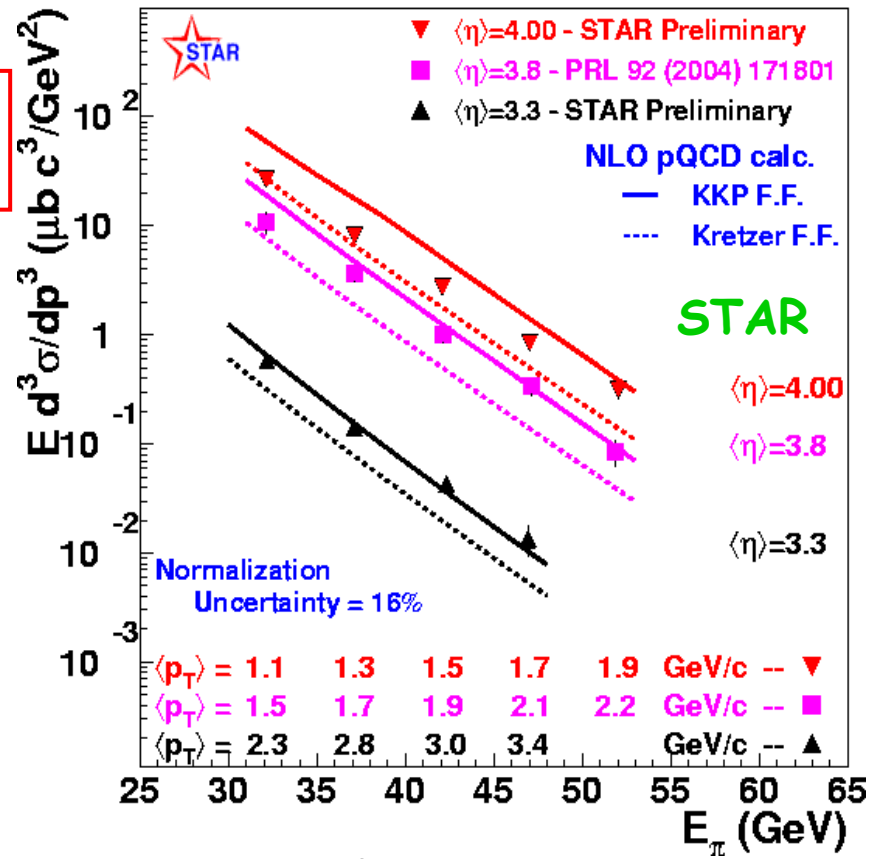
unpol. measurements should always precede A_{LL} measurements

Unpol. "benchmarks" @ RHIC: pions

so far it works extremely well (down to unexpectedly small p_T !!):

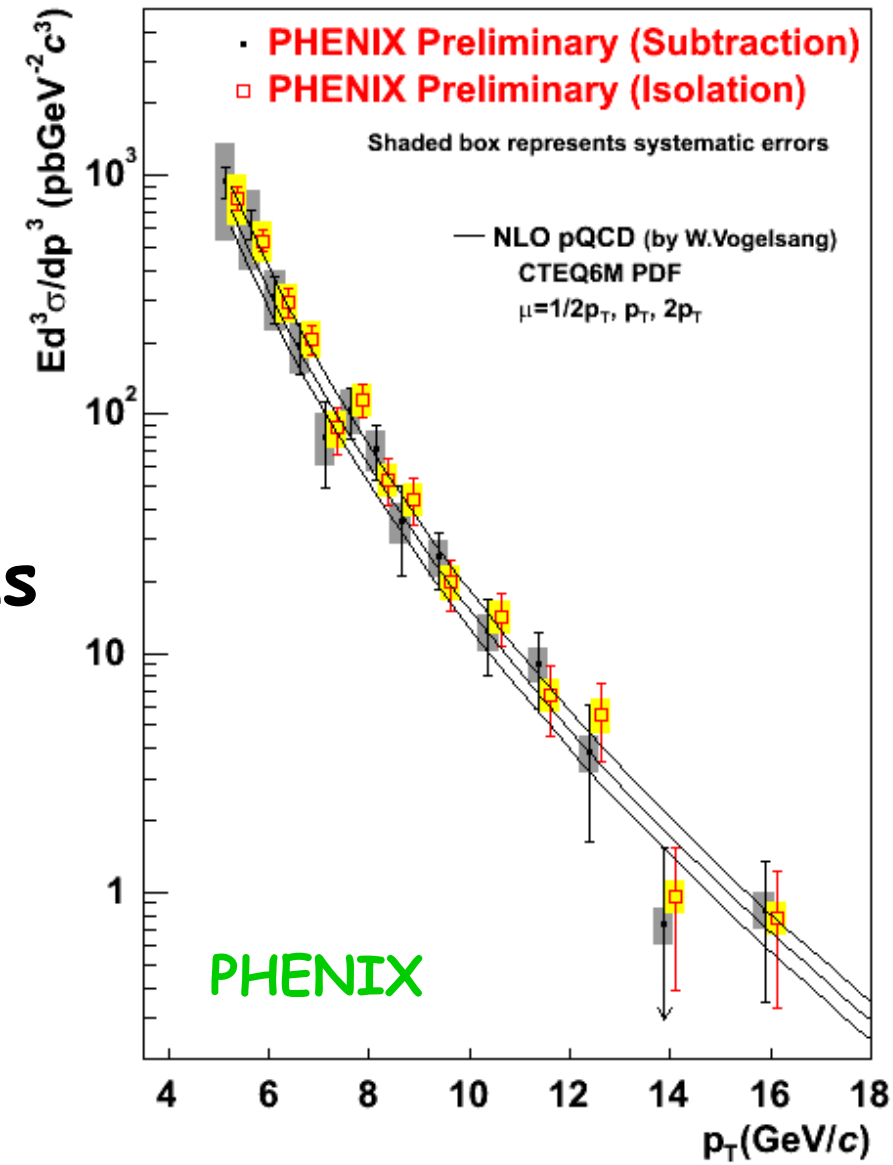


'05 data up to **20 GeV** !!



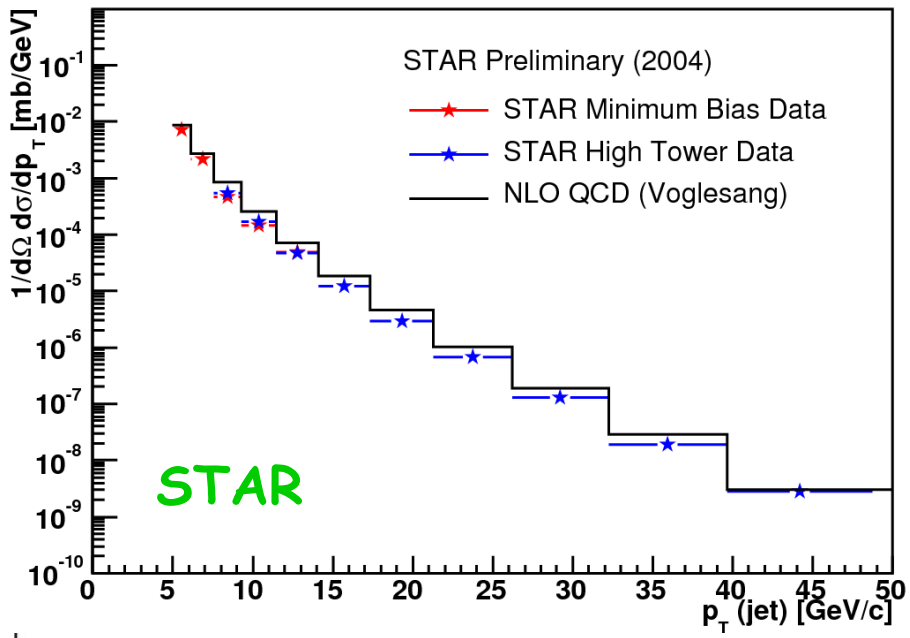
Unpol. "benchmarks" @ RHIC: photons

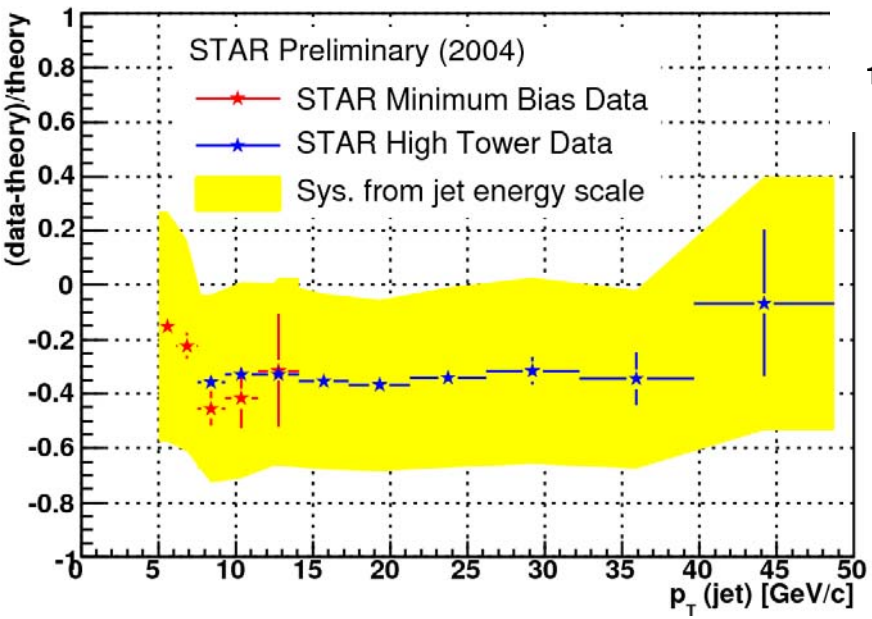
also for prompt photons



Unpol. "benchmarks" @ RHIC: jets

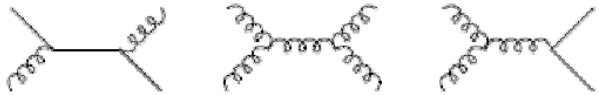
... and jets



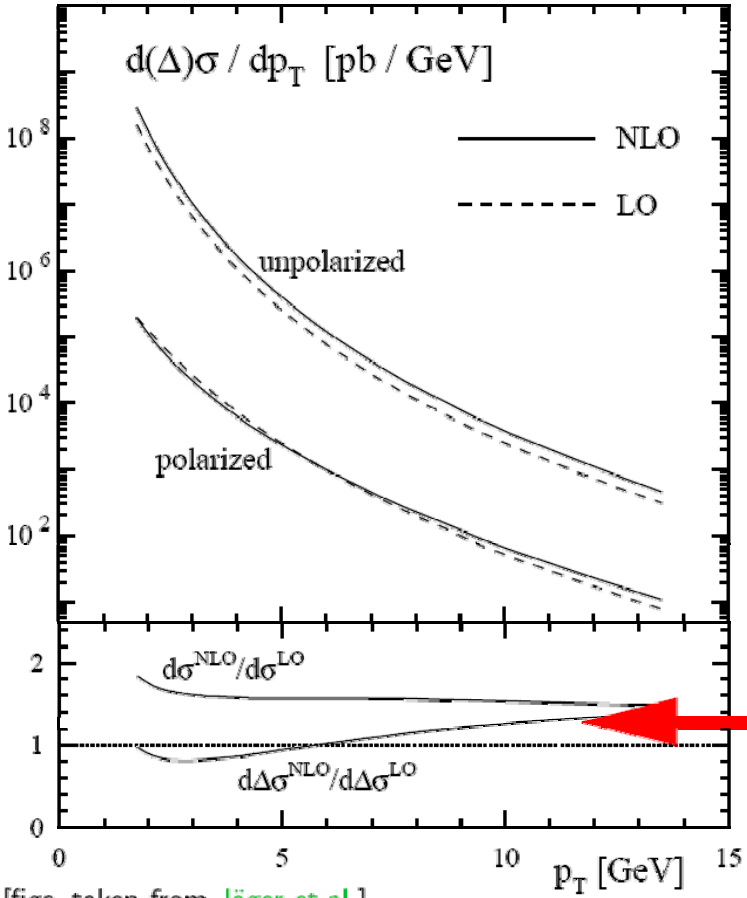


foundation for similar measurements with polarization

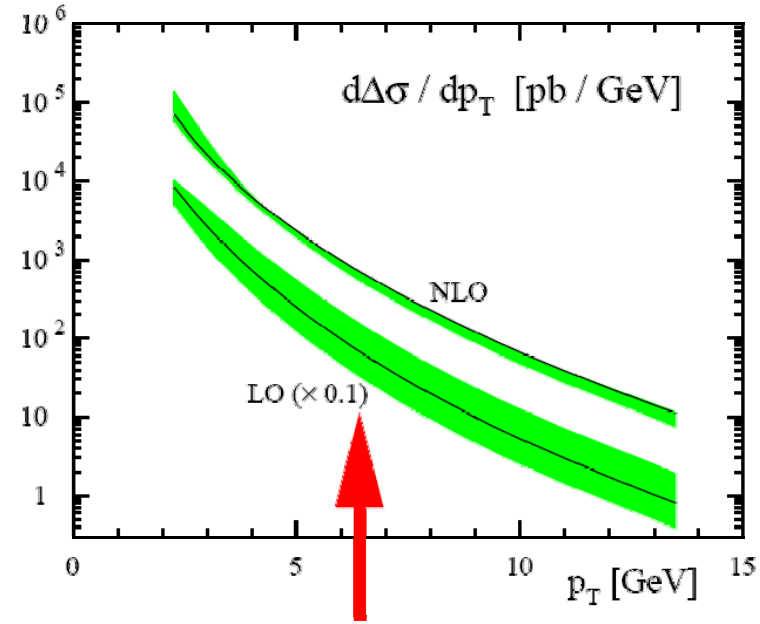
RHIC spin: inclusive pions (xsec & scale dep.)



NLO: Jäger, MS, Schäfer, Vogelsang; de Florian



[figs. taken from Jäger et al.]



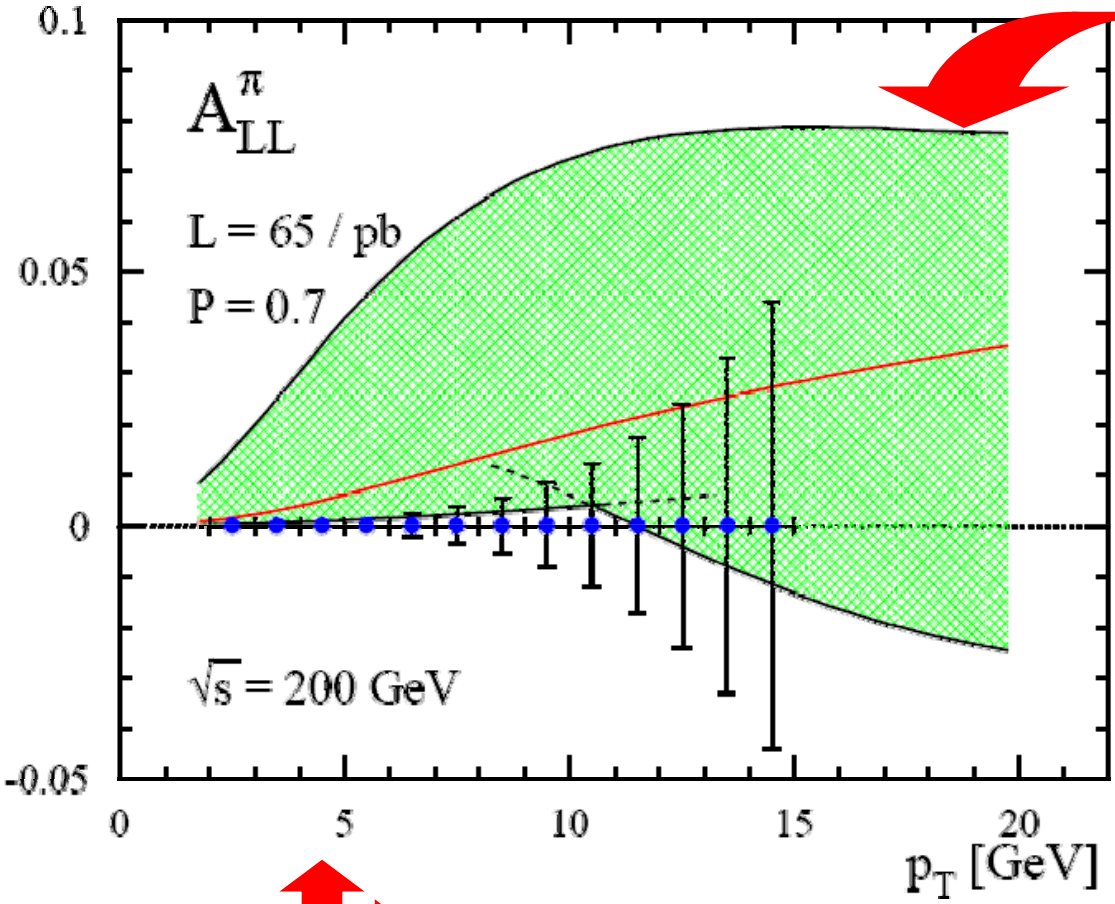
μ_f dependence much reduced in NLO

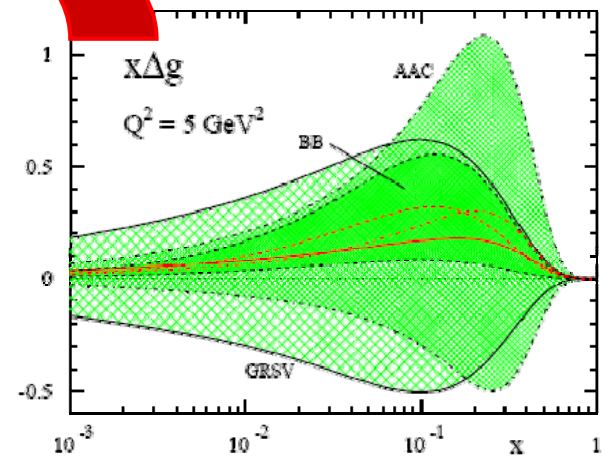
NLO corrections different for $d\Delta\sigma$ and $d\sigma$

$\rightarrow$ do not cancel in $A_{LL} = d\Delta\sigma/d\sigma$

RHIC spin: inclusive pions (A_{LL})

expectations for spin asymmetry:





based on very different gluon polarizations Δg

note: A_{LL} positive at small p_T (dominance of gg scattering)

RHIC spin: inclusive pions (small p_T region)

one can derive a *model-independent* lower bound on A_{LL}: Jäger, Kretzer, MS, Vogelsang

$$\frac{d\Delta\sigma}{dp_T} = \sum_{a,b,c} \Delta f_a(x_a) \otimes \Delta f_b(x_b) \otimes \frac{d\Delta\hat{\sigma}_{ab}(x_a, x_b, z_c)}{dp_T} \otimes D_c^h(z_c)$$

take $x_T^2 \equiv 4p_T^2/S$ moments $\int dx_T^2 (x_T^2)^{N-1} \ldots \rightarrow$ convolutions turn into products

$$\Delta\sigma^\pi(N) = \left(\Delta g^{N+1}\right)^2 \underset{gg}{\mathcal{A}^N} + 2\Delta g^{N+1} \underset{qg}{\mathcal{B}^N} + \underset{qq}{\mathcal{C}^N}$$

this is a parabola in $\Delta g^N \rightarrow$ minimize!

$$\Delta\sigma^\pi(N)\Big|_{\min} = -\frac{\left(\mathcal{B}^N\right)^2}{\mathcal{A}^N} + \mathcal{C}^N \xrightarrow[\text{inverse}]{\text{Mellin}}$$

negative, but tiny lower bound: $A_{LL} = O(-10^{-4})$ at $p_T = 1.5$ GeV

Interlude: resummations

known that

Sterman; Catani, Trentadue; Laenen, Oderda, Sterman; Catani et al.; Sterman, Vogelsang; Kidonakis, Owens; ...

at partonic threshold: $\hat{x}_T = \frac{2p_T}{\sqrt{\hat{s}}} \to 1$

- just enough energy to produce high-p_T parton
- "inhibited" gluon radiation

→ IR cancellation leaves large logarithms from soft gluons

as $\alpha_s^k \ln^{2k}(1-\hat{x}_T^2)$ at the kth order:

$$p_T^3 \frac{d\hat{\sigma}_{ab}}{dp_T} = p_T^3 \frac{d\hat{\sigma}_{ab}^{\text{Born}}}{dp_T} \left[1 + \underbrace{\mathcal{A}_1 \alpha_s \ln^2\left(1-\hat{x}_T^2\right) + \mathcal{B}_1 \alpha_s \ln\left(1-\hat{x}_T^2\right)}_{\text{NLO}} + \ldots + \mathcal{A}_k \alpha_s^k \ln^{2k}\left(1-\hat{x}_T^2\right) + \ldots \right] + \ldots$$

- resummation of thres. logs worked out for most processes of interest
- resummed logs = well defined class of higher-order corrections
- at fixed target energies of much phenomenological relevance

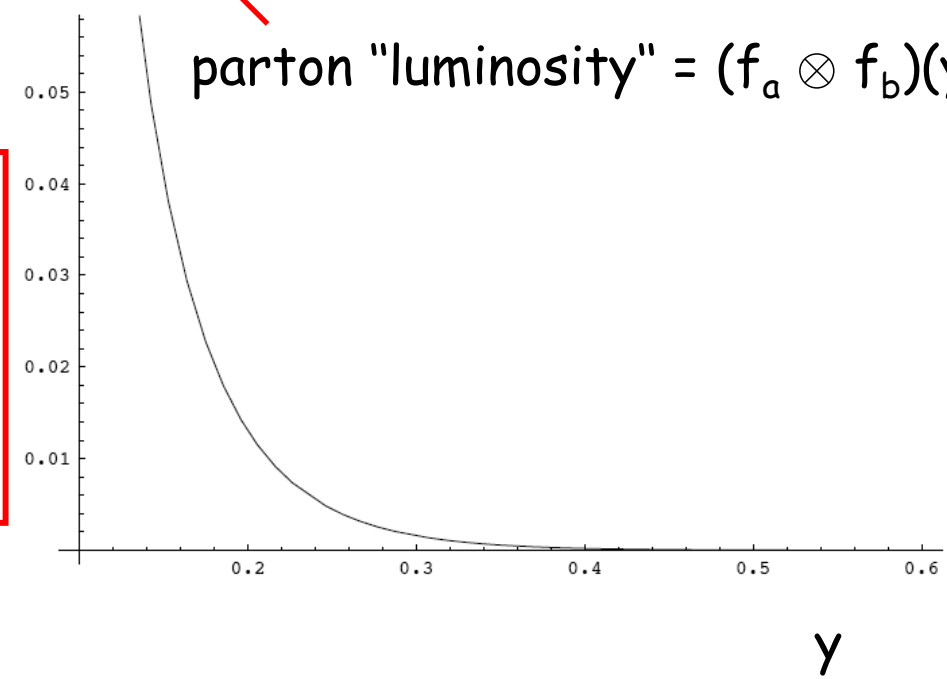
Interlude: resummations

what about threshold logarithms at collider energies?

recall: convoluted form of cross section

$$\frac{d\sigma}{dp_T} = \sum_{a,b} \int_{x_T^2}^{1} \frac{dy}{y} \mathcal{L}_{ab}(y) \frac{d\hat{\sigma}_{ab}\left(x_T^2/y\right)}{dp_T}$$

parton "luminosity" = $(f_a \otimes f_b)(y)$

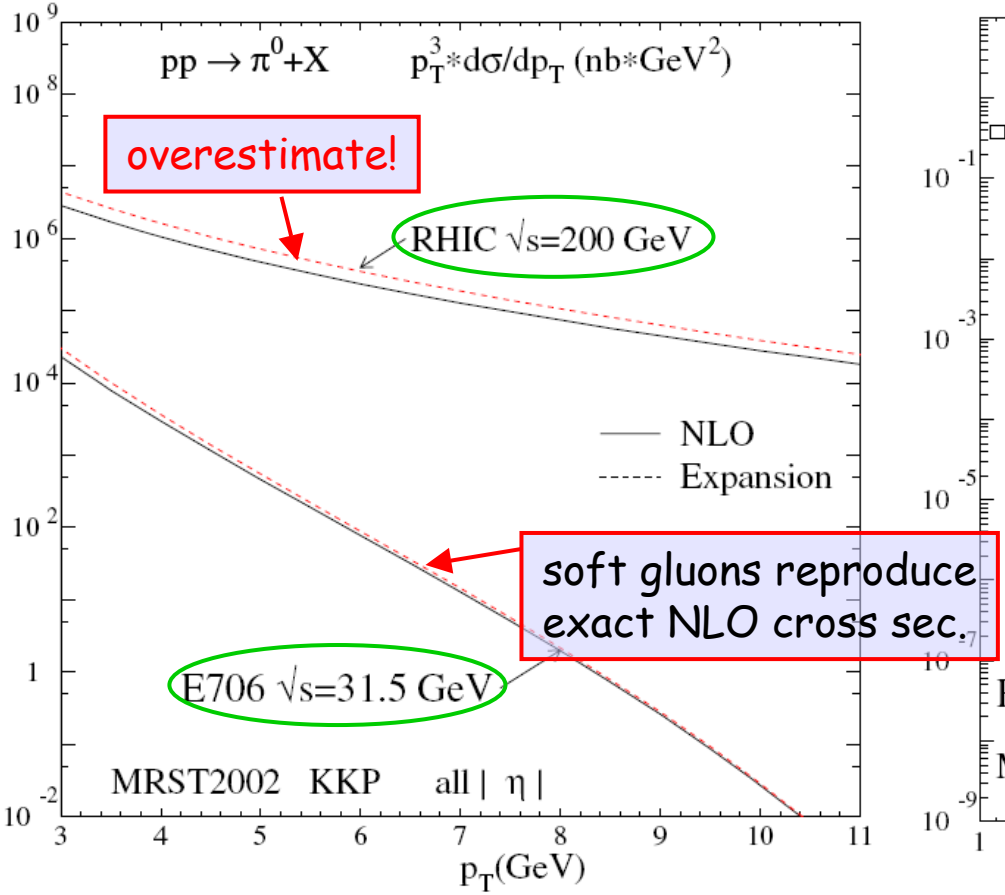


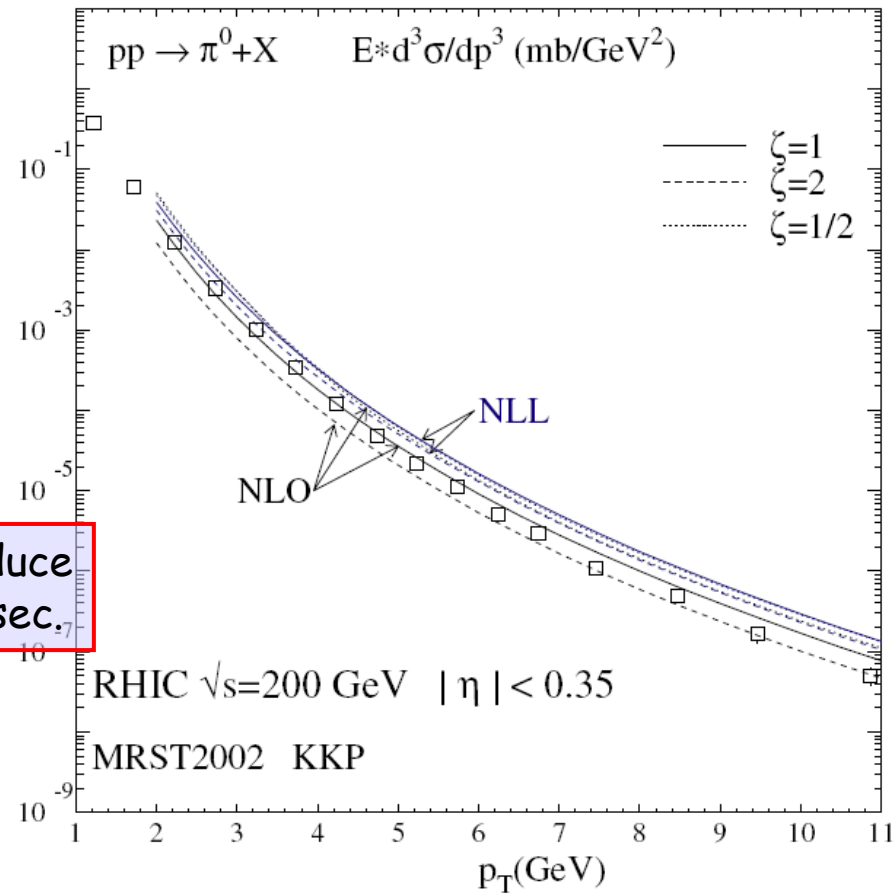
threshold regime may play an important role also at colliders

but:
beware of subleading terms

Resummation effects at RHIC

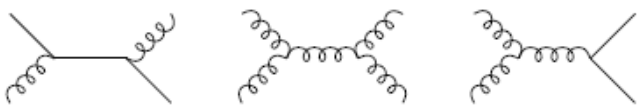
de Florian, Vogelsang



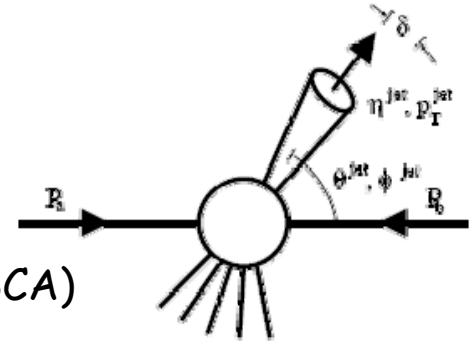


- subleading terms (w.r.t. LL & NLL) important @ RHIC
- NLL result should not be taken too literally (overestimate)

RHIC spin: inclusive jets

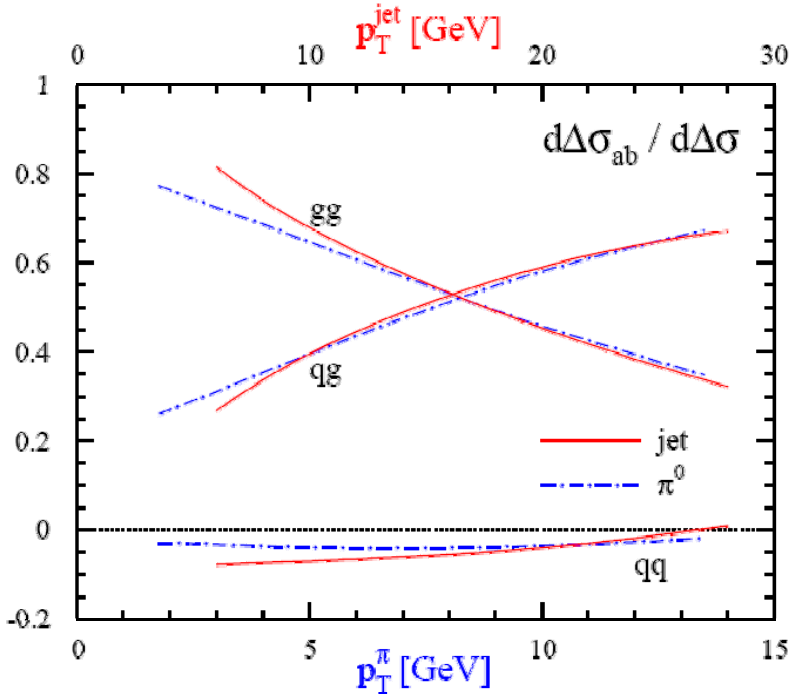


NLO: de Florian et al. (MC);
Jäger, MS, Vogelsang (analyt., SCA)



jet = bunch of particles in a small pencil-like cone; all final-state sing. cancel

jet production proceeds through the same partonic subprocesses as π-production:



π's have roughly $\langle z \rangle \simeq 0.5$:

$\rightarrow$ π with $p_T \simeq$ jet with $2p_T$

comparison to hadrons:

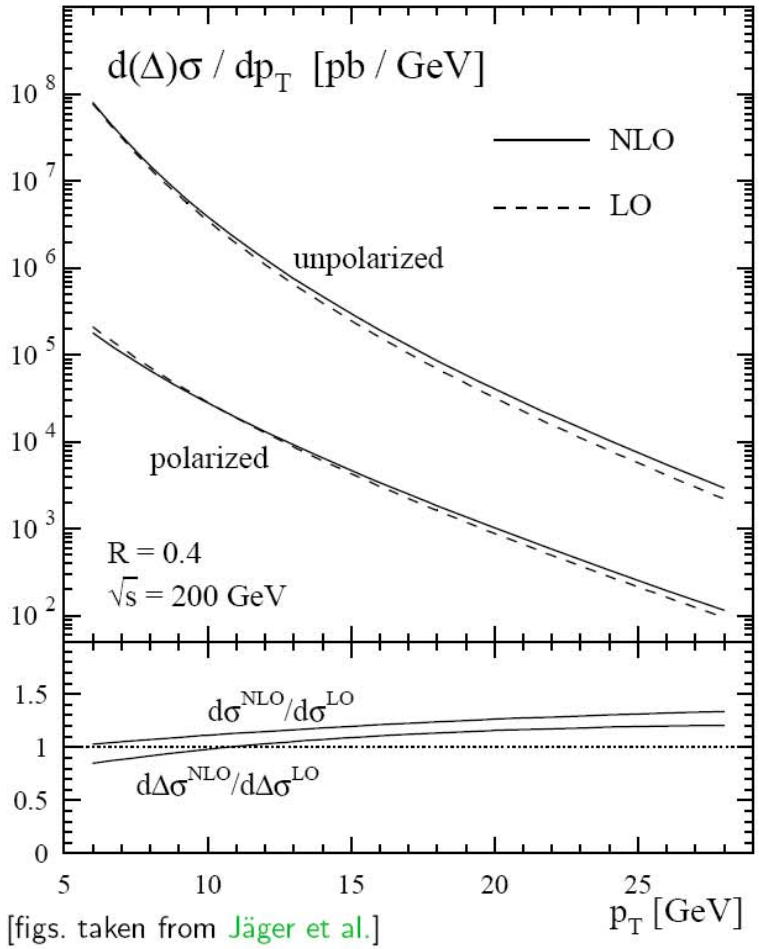
✓ much higher rates

✓ no uncertainties from $D(z)$

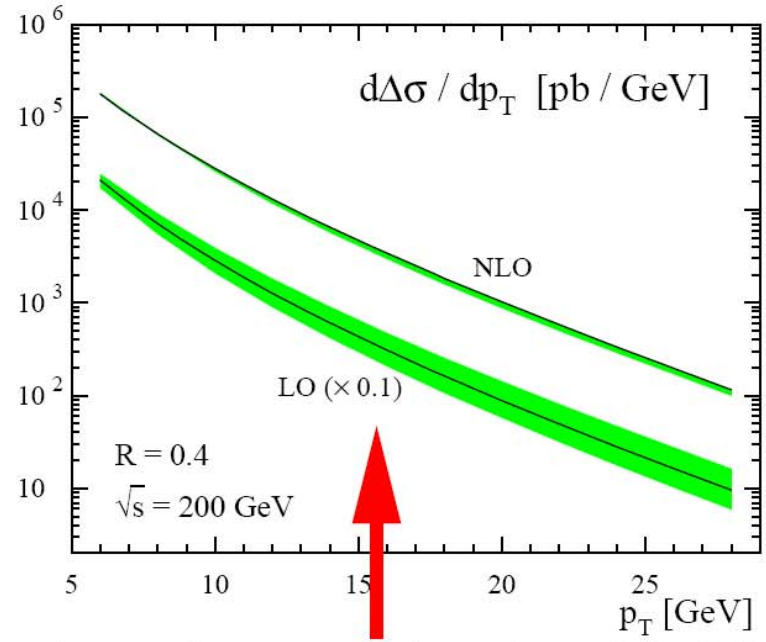
× dependence on precise definition of jet

RHIC spin: inclusive jets (xsec & scale dep.)

pQCD results for jet-production at $\sqrt{S} = 200\,\mathrm{GeV}$, $R_{\mathrm{cone}} = 0.4$ (SCA), $|\eta| \leq 1$:



[figs. taken from Jäger et al.]

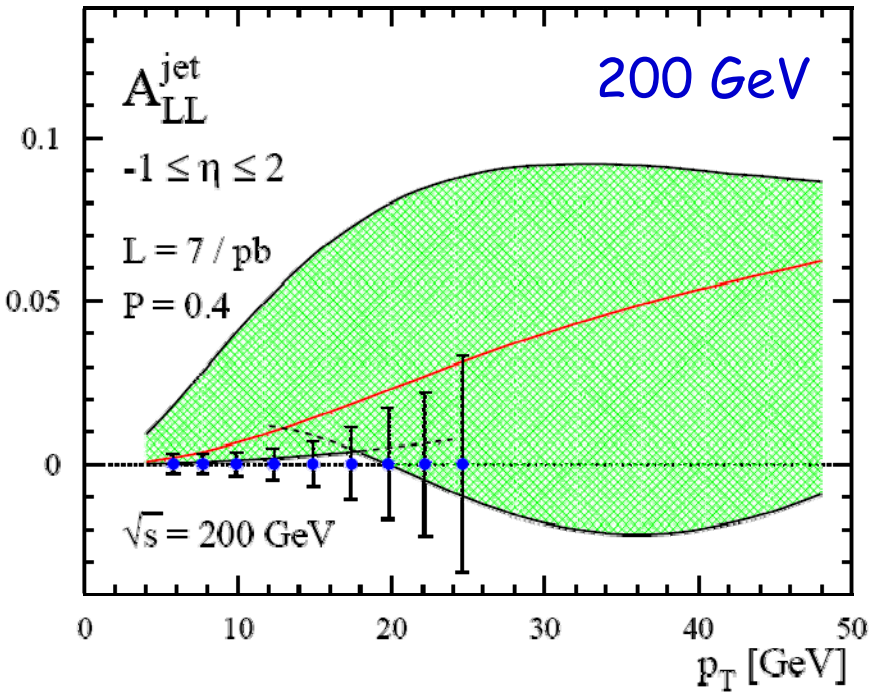


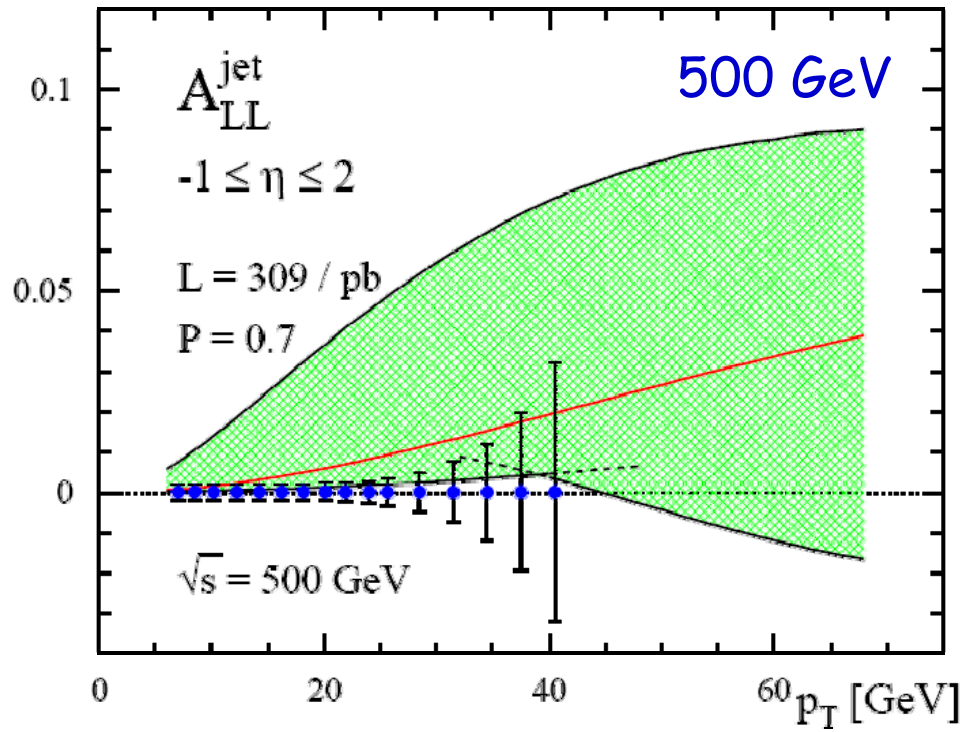
μ_f dependence much reduced in NLO

theoretical uncertainties even smaller than for hadrons

RHIC spin: inclusive jets (A_{LL})

expectations for spin asymmetries:





again: no sensitivity to sign of Δg at *small* p_T

III. Towards a global analysis

Mellin technique

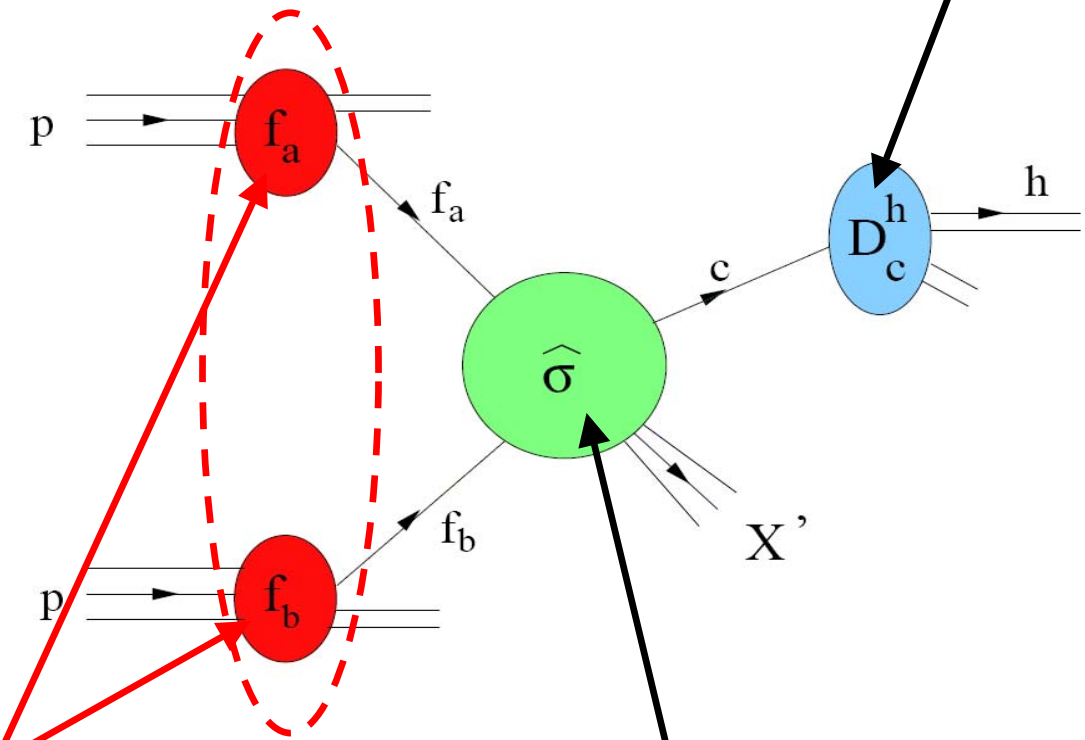
Extraction of spin-dependent pdfs

the strategy is simple ...

extract from e^+e^- data

measure!

$$d(\Delta)\sigma \simeq$$

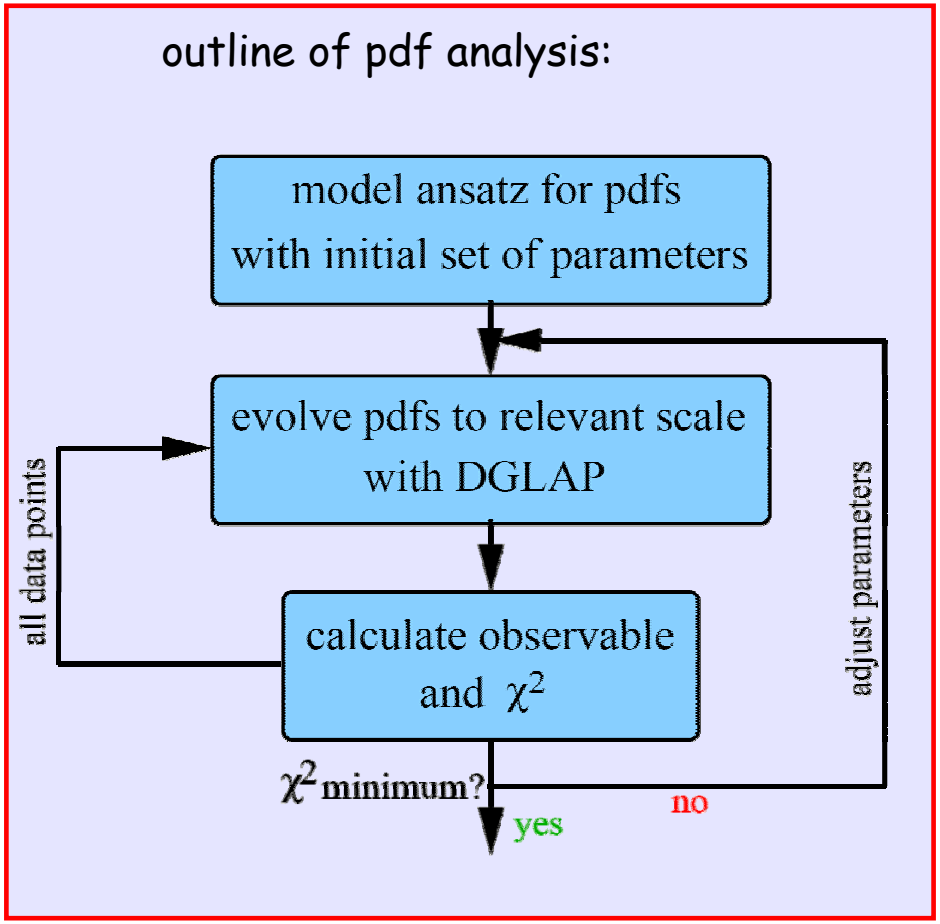


learn about hadronic/spin structure

compute as a power series in α_s in pQCD

Extraction of spin-dependent pdfs

run a χ^2 minimization procedure to determine pdfs



objective:

- extract each and every aspect of helicity-dep. pdfs
- error estimate on each pdf

pdf analysis: complications

highly non-trivial exercise:

- recall: pdfs reside inside complicated convolutions

$$d\Delta\sigma = \sum_{a,b} \int \Delta f_a \otimes \Delta f_b \otimes d\Delta\hat{\sigma}_{ab}$$

→ not directly measurable; no 1:1 relation between data and Δf

- all processes should be analyzed in NLO (or beyond):
  - less dependent on unphysical scales (i.e. smaller theor. errors)
  - NLO corrections are often sizable
  - NLO closer to experiment (jet definition, cuts, ...)
- not a priori clear which/where processes can be described by pQCD

→ unpol. benchmark measurements !

pdf analysis: complications & a way out

main technical challenge of a NLO analysis:

- NLO expressions for most processes are fairly complex
- many different processes needed to pin down all aspects of pdfs
- usual "K-factor trick" $d\Delta\sigma^{\rm NLO} = K \cdot d\Delta\sigma^{\rm LO}$ often is a bad approximation

computing time for a multi-dim. fit becomes excessive

possible way out: get rid of convolutions $\longrightarrow$ Mellin-N moments

$$\int_0^1 dx\, x^{N-1} \left[\int_x^1 \frac{dy}{y} \xi(y) \zeta(x/y) \right] = \int_0^1 dx\, x^{N-1} \left[\int_0^1 dy \int_0^1 dz\, \xi(y) \zeta(z) \delta(x - yz) \right]$$

$$= \int_0^1 dy\, y^{N-1} \xi(y) \int_0^1 dz\, z^{N-1} \zeta(z)$$

$$= \xi^N \zeta^N \quad \text{simple product}$$

pdf analysis: Mellin technique

MS, Vogelsang;
see also Kosower; Berger, Graudenz, Hampel, Vogt

- apply to convolutions in cross section formula:

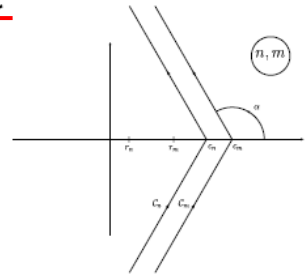
$$d\Delta\sigma \simeq \sum_{a,b} \int dx_a\, dx_b\, \Delta f_a(x_a) \otimes \Delta f_b(x_b) \otimes d\Delta\hat{\sigma}_{ab}(x_a, x_b)$$

Mellin inverse: $\int_{\mathcal{C}_n} dn\, x_a^{-n}\, \Delta f_a^n$ $\quad$ $\int_{\mathcal{C}_m} dm\, x_b^{-m}\, \Delta f_b^m$

$$= \sum_{a,b} \int_{\mathcal{C}_n} dn \int_{\mathcal{C}_m} dm\, \Delta f_a^n\, \Delta f_b^m \left[\underbrace{\int dx_a\, dx_b\, x_a^{-n}\, x_b^{-m}\, \Delta\hat{\sigma}_{ab}(x_a, x_b)}_{\equiv\, \Delta\tilde{\sigma}_{ab}^{n,m}} \right]$$

- pre-calculate time-consuming $\Delta\tilde{\sigma}_{ab}^{n,m}$ on a suitable $n \times m$ grid for all subprocesses and data pts. *once and forever* before the fit

- fast numerical inverse Mellin transformation $\int_{\mathcal{C}_n} dn \int_{\mathcal{C}_m} dm$ on $n \times m$ grids in complex n, m plane

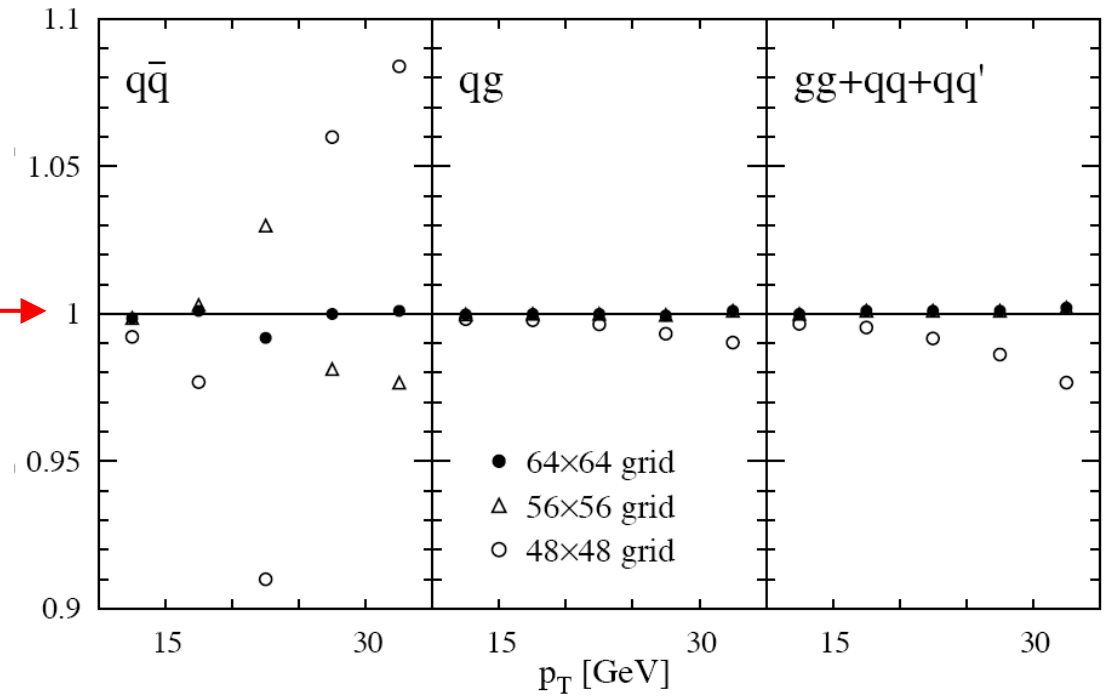


Mellin technique: accuracy & performance

precision:

(example: prompt photons)

64×64 grids do a very good job



performance:

- generation of grids: ~ 12÷24 hrs/grid; parallelization possible
- after that: 1000 eval. of cross section only take about 10 sec

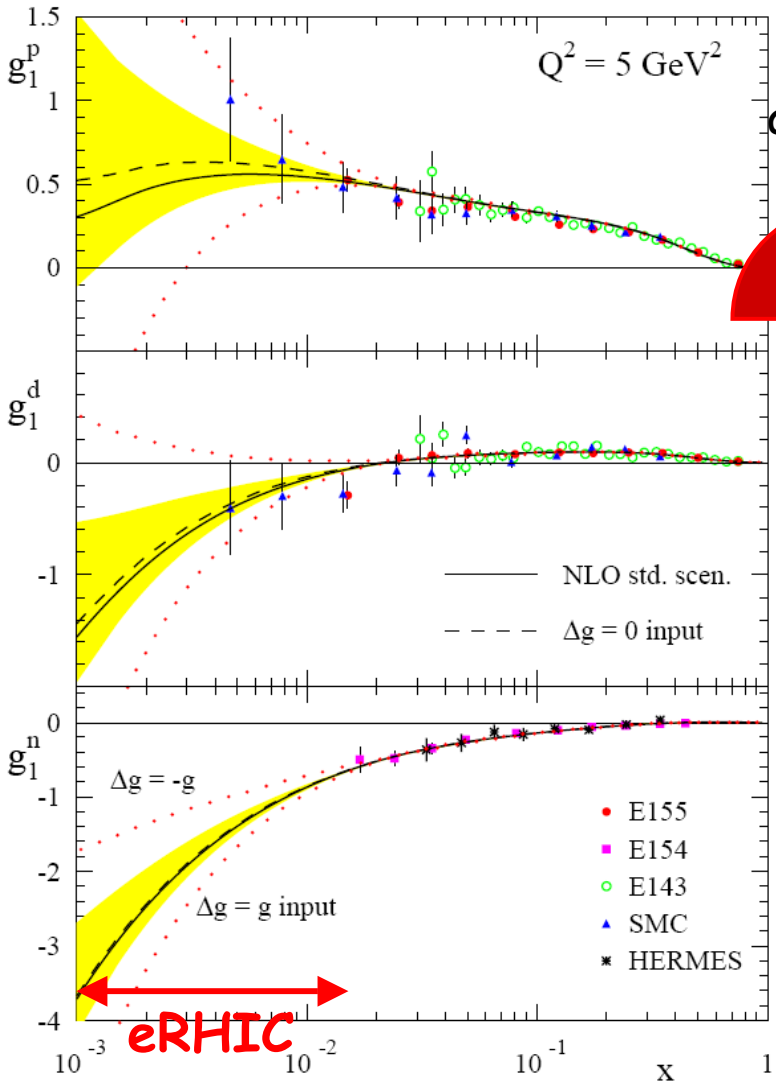
ideal tool for global analysis

IV. Δg revisited

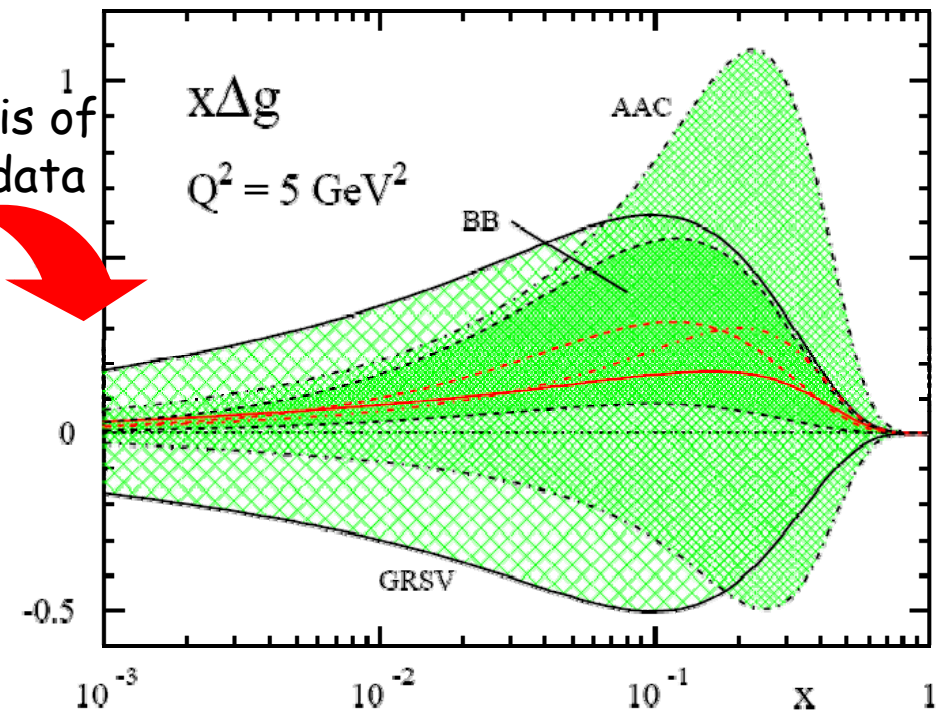
impact of the new data

Δg as of earlier this year ...

unconstrained

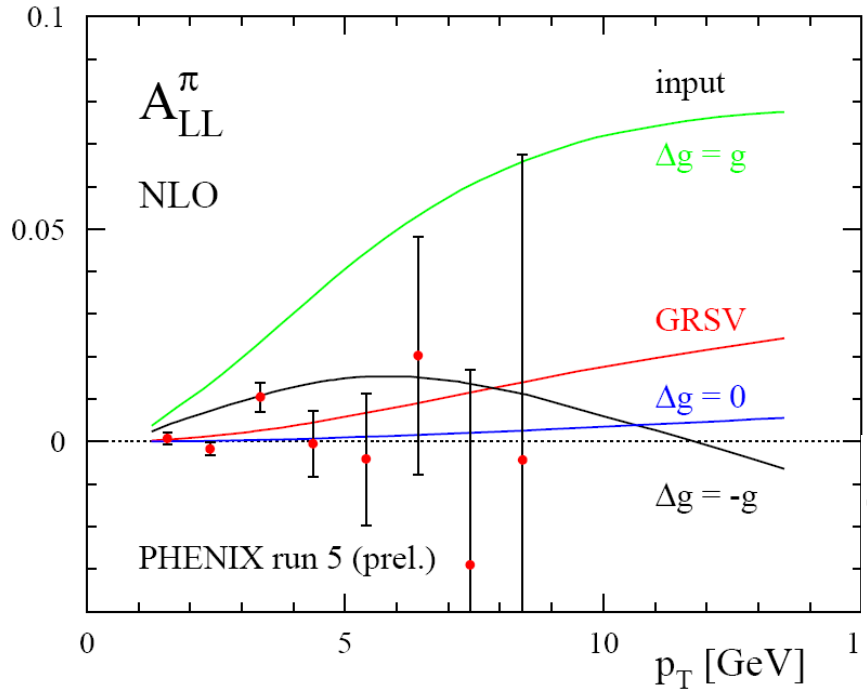


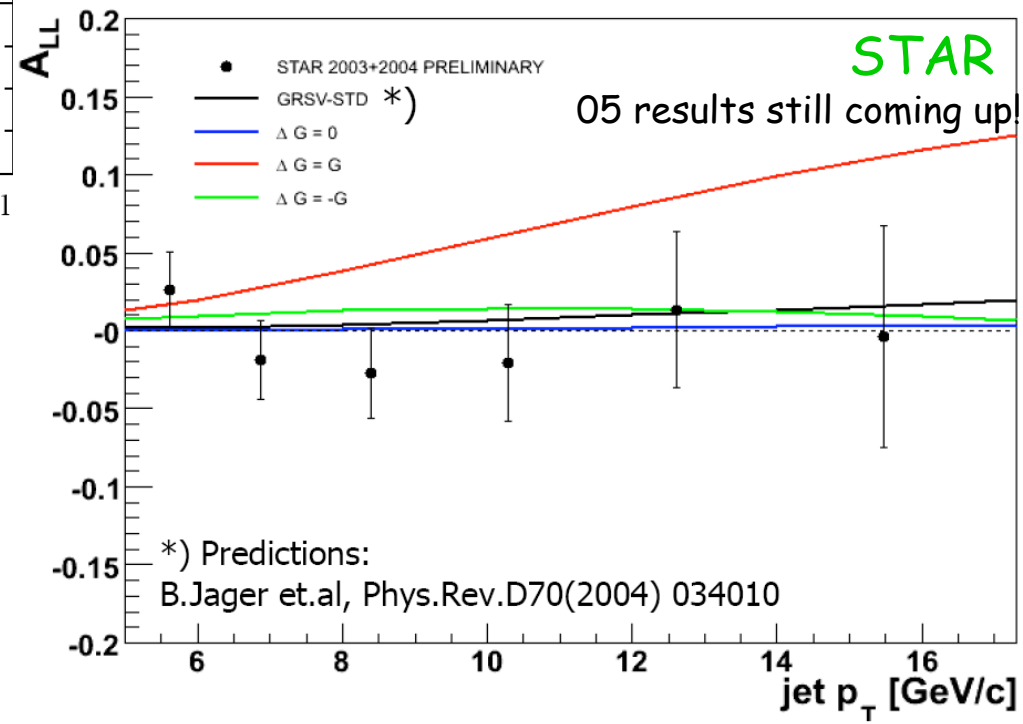
analysis of DIS data



- Asymmetry Analysis Collaboration (Hirai, Kumano, Saito) (set up like unpol. CTEQ fit)
- Glück, Reya, MS, Vogelsang (errors set by positivity bound)
- Blümlein & Böttcher (statistical errors)

new data from RHIC: A_{LL} for pions & jets





plus: 2-hadrons COMPASS
HERMES

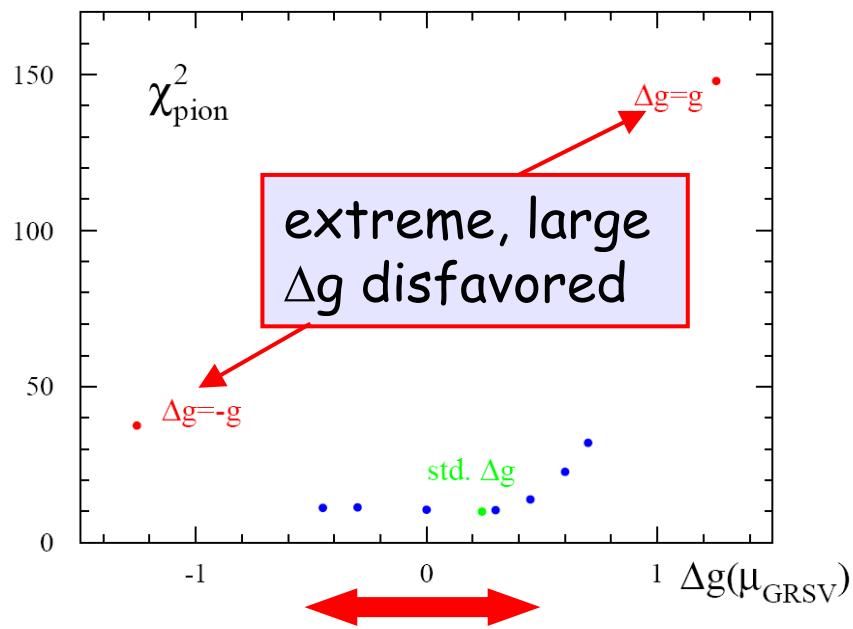
(no NLO available yet!)

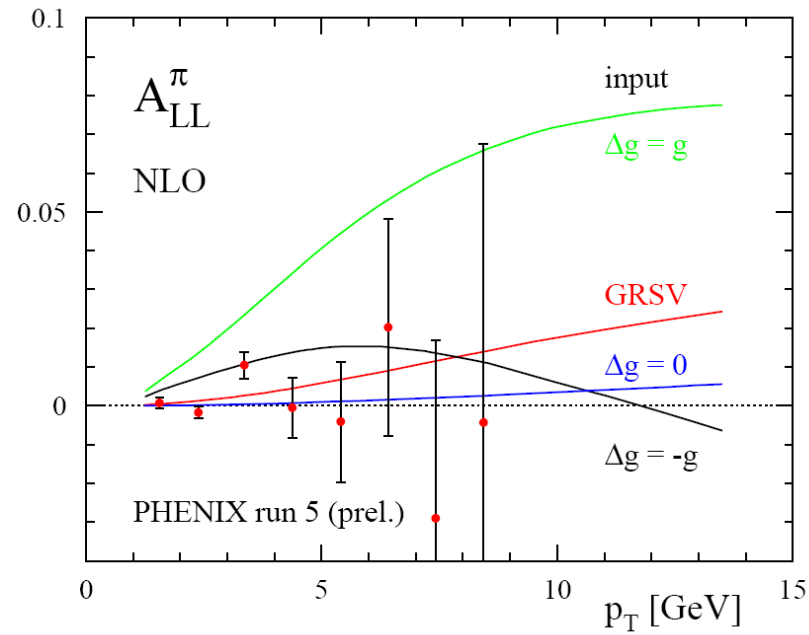
new data from RHIC: implications

MS, Vogelsang

global NLO analysis still ongoing ...

status: Mellin technique set up for DIS, SIDIS, pions
(many subprocesses for pions → lots of grids; jets much easier!)





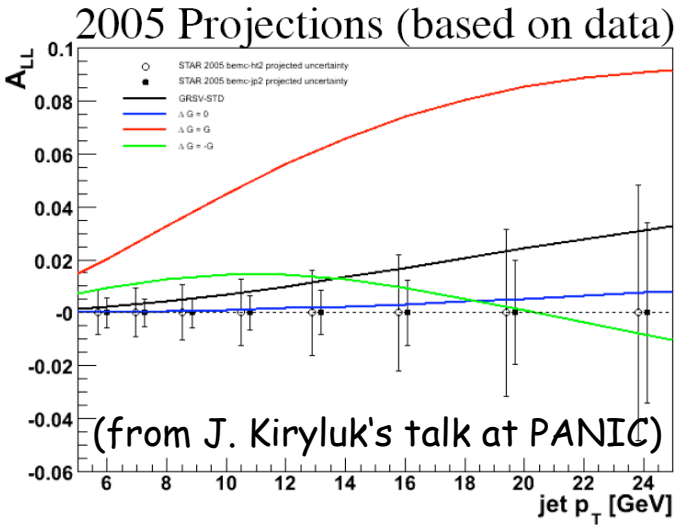
very flat χ^2 distr. around zero

bad news for anomaly aficionados

analysis of new data from RHIC: to-do list

- include STAR jet 05-data once available (very easy to do in Mellin technique)

  → combined constraint on Δg



- systematically check more sophisticated gluons in the fit (one or more nodes & non-standard shapes)

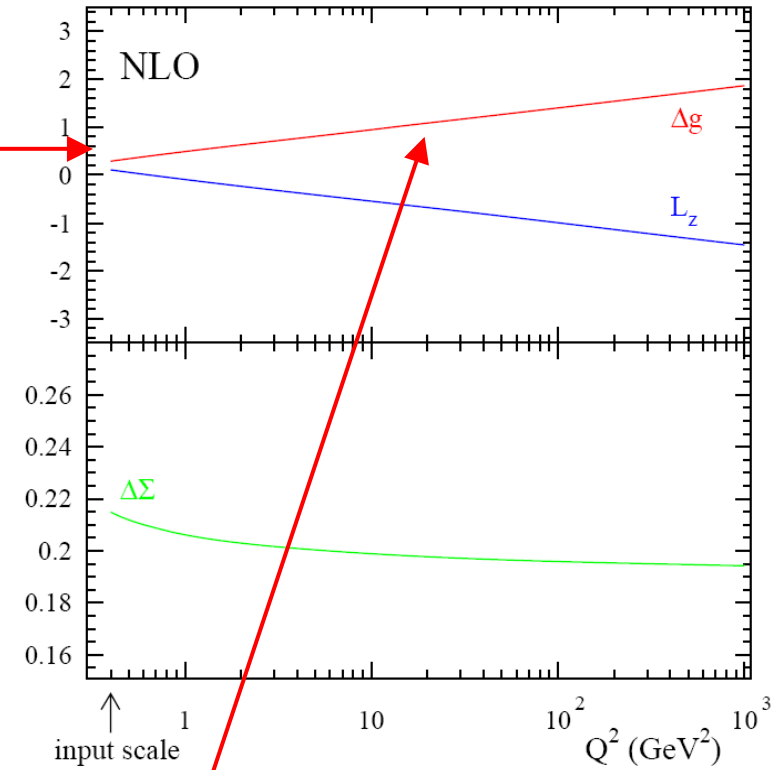
  → how large can Δg be ?

- proper error propagation & error correlations in global analysis (AAC already embarked on a proper CTEQ-like error analysis)

Δg semantics: what does "large" mean?

we always talk about things like "large Δg" - but what does it mean?

we should probably compare with $\frac{1}{2}$ - the spin of the proton



pion data: $-0.5 \lesssim \Delta g(\text{input}) \lesssim 0.5$

still pretty large!!

"anomaly inspired" Δg's are *really* large:

$\Delta g(\text{input}) \gtrsim 1$

beware of pQCD evolution: small Δg can turn asymptotically into large Δg

however, only $J_g = \Delta g + L_g$ is phys. meaningful

Ji

Summary

- exp. results sensitive to Δg start to emerge
- pQCD calculations will now be challenged by experiment

exciting

so – what can stop us now from making rapid progress ?

more stupid budget cuts