Aspects of Quark Orbital Angular Momentum

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Motivation

- spin sum rule

$$\frac{1}{2} = \sum_q \frac{1}{2}\Delta q + L_q + \Delta g + L_g$$

- know that $\sum_q \Delta q$ small ($\longrightarrow$ “spin crisis”) and that $|\Delta g| \leq g$
- How large is L_q?
- today: constraints on L_q from $\kappa \neq 0$

The anomalous magnetic moment

- Matrix element for the Pauli form factor F_2 involves nucleon spin-flip

$$\left\langle P+\Delta,\uparrow \left| \bar{q}\gamma^{+}q \right| P,\downarrow \right\rangle = -\frac{\Delta_x - i\Delta_y}{2M} F_2$$

$\hookrightarrow$ initial and final state differ by one unit of angular momentum

- $\bar{q}\gamma^{+}q$ chirally even (does not flip quark helicity)
- $\bar{q}\gamma^{+}q$ acts only on quarks

$\hookrightarrow$ quark <u>orbital</u> angular momentum (OAM) must change by one unit

The anomalous magnetic moment

- Overlap integral representation (light-cone wave functions) for F_2 requires interference between wave function components that differ by one unit of OAM

$\hookrightarrow$ $\kappa \equiv F_2(0) \neq 0 \quad \Rightarrow \quad$ nucleon must have LC-wave function components with $L_z \neq 0$.

$\hookrightarrow$ LC wave function models $\Rightarrow$ constraints on wave function components with $L_z \neq 0$

- today: attempt to formulate model-independent constraints on L_z

The GPD $E(x,0,0)$

- Matrix element for generalized parton distribution $E_q(x,0,-\mathbf{\Delta}_\perp^2)$ involves nucleon spin-flip ($F_2 = \int dx E$)

$$\langle P{+}\Delta,\uparrow|O_q(x,\mathbf{0}_\perp)|P,\downarrow\rangle = -\frac{\Delta_x - i\Delta_y}{2M} E_q(x,0,-\mathbf{\Delta}_\perp^2)$$

$$O_q(x,\mathbf{b}_\perp) = \int \frac{dx^-}{4\pi} e^{ip^+x^-x} \bar{q}(0^-,\mathbf{b}_\perp)\,\gamma^+ q(x^-,\mathbf{b}_\perp)$$

$\hookrightarrow$ initial and final state differ by one unit of angular momentum

- O_q chirally even (does not flip quark helicity)
- O_q acts only on quarks

$\hookrightarrow$ quark <u>orbital</u> angular momentum (OAM) must change by one unit

The GPD $E(x, 0, 0)$

- Overlap integral representation (light-cone wave functions) for $E_q(x, 0, 0)$ requires interference between wave function components that differ by one unit of OAM (Brodsky, Diehl,..)

$\hookrightarrow$ $E_q(x, 0, 0) \neq 0 \quad \Rightarrow \quad$ nucleon must have LC-wave functions components with $L_z \neq 0$.

$\hookrightarrow$ LC wave function models $\Rightarrow$ constraints on wave function components with $L_z \neq 0$

- today: attempt to formulate model-independent constraints on L_z

OAM decomposition for $E(x,0,0)$

- consider nucleon localized in $\perp$ direction

$$\left|p^+,\mathbf{R}_\perp,\downarrow\right\rangle = \mathcal{N}\int d^2\mathbf{p}_\perp \left|p^+,\mathbf{p}_\perp,\downarrow\right\rangle$$

- introduce quark field operators in impact parameter space
$q(xp^+,\mathbf{b}_\perp,\lambda) = \int dx^- e^{ip^+x^-x} q_+(x^-,\mathbf{b}_\perp,\lambda)$
- convenient representation for $E(x,0,0)$

$$\left\langle p^+,\mathbf{R}_\perp,\uparrow\right| B_q^+(x) \left|p^+,\mathbf{R}_\perp,\downarrow\right\rangle = \frac{1}{2M} E_q(x,0,0).$$

with

$$B_q^+(x) \equiv \sum_\lambda \int d^2\mathbf{b}_\perp \left(b_x + i b_y\right) q^\dagger(xp^+,\mathbf{b}_\perp) q(xp^+,\mathbf{b}_\perp,\lambda)$$

OAM decomposition for $E(x, 0, 0)$

- perform angular decomposition

$$b^\dagger(xp^+, \mathbf{b}_\perp) = \sum_m b_m^\dagger(xp^+, \mathbf{b}_\perp)$$

where

$$b_m^\dagger(xp^+, \mathbf{b}_\perp) = e^{im\phi_b} \int_0^{2\pi} \frac{d\phi_b'}{2\pi} e^{-im\phi_b'} b^\dagger(xp^+, \mathbf{b}_\perp')$$

Polar coordinates $b_x = |\mathbf{b}_\perp| \cos\phi$ and $b_y = |\mathbf{b}_\perp| \sin\phi$, and $b_x' = |\mathbf{b}_\perp| \cos\phi_b'$ and $b_y' = |\mathbf{b}_\perp| \sin\phi_b'$ respectively.

- In this basis, it becomes obvious that B_+^q changes quark OAM by one unit

$$B_+^q = \sum_m \int d^2\mathbf{b}_\perp \, (b_x + ib_y) \, b_{m+1}^\dagger(xp^+, \mathbf{b}_\perp) b_m(xp^+, \mathbf{b}_\perp).$$

OAM decomposition for $E(x,0,0)$

- Apply Cauchy-Schwarz inequality ($\langle\psi|\phi\rangle \leq \sqrt{\langle\psi|\psi\rangle\langle\phi|\phi\rangle}$) to matrix element for $E_q(x,0,0)$

$$\frac{E_q(x,0,0)}{2M} = \sum_m \int d^2\mathbf{b}_\perp \,(b_x+ib_y)\,\langle\uparrow|\, b^\dagger_{m+1}(xp^+,\mathbf{b}_\perp) b_m(xp^+,\mathbf{b}_\perp)\,|\downarrow\rangle$$

$$\leq \sum_m \int d^2\mathbf{b}_\perp \,|\mathbf{b}_\perp|\, \sqrt{\langle\uparrow|\, b^\dagger_{m+1}(xp^+,\mathbf{b}_\perp) b_{m+1}(xp^+,\mathbf{b}_\perp)\,|\uparrow\rangle}$$

$$\times \sqrt{\langle\downarrow|\, b^\dagger_{m}(xp^+,\mathbf{b}_\perp) b_{m}(xp^+,\mathbf{b}_\perp)\,|\downarrow\rangle}$$

- next we apply the Cauchy-Schwarz inequality ($\sum_m a_m b_m \leq \sqrt{\left(\sum_m a_m a_m\right)\left(\sum_n b_n b_n\right)}$ to the integral/sum ....

OAM decomposition for $E(x,0,0)$

- Application of the CS-ineq. to above integral/sum yields

$$\begin{aligned}\frac{E_q(x,0,0)}{2M} &\leq \sum_{m\geq 0}\sqrt{q^{\uparrow}_{m+1}(x)b^{2,\downarrow}_{m}(x)}+\sum_{m<0}\sqrt{b^{2,\uparrow}_{m+1}(x)q^{\downarrow}_{m}(x)}\\ &= 2\sum_{m\geq 0}\sqrt{q^{\uparrow}_{m+1}(x)b^{2,\uparrow}_{-m}(x)}\end{aligned}$$

- distribution of quarks with $L_z=m$ in target with spin $\uparrow$

$$q^{\uparrow}_{m}(x)\equiv\int d^2\mathbf{b}_\perp \left\langle p^+,\mathbf{0}_\perp,\uparrow\right| b^\dagger_m(xp^+,\mathbf{b}_\perp)b_m(xp^+,\mathbf{b}_\perp)\left|p^+,\mathbf{0}_\perp,\uparrow\right\rangle$$

- $\mathbf{b}^2_\perp$ weighted distribution of quarks with $L_z=m$

$$b^{2\,\uparrow}_{\ m}(x)\equiv\int d^2\mathbf{b}_\perp \left\langle p^+,\mathbf{0}_\perp,\uparrow\right| b^\dagger_m(xp^+,\mathbf{b}_\perp)b_m(xp^+,\mathbf{b}_\perp)\left|p^+,\mathbf{0}_\perp,\uparrow\right\rangle \mathbf{b}^2_\perp .$$

OAM decomposition for $E(x,0,0)$

- If one makes assumption that only $m=0$ and $m=1$ contribute then
$$\frac{E_q(x,0,0)}{2M} \leq \sum_{m\geq 0} 2 \sum_{m\geq 0} \sqrt{q^{\uparrow}_{m+1}(x) b^{2,\uparrow}_{-m}(x)}$$
reduces to
$$\left(\frac{E_q(x,0,0)}{4M}\right)^2 \leq q_1^{\uparrow}(x) b_0^{2,\uparrow}(x)$$
- without this assumption, apply CS ineq. again (to the sum), yielding
-
$$\left(\frac{E_q(x,0,0)}{4M}\right)^2 \leq \left(\sum_{m\geq 0} q^{\uparrow}_{m+1}(x)\right)\left(\sum_{n\geq 0} b^{2,\uparrow}_{-n}(x)\right) = q_{L_z\geq 1}(x)\, b^2_{L_z\leq 0}(x).$$

Discussion

- Novel lower bound on wave function components with $L_z \neq 0$

$$\left(\frac{E_q(x,0,0)}{4M}\right)^2 \leq q_{L_z\geq 1}(x)\, b^2_{L_z\leq 0}(x).$$

$\hookrightarrow$ point-like nucleons ($b^2 \to 0$) cannot have a nonvanishing anomalous magnetic moment (see also Brodsky & Schlumpf)

- depending on the $\perp$ size, can make prediction for $L_z \neq 0$

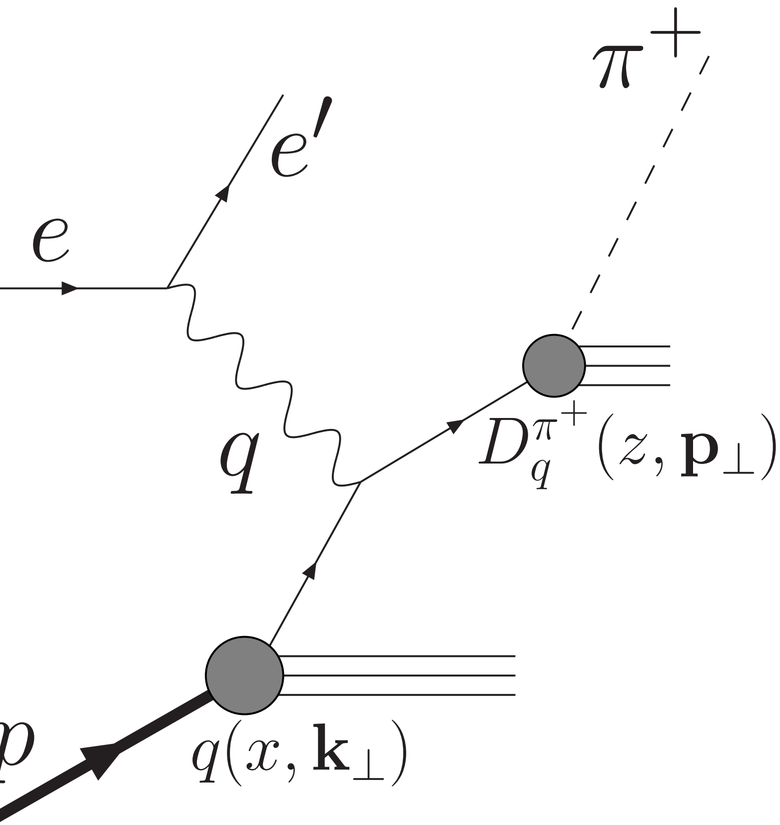
$$\frac{q_{L_z\geq 1}(x)}{q(x)} \geq \text{few}\,\%$$

- $\kappa \neq 0$ and $q_{L_z\geq 1}(x) = 0$ only possible for $b^2 \to \infty$
- numerical size of lower bound disappointlingly small even though κ is large, since κ arises from s-p interference (linear in p-wave component), while $q_{L_z=1}(x)$ involves the square of the p-wave component

Discussion

- $\hookrightarrow$ compare: tiny d-wave component in deuterium leads to relatively large quadrupole moment.
- Still possible: maybe stronger inequalities can be derived is CS-ineq. is applied less often...

How about SSAs?



- use factorization (high energies) to express momentum distribution of outgoing π^+ as convolution of
  - momentum distribution of quarks in nucleon

  $\hookrightarrow$ unintegrated parton density $q(x, \mathbf{k}_\perp)$
  - momentum distribution of π^+ in jet created by leading quark q

  $\hookrightarrow$ fragmentation function $D_q^{\pi^+}(z, \mathbf{p}_\perp)$

- average $\perp$ momentum of pions obtained as sum of
  - average $\mathbf{k}_\perp$ of quarks in nucleon (Sivers effect)
  - average $\mathbf{p}_\perp$ of pions in quark-jet (Collins effect)

How about SSAs?

- situation for SSA very similar:
  - described by matrix element of chirally even operator
  - nucleon spin flip
- same wave function components that enter overlap representation for κ also enter in overlap rep. for SSA (Brodsky, Hwang, Schmidt)

$$q(x,\mathbf{k}_\perp) \propto \int \frac{d\xi^- d^2\xi_\perp}{(2\pi)^3} e^{ip\cdot\xi} \left\langle P,S \left| \bar{q}(0) U_{[0,\infty]} \gamma^+ U_{[\infty,\xi]} q(\xi) \right| P,S \right\rangle \Big|_{\xi^+=0}$$

with $U_{[0,\infty]} = P\exp\left(ig\int_0^\infty d\eta^- A^+(\eta)\right)$

- does measurement of nonzero Sivers effect (HERMES) also provide rigorous constraint on quark OAM?

How about SSAs?

$$q(x,\mathbf{k}_\perp) \propto \int \frac{d\xi^- d^2\xi_\perp}{(2\pi)^3} e^{ip\cdot\xi} \left\langle P,S \left| \bar{q}(0) U_{[0,\infty]} U_{[\infty,\infty]} \gamma^+ U_{[\infty,\xi]} q(\xi) \right| P,S \right\rangle \Big|_{\xi^+=0}$$

with $U_{[0,\infty]} = P\exp\left(ig\int_0^\infty d\eta^- A^+(\eta)\right)$

- operator describing SSA involves acts on both quarks and glue
- $\hookrightarrow$ either quark OAM or gluon OAM (or both) can change in the matrix element
- $\hookrightarrow$ SSA can arise from interference between wave function components differing in either L_q or L_g (or both)
- Even though model builders agree that Sivers requires some form of quark OAM, unable to establish rigorous bound on L_q

How about SSAs?

$$q(x,\mathbf{k}_\perp) \propto \int \frac{d\xi^- d^2\xi_\perp}{(2\pi)^3} e^{ip\cdot\xi} \left\langle P,S \left| \bar{q}(0) U_{[0,\infty]} U_{[\infty,\infty]} \gamma^+ U_{[\infty,\xi]} q(\xi) \right| P,S \right\rangle \Big|_{\xi^+=0}$$

with $U_{[0,\infty]} = P\exp\left(ig\int_0^\infty d\eta^- A^+(\eta)\right)$

- However, if FSI is treated in perturbation theory (Brodsky, Hwang, Schmidt), A^μ expressed in terms of color charge density of quarks

$\hookrightarrow$ SSA expressed in color-density-density correlations in $\perp$ plane (MB, hep-ph/0311013)

$\hookrightarrow$ apply to valence wave functions

$\hookrightarrow$ again find that $L_q \neq 0$ is essential

Summary

- $\kappa \neq 0$ requires wave function components with $L_q \neq 0$
- derived model independent lower bound on distribution of quarks with $L_z \neq 0$

$$\left(\frac{E_q(x,0,0)}{4M} \right)^2 \leq q_{L_z \geq 1}(x)\, b^2_{L_z \leq 0}(x).$$

- no rigorous statement on <u>net</u> L_q
- nonzero Sivers (HERMES) proves that there must be nonzero wave function components with $L_q + L_g \neq 0$
- However, strictly speaking, no rigorous bound on L_q alone from Sivers
- for those interested in even more details ...
  MB + G.Schnell, hep-ph/0510249.

OAM decomposition for $E(x,0,0)$

- Apply Cauchy-Schwarz inequality to matrix element for $E_q(x,0,0)$

$$
\begin{aligned}
\frac{E_q(x,0,0)}{2M} &= \sum_m \int d^2\mathbf{b}_\perp \, (b_x + i b_y) \left\langle \uparrow \right| b^\dagger_{m+1}(xp^+,\mathbf{b}_\perp) b_m(xp^+,\mathbf{b}_\perp) \left| \downarrow \right\rangle \\
&\le \sum_m \int d^2\mathbf{b}_\perp \, |\mathbf{b}_\perp| \sqrt{\left\langle \uparrow \right| b^\dagger_{m+1}(xp^+,\mathbf{b}_\perp) b_{m+1}(xp^+,\mathbf{b}_\perp) \left| \uparrow \right\rangle} \\
&\qquad \times \sqrt{\left\langle \downarrow \right| b^\dagger_{m}(xp^+,\mathbf{b}_\perp) b_{m}(xp^+,\mathbf{b}_\perp) \left| \downarrow \right\rangle}
\end{aligned}
$$

- next we apply the Cauchy-Schwarz inequality to the integral/sum ....

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$$
\begin{aligned}
\frac{E_q(x,0,0)}{2M} &= \int d^2\mathbf{b}_\perp \, (b_x + i b_y) \left\langle \uparrow \right| b_1^\dagger(xp^+,\mathbf{b}_\perp) b_0(xp^+,\mathbf{b}_\perp) \left| \downarrow \right\rangle + ... \\
&\leq \int d^2\mathbf{b}_\perp \, |\mathbf{b}_\perp| \sqrt{\left\langle \uparrow \right| b_1^\dagger(xp^+,\mathbf{b}_\perp) b_1(xp^+,\mathbf{b}_\perp) \left| \uparrow \right\rangle} \\
&\qquad\qquad\qquad \times \sqrt{\left\langle \downarrow \right| b_0^\dagger(xp^+,\mathbf{b}_\perp) b_0(xp^+,\mathbf{b}_\perp) \left| \downarrow \right\rangle} + ...
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$$
\begin{aligned}
\frac{E_q(x,0,0)}{2M} &= \int d^2\mathbf{b}_\perp \, (b_x + i b_y) \left\langle \uparrow \right| b_0^\dagger(xp^+,\mathbf{b}_\perp) b_{-1}(xp^+,\mathbf{b}_\perp) \left| \downarrow \right\rangle + ... \\
&\leq \int d^2\mathbf{b}_\perp \, |\mathbf{b}_\perp| \sqrt{\left\langle \uparrow \right| b_0^\dagger(xp^+,\mathbf{b}_\perp) b_0(xp^+,\mathbf{b}_\perp) \left| \uparrow \right\rangle} \\
&\qquad\qquad\qquad \times \sqrt{\left\langle \downarrow \right| b_{-1}^\dagger(xp^+,\mathbf{b}_\perp) b_{-1}(xp^+,\mathbf{b}_\perp) \left| \downarrow \right\rangle} + ...
\end{aligned}
$$

- next we apply the Cauchy-Schwarz inequality to the integral/sum ....

OAM decomposition for $E(x,0,0)$

- Apply Cauchy-Schwarz inequality to matrix element for $E_q(x,0,0)$

$$\frac{E_q(x,0,0)}{2M} = \int d^2\mathbf{b}_\perp \, (b_x + i b_y) \, \langle\uparrow| \, b_2^\dagger(xp^+,\mathbf{b}_\perp) b_1(xp^+,\mathbf{b}_\perp) \, |\downarrow\rangle + ...$$

$$\leq \int d^2\mathbf{b}_\perp \, |\mathbf{b}_\perp| \sqrt{\langle\uparrow| \, b_2^\dagger(xp^+,\mathbf{b}_\perp) b_2(xp^+,\mathbf{b}_\perp) \, |\uparrow\rangle}$$

$$\times \sqrt{\langle\downarrow| \, b_1^\dagger(xp^+,\mathbf{b}_\perp) b_1(xp^+,\mathbf{b}_\perp) \, |\downarrow\rangle} + ...$$

- next we apply the Cauchy-Schwarz inequality to the integral/sum ....

OAM decomposition for $E(x,0,0)$

- Apply Cauchy-Schwarz inequality to matrix element for $E_q(x,0,0)$

$$\frac{E_q(x,0,0)}{2M} = \int d^2\mathbf{b}_\perp \, (b_x + i b_y) \left\langle \uparrow \right| b^\dagger_{-1}(xp^+, \mathbf{b}_\perp) b_{-2}(xp^+, \mathbf{b}_\perp) \left| \downarrow \right\rangle + ...$$

$$\leq \int d^2\mathbf{b}_\perp \, |\mathbf{b}_\perp| \sqrt{\left\langle \uparrow \right| b^\dagger_{-1}(xp^+, \mathbf{b}_\perp) b_{-1}(xp^+, \mathbf{b}_\perp) \left| \uparrow \right\rangle}$$

$$\times \sqrt{\left\langle \downarrow \right| b^\dagger_{-2}(xp^+, \mathbf{b}_\perp) b_{-2}(xp^+, \mathbf{b}_\perp) \left| \downarrow \right\rangle} + ...$$

- next we apply the Cauchy-Schwarz inequality to the integral/sum ....