What exactly are parton densities? Are they universal?

John Collins (Penn State)

[+ Andreas Metz (Bochum), in PRL 93, 252001 (2004)]

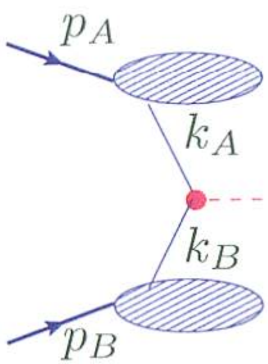
Why pdfs matter?

- Factorization:

$$\sigma \simeq \text{non-pert. universal factors} \otimes \text{hard scattering}$$

pdfs etc: Universal $\Longrightarrow$ measure then use (with DGLAP, perturbative)
Perturbative $\Longrightarrow$ approx. predictable from first principles

- Critical to much HEP
- E.g., Higgs at LHC:

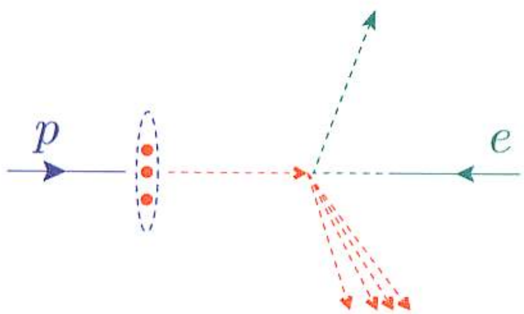


Issues

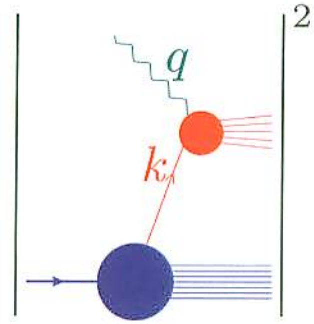
- Use of *exactly* on-shell partons.
- Actual IR/collinear divergences appear vital to methods/justification of NLO etc
- Summary of this talk:
  - Parton model justification of pdf
  - QCD complications $\Longrightarrow$ Wilson lines and non-universality
  - Spin & Sivers function
  - Implications
- Focus on unintegrated pdfs, $f(x, k_T)$, not NLO, NNLO etc

Why pdf? (Basic idea)

- Hard scattering in DIS



$\Longrightarrow$ (?)



- Light-front coordinates: $k^{\pm} = (k^0 \pm k^z)/\sqrt{2}$:

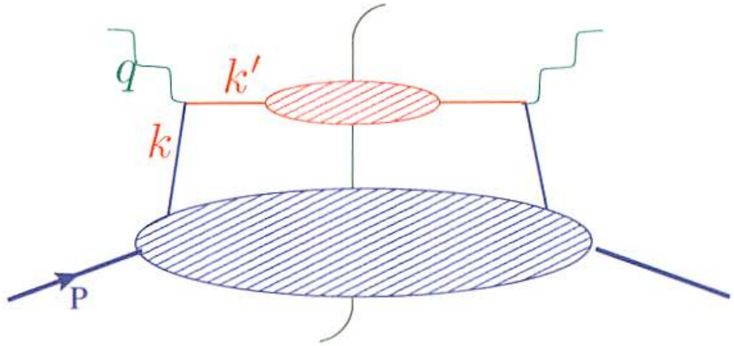
$$k^\mu = (k^+,\, k^-,\, k_T)$$

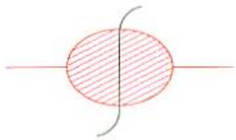
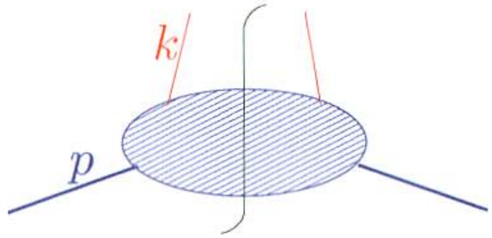
$$q^\mu = \left(-xp^+,\, \frac{Q^2}{2xp^+},\, 0_T\right) \qquad \text{(Brickwall/Breit frame)}$$

$$k'^\mu = k^\mu + q^\mu = \left(k^+ - xp^+,\, \frac{Q^2}{2xp^+} + \text{small, small}\right)$$

- Small k' virtuality $\Longrightarrow$ small $k'^+ \Longrightarrow k^+ \simeq xp^+$

- Short circuit integrals over small components (& take large Dirac matrices):

$$F_2 \simeq \int \frac{d^4k}{(2\pi)^4}$$ 

$\simeq$ vertex $\times \int dk'^{+}$  $\times \int dk^{-}\, d^2k_T$ 

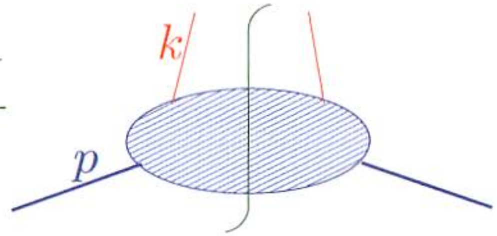
$=$ (LO) on-shell $eq \to eq$ $\times$ pdf

- In hard scattering (top of graph), use approximated k:

$$k^{\mu} \simeq \left(xp^{+},\, 0,\, 0_T\right), \qquad k^{\mu} + q^{\mu} \simeq \left(0,\, Q^2/(2xp^{+}),\, 0_T\right)$$

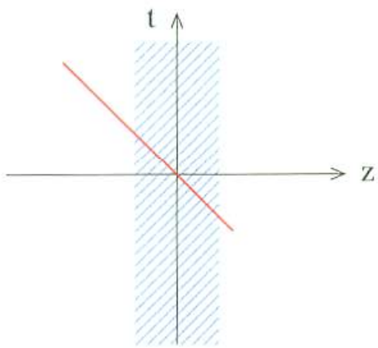
Pdf definition: candidate 1

- Main definition: $f(x) = \int \frac{\mathrm{d}k^- \, \mathrm{d}^2 k_T}{(2\pi)^4} \, \mathrm{Tr} \, \frac{\gamma^+}{2}$



$$= \int \mathrm{d}y^- \, e^{-ixp^+y^-} \, \langle p | \, \bar{\psi}(0, y^-, 0_T) \, \frac{\gamma^+}{2} \, \psi(0) \, | p \rangle$$

- Proton rest frame:



- Light front quantization $\Longrightarrow$ number density $f(x) = \int \mathrm{d}^2 k_T \, \langle p | a_k^\dagger a_k | p \rangle \, / \, \langle p | p \rangle$

$\Longrightarrow$ Unintegrated density:

$$P(x, k_T) = \langle p | a_k^\dagger a_k | p \rangle \, / \, \langle p | p \rangle$$

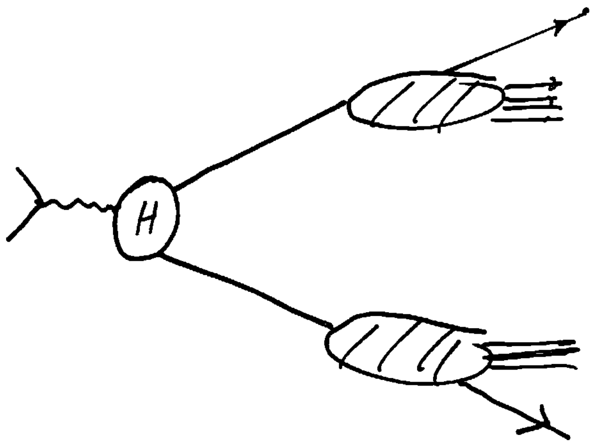
$$= \int \mathrm{d}y^- \, \mathrm{d}^2 y_T \, e^{-ixp^+y^- + ik_T \cdot y_T} \, \langle p | \, \bar{\psi}(0, y^-, y_T) \, \frac{\gamma^+}{2} \, \psi(0) \, | p \rangle$$

But:

- Gauge invariance?
- Other regions, extra gluon exchanges
- Solution:
  - Add up gluons (Ward identities)
  - Obtain Wilson lines in operator definitions
- [UV divergence in integrated pdf and corrections to hard scattering]

Regions for leading power

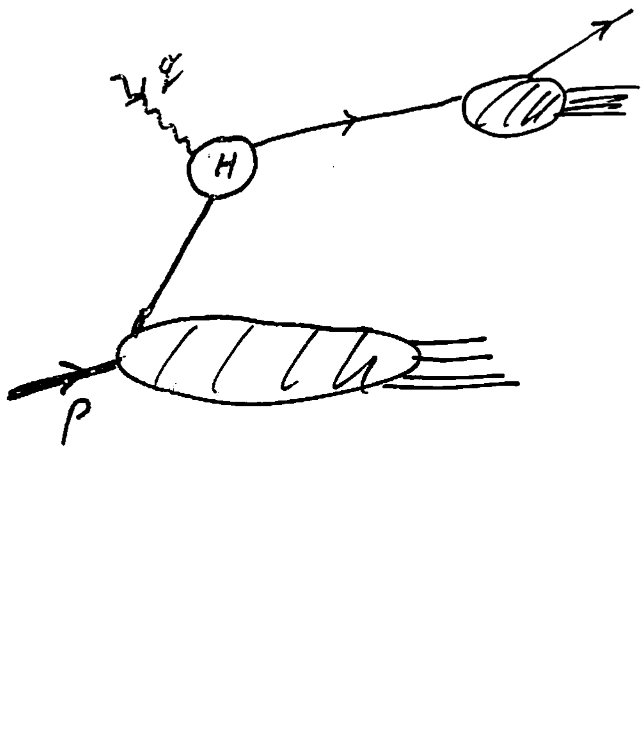
e^+e^-:



B

A

DIS

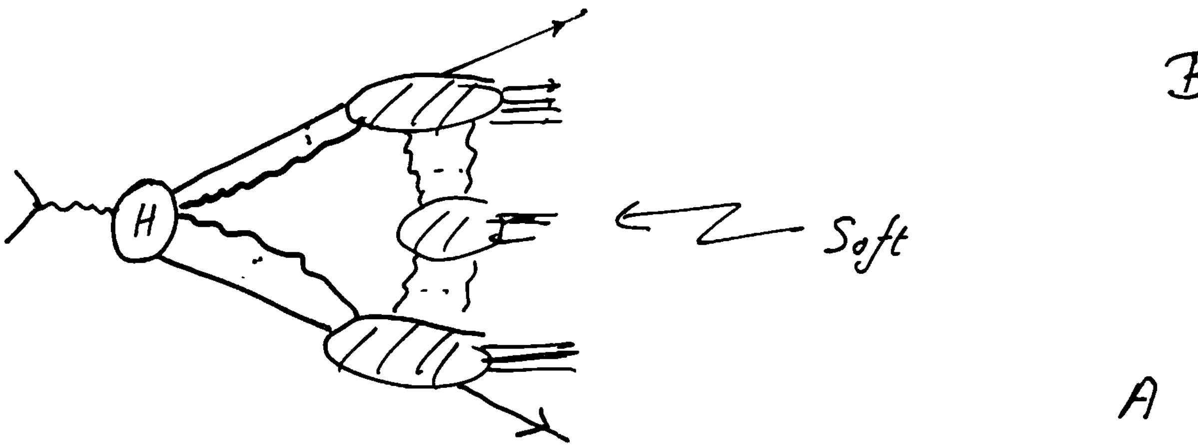


B

A

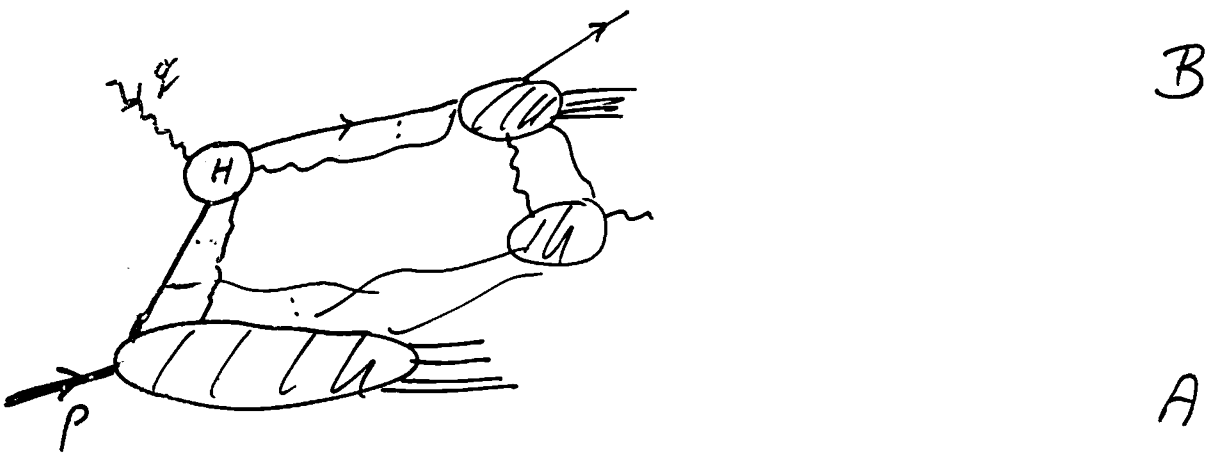
Regions for leading power

e^+e^-:



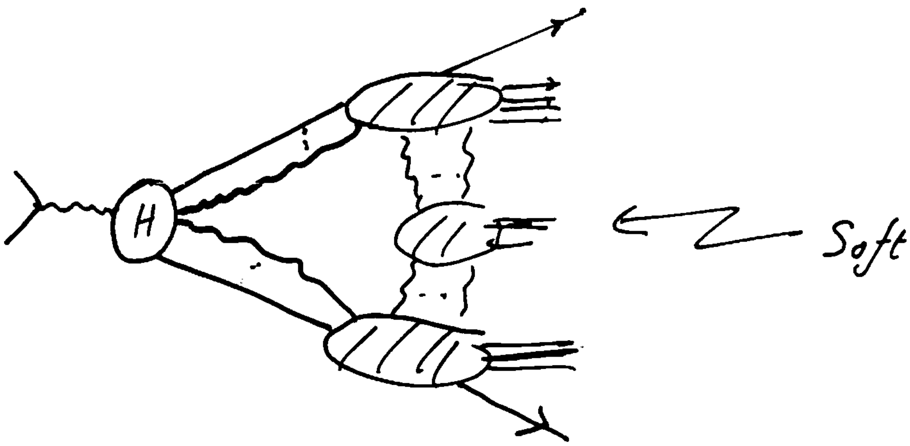
$\Rightarrow \begin{cases} \text{Extra: soft} \\ \qquad\quad \text{collinear to hard} \\ \text{Also high } q_T \text{ \& extra jets} \end{cases}$

DIS



Regions for leading power

(e^+e^-:)

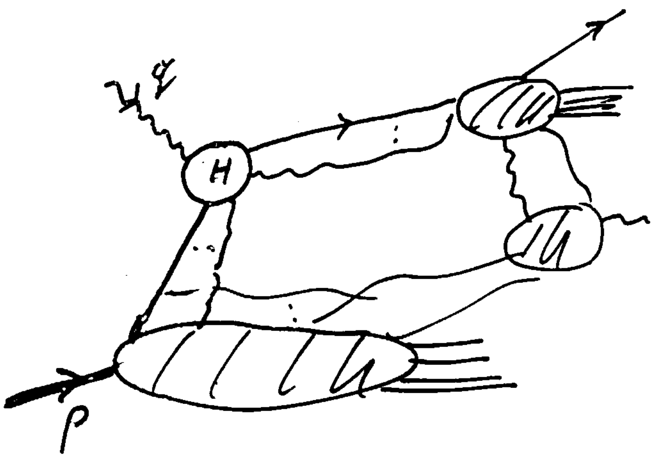


B

A

$\Rightarrow$ Extra: soft ← (f.s.)
collinear to hard
Also high q_T & extra jets

(DIS)

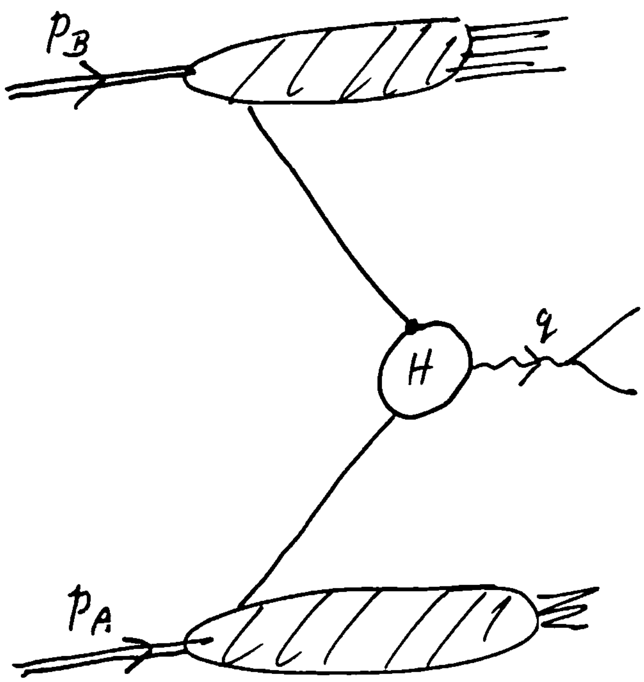


B

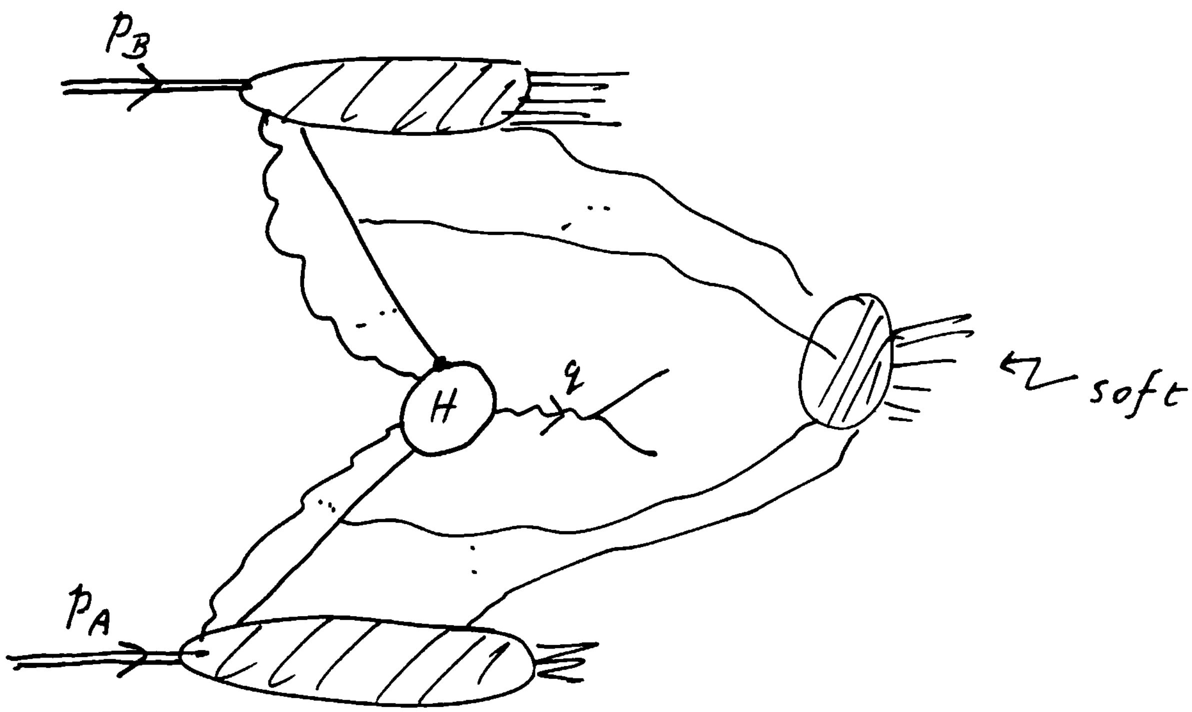
A

Soft is: f.s. for jet
i.s. & f.s. for target

Drell-Yan



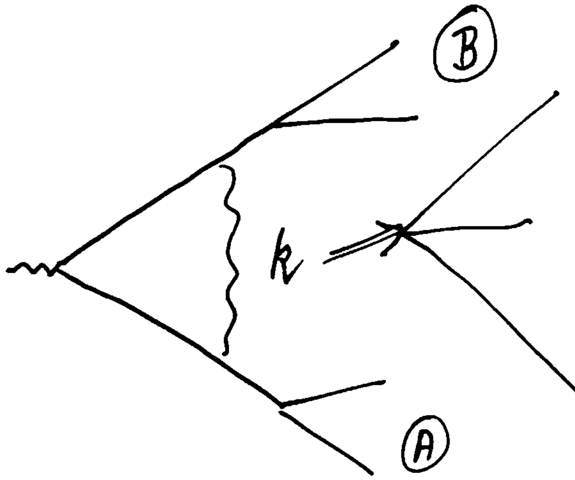
Drell-Yan



Soft: c.s. & f.s. on A & B sides.

– Issues: • Extra high q_T partons $\Rightarrow$ NLO in hard sc. &c.
Not today

$\Longrightarrow$ • Extra gluon exchanges: kinematics, ...



$y \sim y_B$ $\longrightarrow$ collinear to B

$y \sim 0$ or $k^\pm \ll Q$ $\longrightarrow$ soft

& $|k^+ k^-| \ll k_T^2$ $\longrightarrow$ soft & 'Glauber'

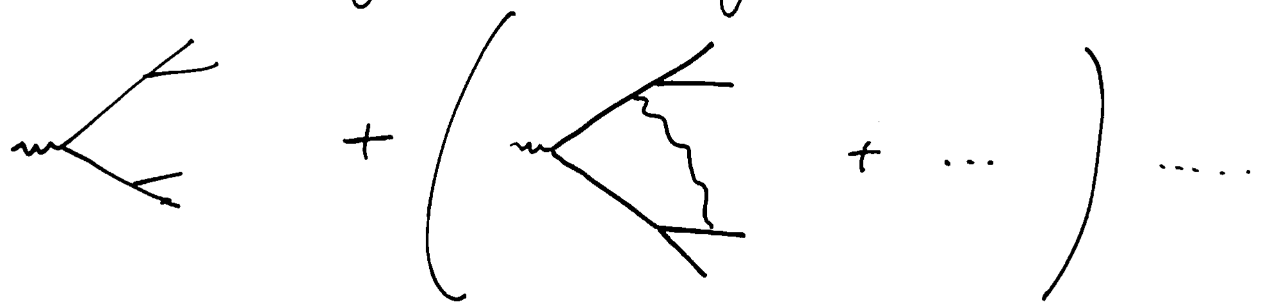
$y \sim y_A$ $\longrightarrow$ collinear to A

– Previous work: • cancellation in std. processes

• CSS &c for k_T-dep^t processes

• Proof lacking for D-Y w/ k_T-pdfs.

Examine 1-gluon exchange (JCC + Metz)



$\oplus$ real glue

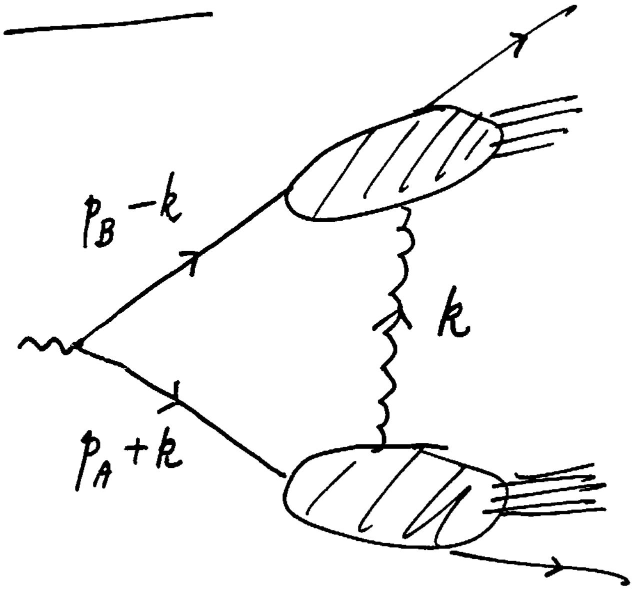
$$A \otimes B \otimes S \sim (1 + \alpha_s A_1 + \dots) \otimes (1 + \alpha_s B_1 + \dots) \times (1 + \alpha_s S_1 + \dots)$$

$$= 1 + \alpha_s (A_1 + B_1 + S_1) + O(\alpha_s^2).$$

$\Longrightarrow$ sum of terms at this order.

Generalization expected.

e^+e^-



(Treat only $k_T \sim m$)

Denoms.:

$$\frac{1}{2k^+k^- - k_T^2 + i\varepsilon} \times \left(\frac{1}{-2p_B^- k^+ + \cdots + i\varepsilon} \times \cdots \right) \times \left(\frac{1}{2p_A^+ k^- + \cdots + i\varepsilon} \times \cdots \right)$$

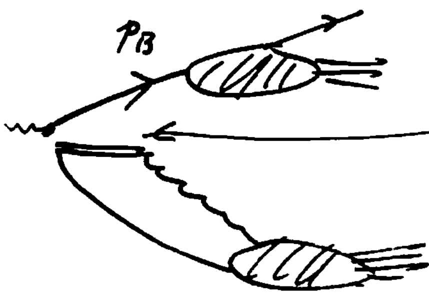
Num.: $B^- A^+$ dominates.

WI: k collinear to $A \Longrightarrow k^+ \sim Q$

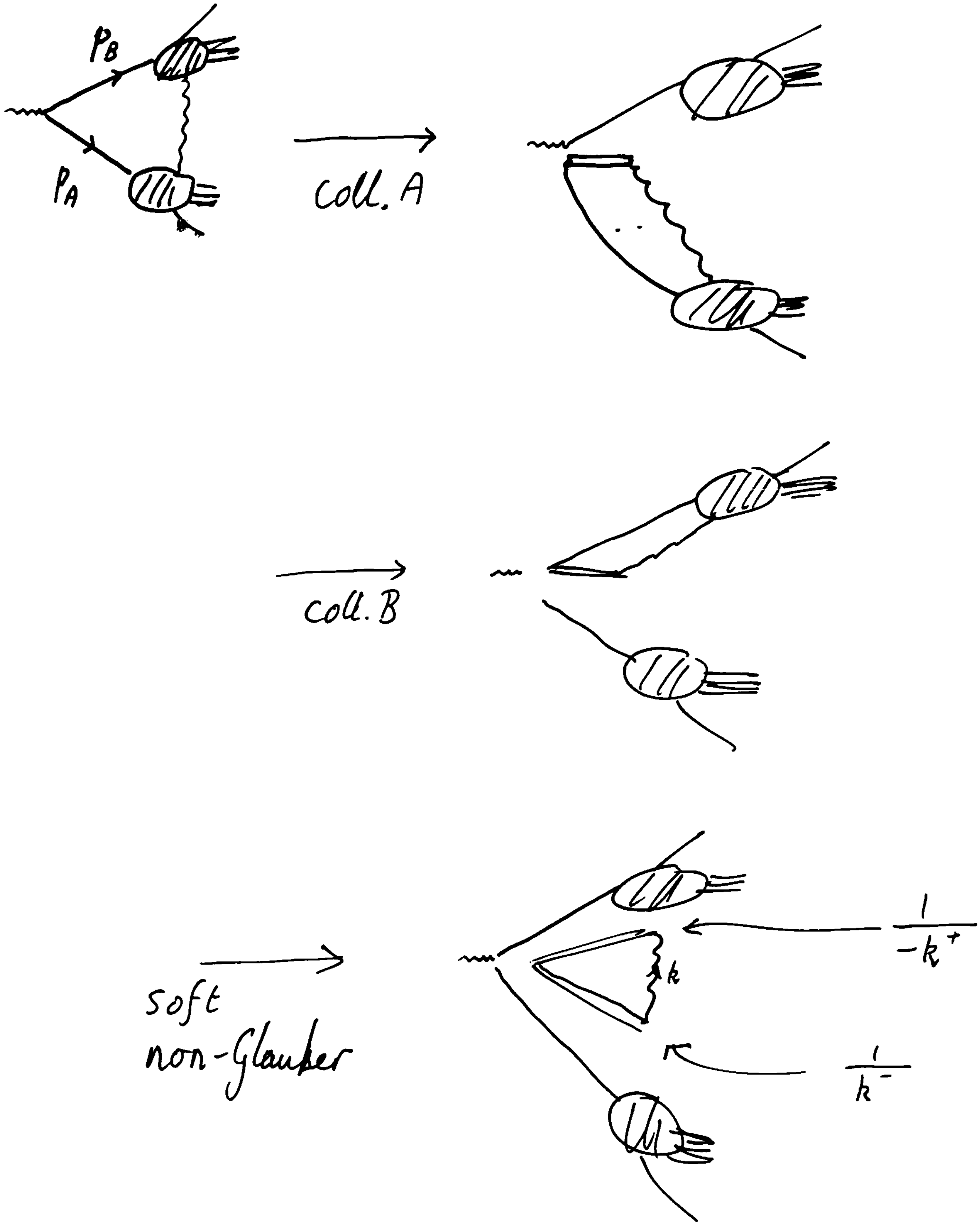
$$\Longrightarrow B^-(k; p_B) \simeq B^-(k^+, 0, 0_T; p_B)$$

$$= k^+ B^-(k^+, 0, 0_T) \quad \frac{1}{k^+ + ?\, i\varepsilon}$$

$\overrightarrow{\text{WI}}$

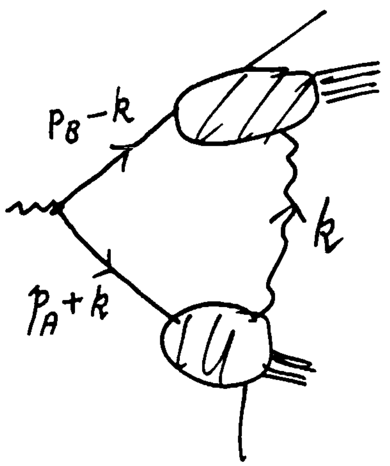


Wilson line



p_B
p_A
coll. A
coll. B
soft
non-Glauber
k
$\frac{1}{-k^+}$
$\frac{1}{k^-}$

Glauber v. non-Glauber soft:



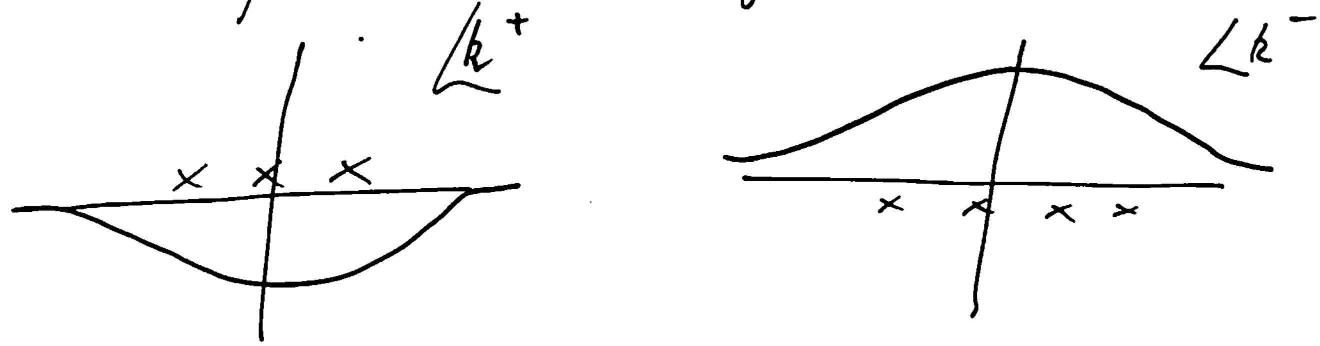
$$\frac{1}{2k^+k^- - k_T^2 + i\varepsilon} \qquad \frac{1}{-2p_B^- k^+ + k_T\cdot m + i\varepsilon} \qquad \frac{1}{2p_A^+ k^- + k_T\cdot m + i\varepsilon}$$

$k^+ \sim k^- \sim k_T \sim m \Rightarrow p_B^- k^+ \,\&\, p_A^+ k^- \sim Qm \gg m^2$

$\Rightarrow$ WI argument OK

$$\boxed{\text{Glauber: } |k^+k^-| \ll k_T^2 \Rightarrow p_B^- k^+,\ p_A^- k^+ \text{ not large.}}$$

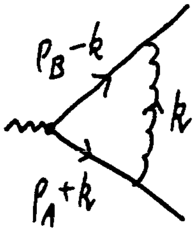
Contour deformation out of Glauber



$\Rightarrow$ need $i\varepsilon$ in WI denoms

$\Rightarrow$ Wilson lines go to future

Double-counting, region overlap: subtractions

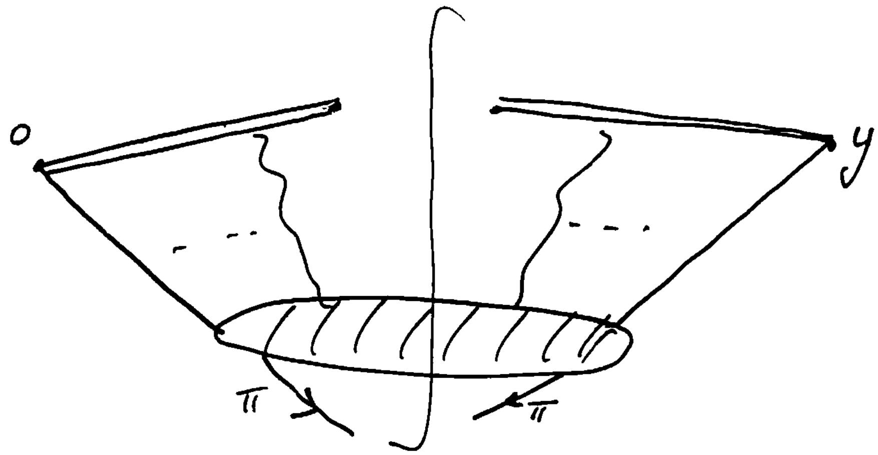


$$
\frac{1}{k^2}\left\{ \frac{1}{-k^+ + i\varepsilon} \, \frac{1}{k^- + i\varepsilon} - (\mathrm{ct}\ y \to +\infty) - (\mathrm{ct}\ y \to -\infty) \right.
$$

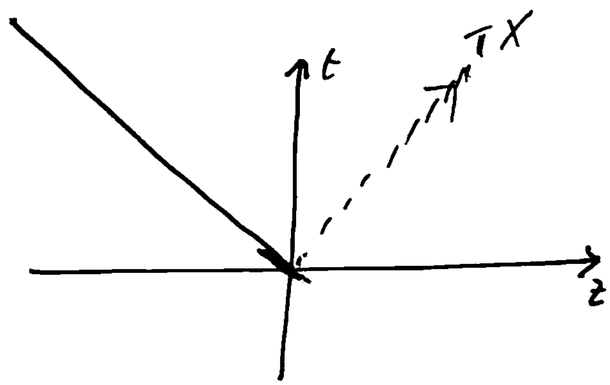
$$
+ \frac{1}{-k^+ + i\varepsilon} \, \frac{2p_A^+}{(p_A+k)^2 - m^2 + i\varepsilon} - \frac{1}{-k^+ + i\varepsilon} \, \frac{1}{+k^- + i\varepsilon} + (\mathrm{ct}\ y \to -\infty)
$$

$$
\left. + \frac{2p_B^-}{(p_B-k)^2 - m^2 + i\varepsilon} \, \frac{1}{k^- + i\varepsilon} - \frac{1}{-k^+ + i\varepsilon} \, \frac{1}{k^- + i\varepsilon} + (\mathrm{ct}\ y \to +\infty) \right\}
$$

Wilson lines & fragmentation fn.



$$\text{FT \& tr.} \sum_X \langle 0 | \, e^{ig\int_y^\infty A\cdot dz} \, \psi(0,y^-,y^T) \, |\pi X\rangle \langle \pi X | \, \bar\psi(0) \, e^{-ig\int_0^\infty A\cdot dz} \, |0\rangle$$



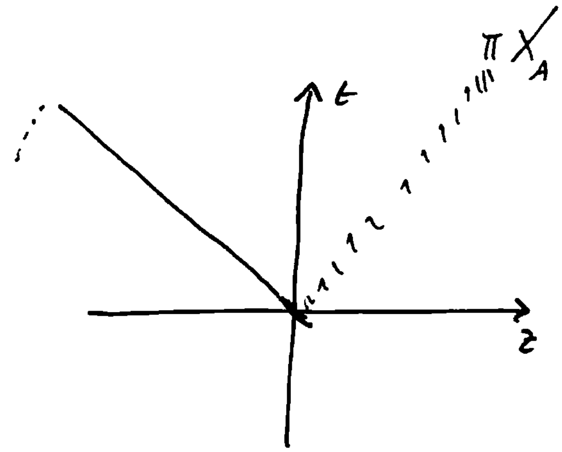
With counterterm to remove $y(\text{gluons}) \to -\infty$ dg ce

w/ effective cutoff y_A.

Or space-like WL $\quad \dfrac{1}{-k^+ + \varepsilon_A k^- + i\varepsilon}$ & c

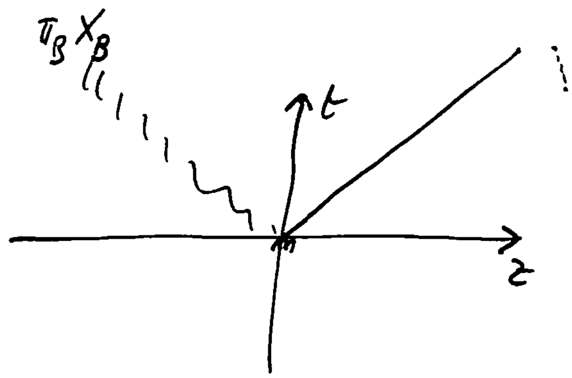
Factors in CSS

Frag. fn A



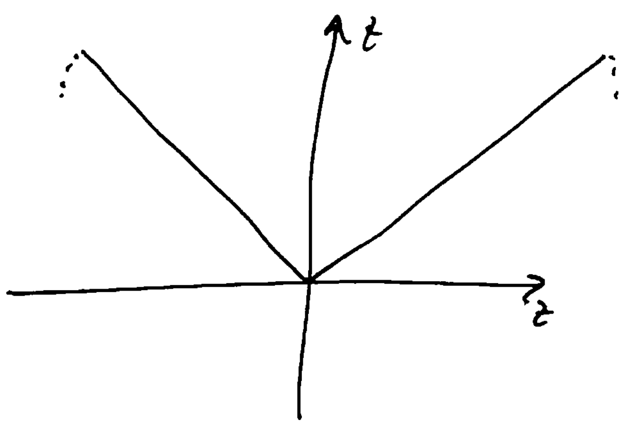
Remove $y \to -\infty$ dgc
Divide by soft facto

Frag. fn B



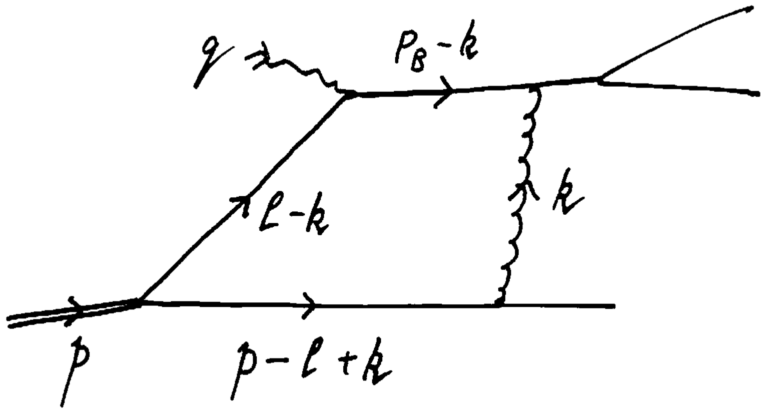
Remove $y \to +\infty$ dgcc
Divide by soft factor

Soft



Remove $y \to \pm\infty$ dgees.

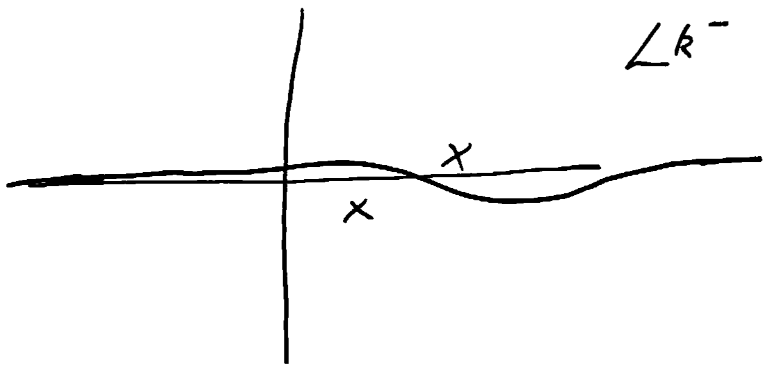
SI DIS example

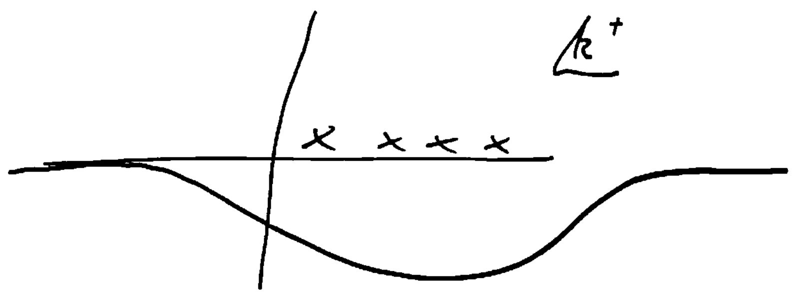


$$\frac{1}{-2p_B^- k^+ + \dots + i\varepsilon} \qquad \frac{1}{-2\ell^+ k^- + \dots + i\varepsilon} \qquad \frac{1}{2(p^+ - \ell^+)k^- + \dots + i\varepsilon}$$

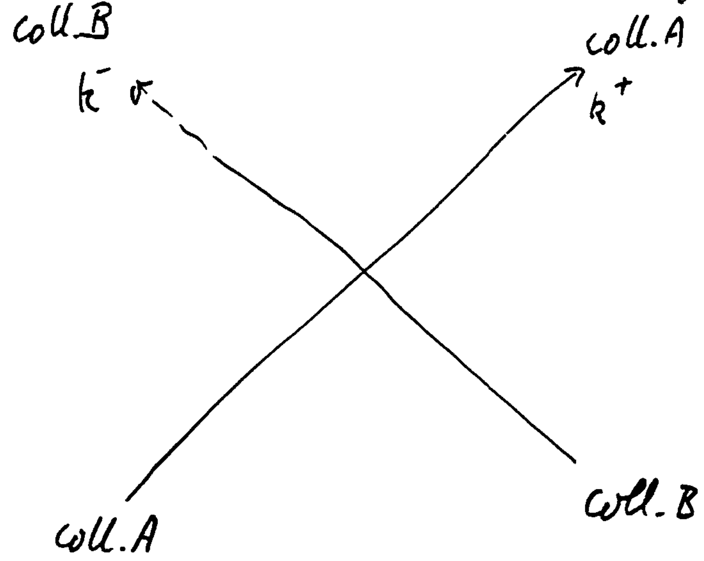
FS IS FS

Trap/pinch in k^-
(Glauber region)

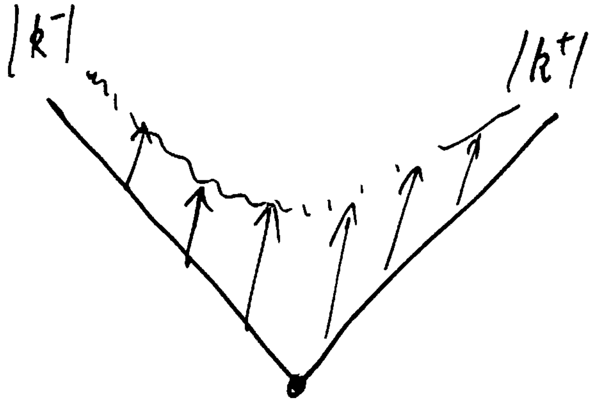




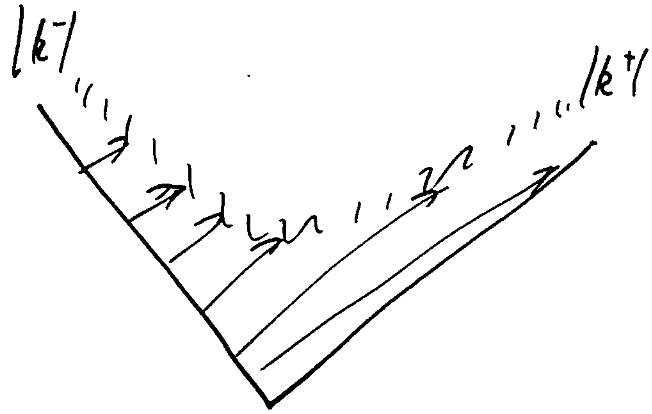
Choice or not for deGlaubering



e^+e^-

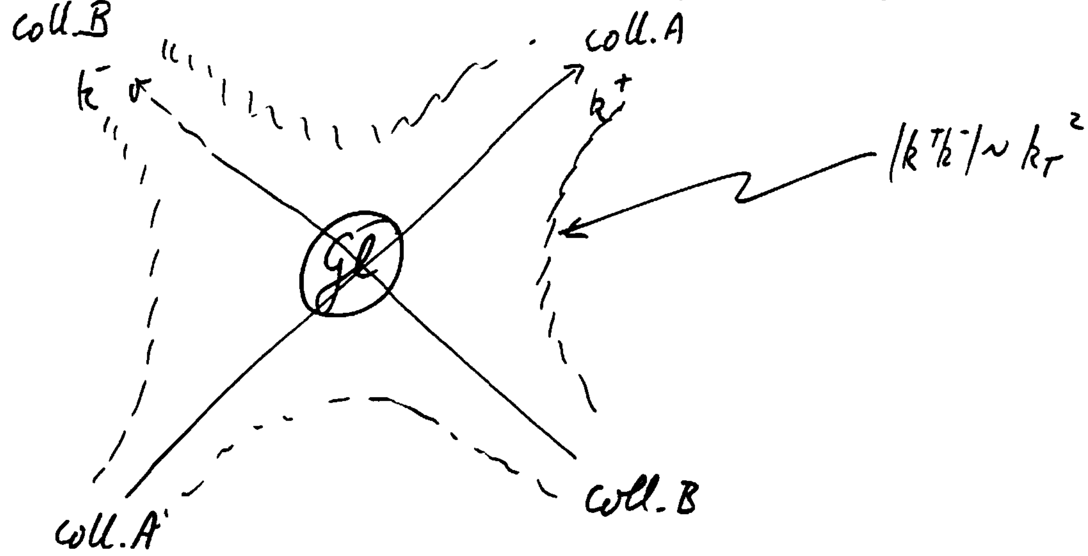


DIS

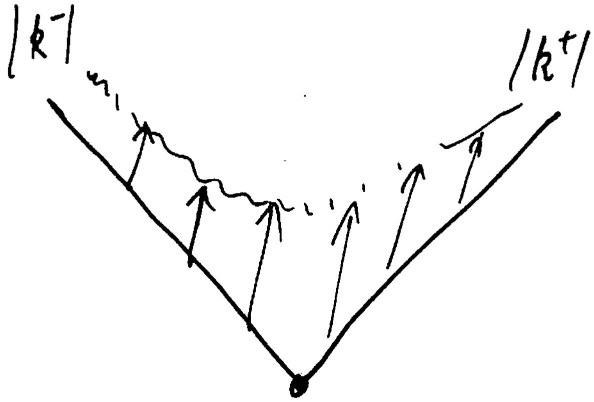


Deform k^+ only.

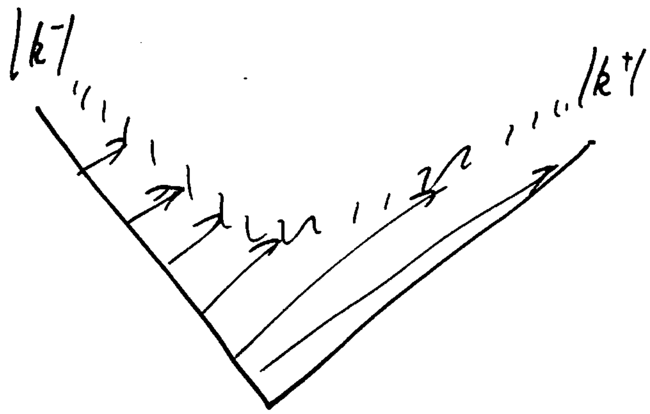
Choice or not for deGlaubering



e^+e^-



DIS



Deform k^+ only.

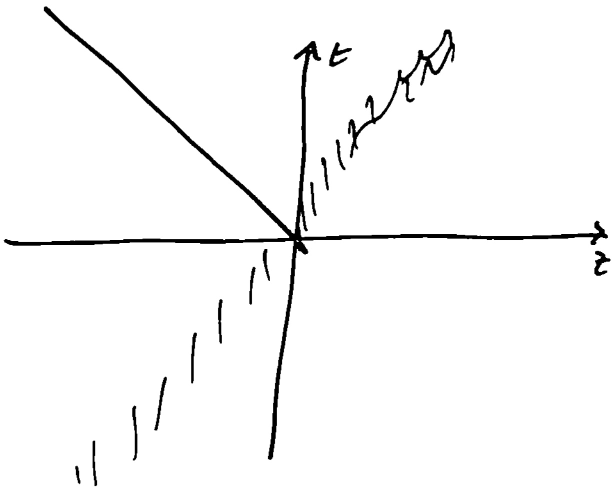
Result

1. Keep frag. fn. same as in e^+e^- : universality!

2. Define pdf with <u>final</u>-state Wilson line

$$\text{F.T. } \langle p | \bar{\psi}(0, y^-, y_T) \, e^{ig\int A\cdot dz} \frac{\gamma^+}{2} \psi(0) | p \rangle$$

& $y \to -\infty$ gluon c.t. (y_A)

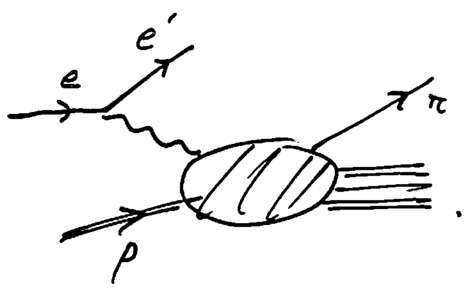


Wilson line:
- approximates outgoing colored quark & its interaction w/ glue in p
- regulator & c.t. : space-like.

CSS factorization

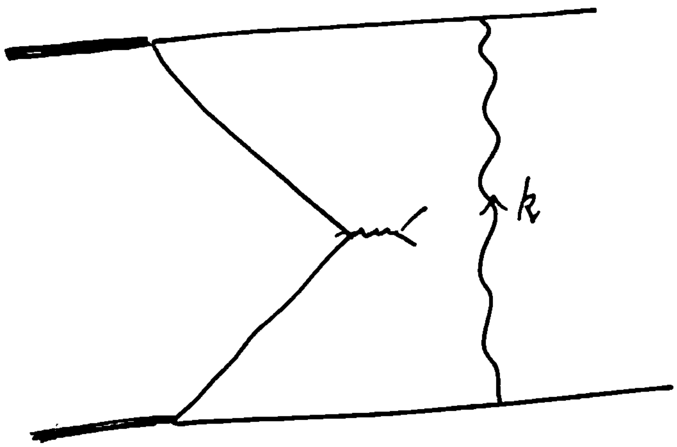
$$\underline{d\sigma (ep \to e\pi X)} = \int H \; f(x, k_{AT}; y_A) \; d(z, k_{BT}; y_B) \; S(q_T - k_{AT} - k_{BT}; y_A, y_B)$$

- - -

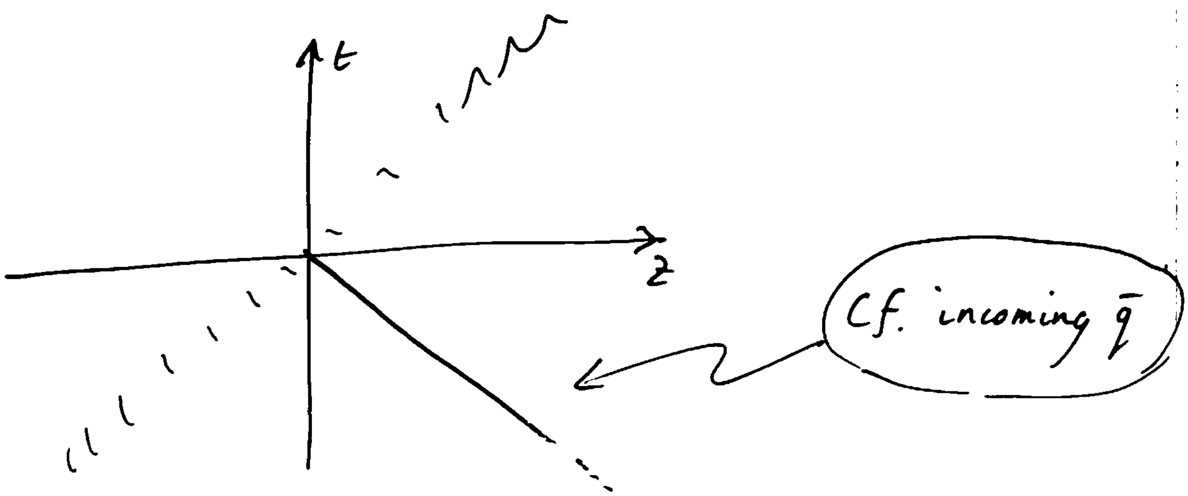


- $\dfrac{d}{dy_A} f = \text{kernel} \times f$
- Etc
- RGEs
- expansion of f & d for large k_T.

Drell-Yan



- Pinch of both k^+ & k^- between i.s. & f.s.
- Inclusive $\Rightarrow$ cancellation of fs poles (CSS)
- $\therefore$ deform as if i.s. poles only
- $\therefore$ pdfs with i.s. Wilson lines



Modified pdf universality

$$\langle p \mid op_{DIS} \mid p \rangle \underset{P\,T}{=\!=} \langle p \mid op_{DY} \mid p \rangle$$

Unpolarized pdfs: same universality OK

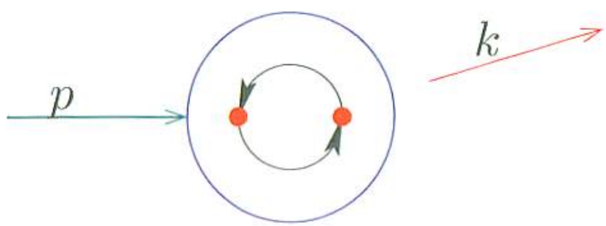
Modified pdf universality

$$\langle P, S_T | \, \mathcal{O}_{DIS} \, | P, S_T \rangle \underset{PT}{=\!=} \langle P, -S_T | \, \mathcal{O}_{DY} \, | P, -S_T \rangle$$

- Unpolarized pdfs: same universality OK
- Transverse polarization reversed by PT
  $\Longrightarrow$ Sivers fn. reverses sign
- All other pdfs universal
- Rapidity cut-off / renormalization c.t. unaffected.

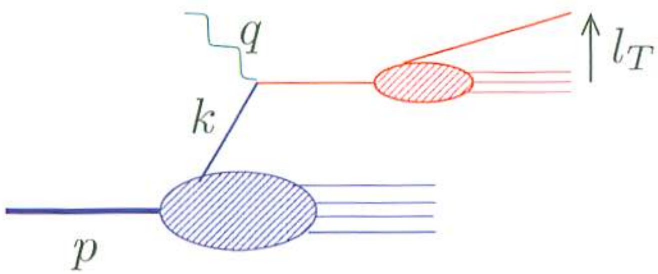
Sivers function

- Meaning: Transverse-spin-dependent k_T:

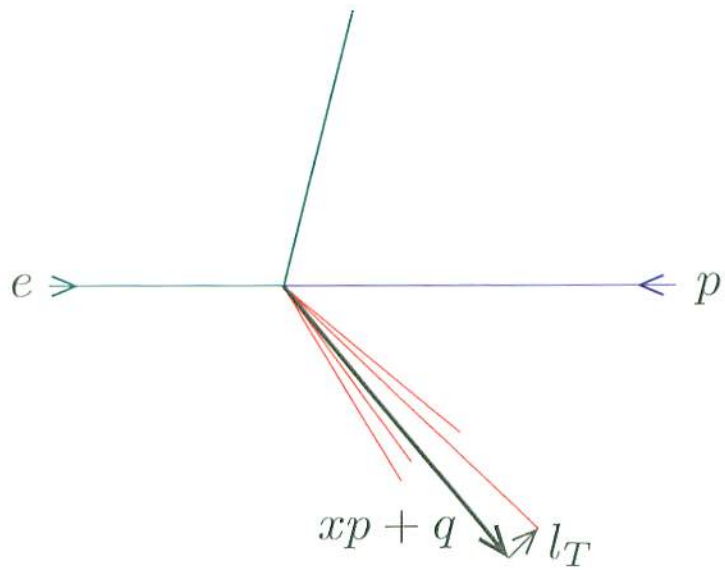


$$\frac{\mathrm{d}N}{\mathrm{d}x\,\mathrm{d}^2k_T} = A(|k_T|, x) + B(|k_T|, x)\sin\phi$$

- SIDIS (with data from HERMES et al.):

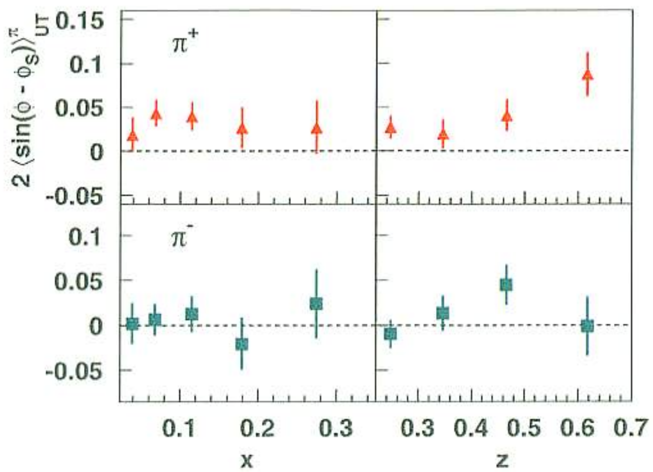


l_T rel. to

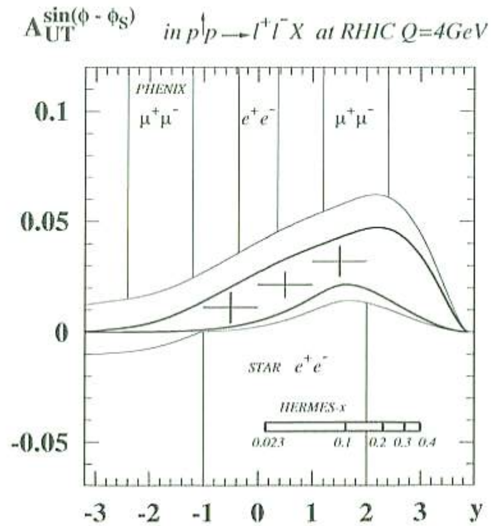


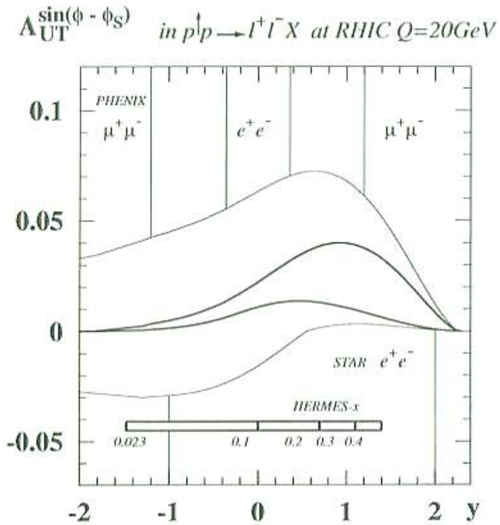
HERMES data $\Longrightarrow$ RHIC Drell-Yan SSA

HERMES:



RHIC:

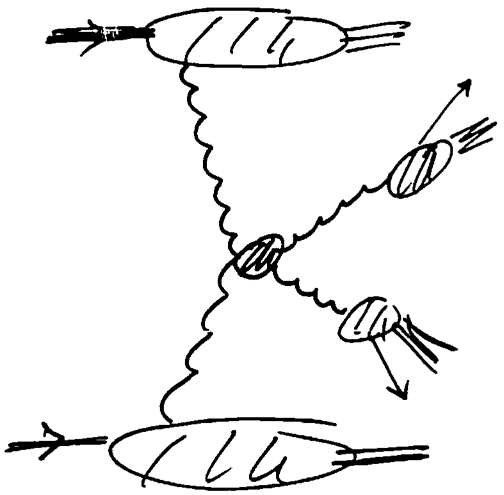


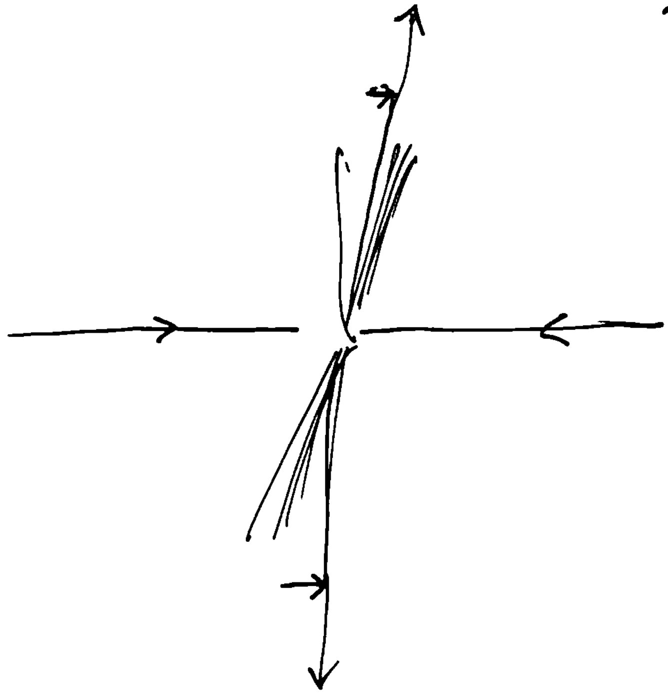


(Data: HERMES, PRL 94 (2005) 012002. Prediction, e.g., JCC+Bochum group, hep-ph/0511272)

Conclusions and outlook

- Wilson lines in pdf $\Leftrightarrow$ space-time in scattering
- Smoking gun: Sivers function sign change. Modified universality
- Use of unintegrated pdfs and frag. fns. à la CSS
- High p_T hadron production in hadron-hadron collisions?
  - Proof of factorization for DY (inclusive). [CSS, Bodwin]
  - No published proof for jet-jet. [As far as I know.]
  - Counterexamples for diffractive hard scattering.
  - MC event generators?





Soft glue attaches in many ways fs & is.

Do DY-style cancellations of fs still work?
Wilson lines?

Probes:
- p_T structure & correlations between jets
- MC & minijets & spectator final state &c.