Gluons in the proton

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Outline

1. Introduction
2. Gluons and proton mass:
   - a “virial” theorem
3. Gluons and proton spin:
   - a model calculation
4. Gluons and strange quark contribution to proton’s magnetic moment:
   - an EFT analysis
5. Summary

The Glue

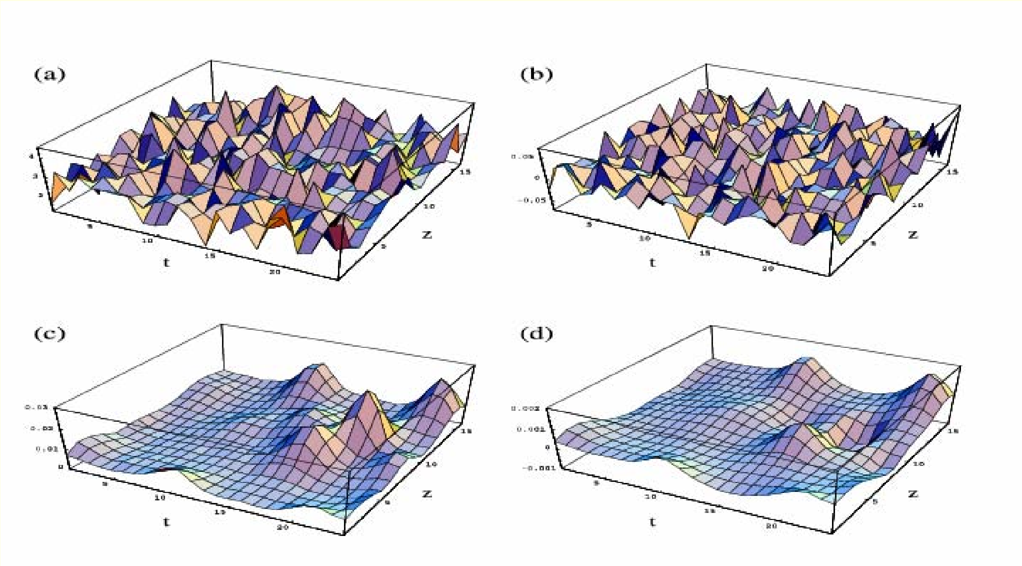
- Mediator of the strong interactions
  - Without of it, the mass of the proton would be the sum of three quark masses!
  - Determine all the essential features of strong interactions (more important than quarks!)
- However, it is actually hard to "see" the glue in the low-energy world
  - Does not couples to electromagnetism
  - Gluon degrees of freedom "missing" in hadronic spectrum.
  - …

"Crouching quarks, hidden glue"

Gluon dominance in the vacuum

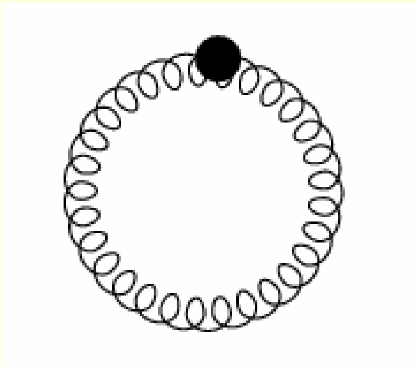
- QCD vacuum has interesting non-perturbative structures,
  - Color confinement
  - Chiral symmetry breaking
- These properties are large due to strong fluctuation gluon fields in the vacuum as in pure glue QCD.

J.Negele et al



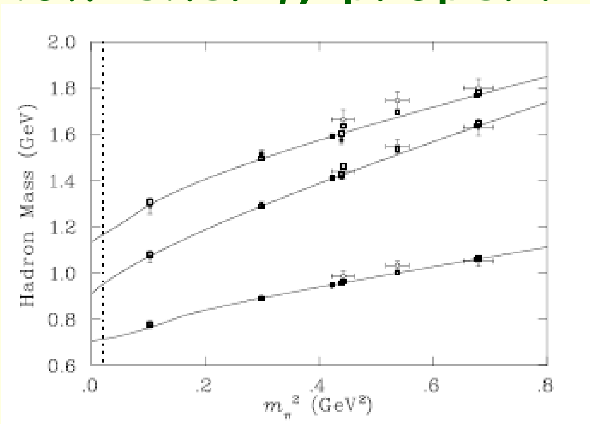
Large N_c QCD (t' Hooft, Witten)

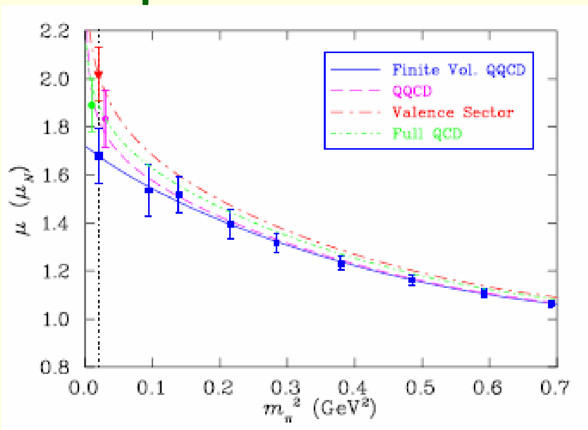
- Many features of QCD seem to be kept in a theory with N_c quark colors, where N_c is large.
- Gluon dominance become obvious in this limit. A scalar gluon operator of *GG* types goes like N_c^2 in the vacuum, because there are N_c^2-1 gluons (the majority wins!).



Chiral symmetry breaking (CSB)

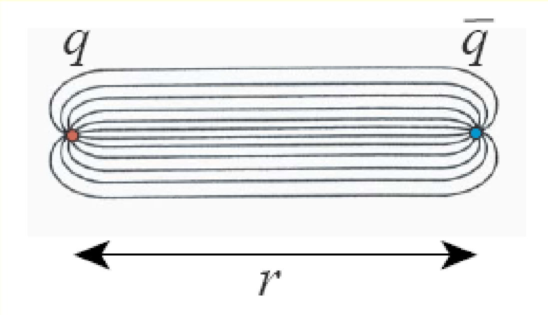
- The left and right-handed quarks can be rotated independently in flavor space.
- This symmetry is broken by the zero-mode of instantons.
- CSB might generates a new mass scale for quarks and responsible for the success of quark models.
- CSB and Goldstone boson physics are critical to the low-energy properties of the proton.

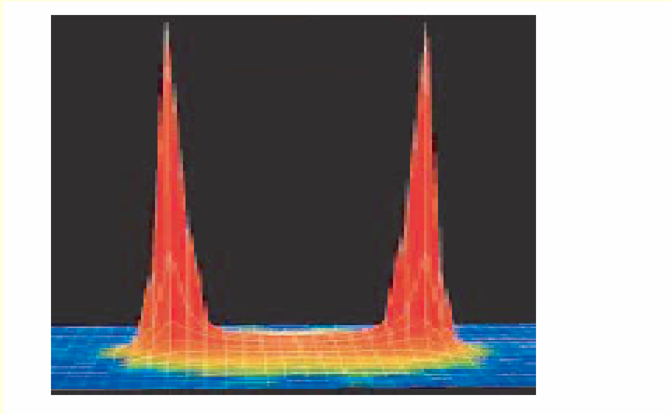




Color confinement

- An amazing property of the QCD vacuum! May be understood from the view of color flux tubes





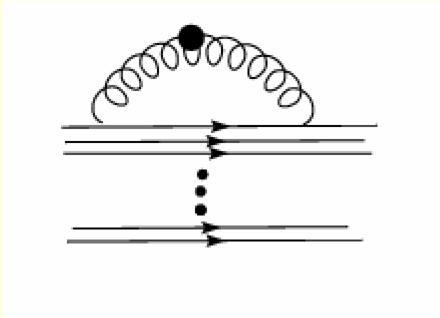
- Flux tubes deplete the vacuum color fields and generate (QCD) strings of constant energy density ✸ needs for an effective string theory?

$1M from Clay Institute of Mathematics

Gluon and proton Mass

Gluons in a proton

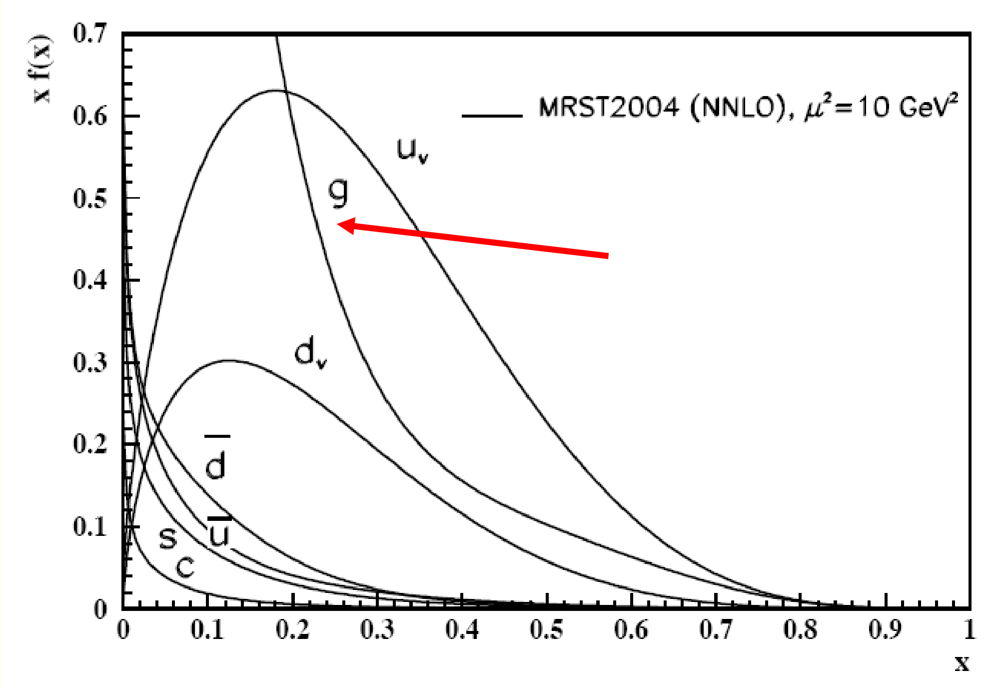
- The proton matrix element of the gluon operators goes like N_c.



  - Introduction of valence quarks yields a relatively *small change* in the background gluon field!
  - Gluon distribution in the nucleon goes like

    $g(x) = N_c^2 f(N_c x)$: fraction of nucleon momentum carried by gluon is a constant!

Gluon parton distribution



- Gluons become dominant in the proton at small *x* (gluon saturation)
- Gluons account for about ½ of the proton momentum.

The proton mass

- One can calculate the proton mass through the expectation value of the QCD hamiltonian,

$$H_{\mathrm{QCD}} = H_q + H_m + H_g + H_a \, .$$

$$H_q = \int d^3\vec{x}\ \bar{\psi}(-i\mathbf{D}\cdot\alpha)\psi,$$ Quark energy

$$H_m = \int d^3\vec{x}\ \bar{\psi}m\psi,$$ Quark mass

$$H_g = \int d^3\vec{x}\ \frac{1}{2}(\mathbf{E}^2+\mathbf{B}^2),$$ Gluon energy

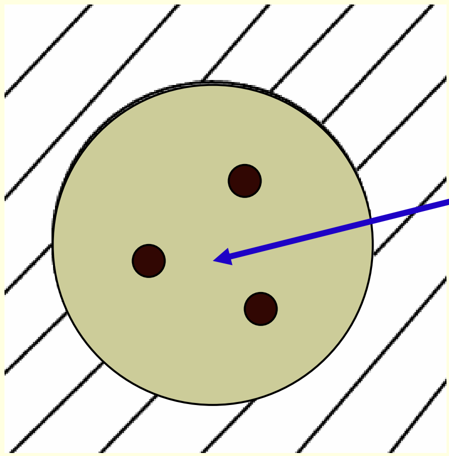
$$H_a = \int d^3\vec{x}\ \frac{9\alpha_s}{16\pi}(\mathbf{E}^2-\mathbf{B}^2).$$ Trace anomaly

A "virial theorem"

- The hamiltonian is just $\int d^3x T^{00}$ (stress-energy tensor of QCD)
- "Virial theorem" (X. Ji, PRL70,1071,1995)
  - The traceless part of the stress-energy tensor accounts for ¾ of the proton mass, and the trace part accounts for ¼.
  - True in the so-called MIT bag model, where the trace part is just the vacuum (dark) energy.

MIT bag model (K. Johnson et al., 1975)

- Quarks are confined in a 3D cavity in which the vacuum gluon fields are depleted.



"false vacuum" energy density B

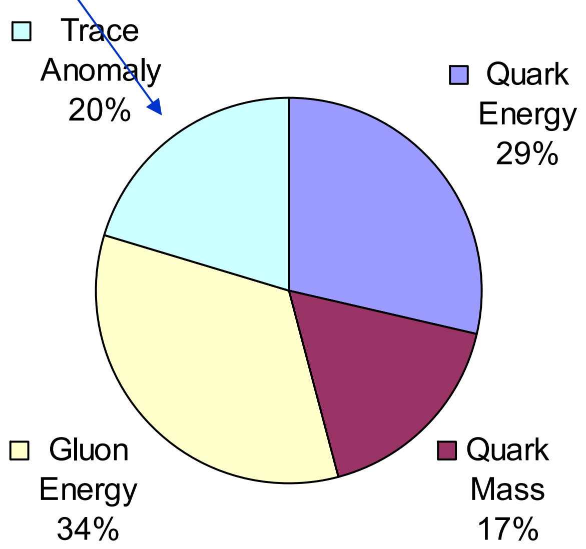
- The quarks inside the cavity obey the free Dirac equation. The pressure generated from mechanical motion balances the negative pressure from "cosmological constant".

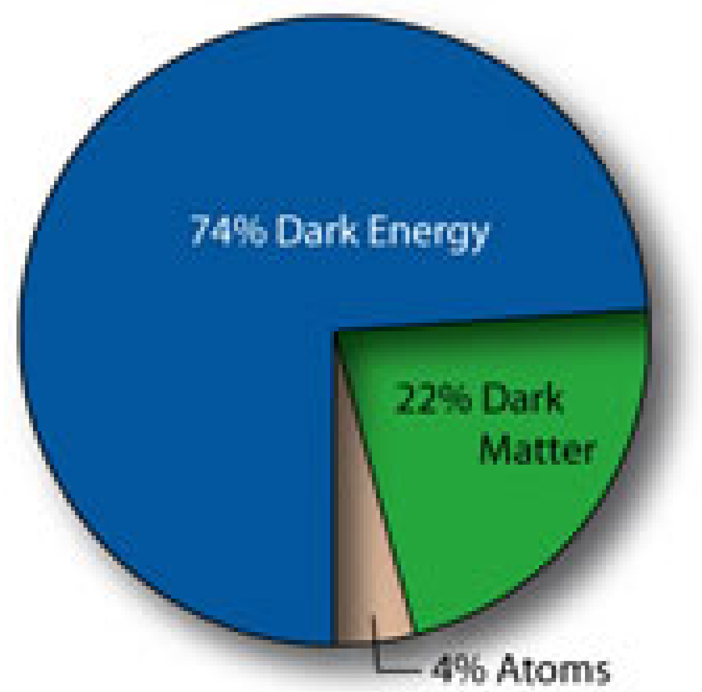
Other ingredients

- The traceless part of the quark contribution can be determined through nucleon momentum sum rule
  - $\int dx\, x\, q(x)$ = fraction of the momentum by quarks
- The quark mass contribution to the proton mass can be determined from
  - Pion-N sigma term $\overline{m}\langle p|\bar{u}u+\bar{d}d|p\rangle$
  - Chiral perturbation theory for masses of baryon octect $m_s\langle p|\bar{s}s|p\rangle$

Mass budget of baryonic matter

Dark-Energy





Energy density of the universe

Color electric & magnetic fields

- One can solve the color electric and magnetic fields in the nucleon from the above

$$\langle P|\mathbf{E}^2|P\rangle = 1700 \text{ MeV},$$
$$\langle P|\mathbf{B}^2|P\rangle = -1050 \text{ MeV}.$$

  - The color electric field is stronger in the proton than that in the vacuum (strong coulomb field?)
  - The color magnetic field produced by the motion of valence quark is also strong, with the surprising feature that it almost cancels that field in the vacuum.

  (A key to color confinement?)

Gluons and proton spin

Spin of the proton in QCD

- The spin of the nucleon can be decomposed into contributions from quarks and gluons

$$J = 1/2 = J_q(\mu) + J_g(\mu)$$

- Decomposition of quark contribution

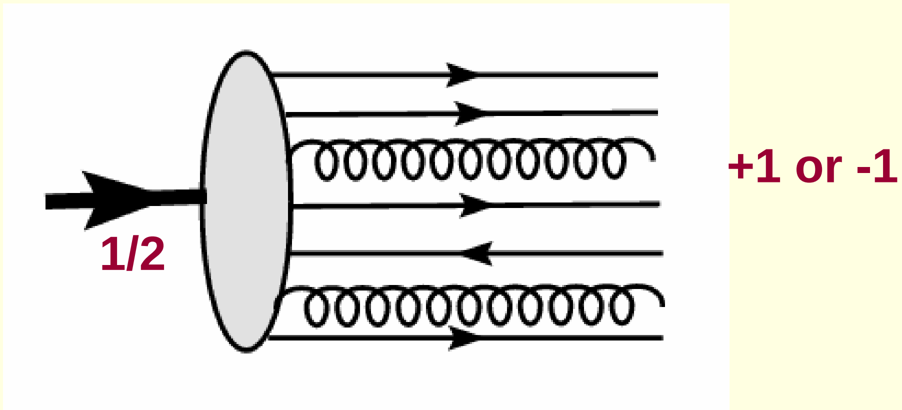
$$J_q = \sum_f [\frac{1}{2}(\Delta q^v{}_f + \Delta q^s{}_f) + L_{qf}]$$

- Decomposition of gluon contribution

$$J_g = \Delta g + L_g$$

Gluon helicity distribution

- In a polarized proton, the gluon parton may have helicity $\pm$ 1. Introduce their densities $g_{\pm}(x)$



- Gluon helicity distribution is

$$\Delta g(x) = g_{+}(x) - g_{-}(x)$$

- The total gluon helicity is $\Delta G = \int dx\, \Delta g(x)$

Size of ΔG?

- Thought to be large because of the possible role of axial anomaly $-(\alpha_s/2\square)\Delta G$ (Altarelli & Ross, 1988)
  - 2-4 units of hbar!
  - Theoretical controversies…
- Of course, the gluon contribute the proton spin directly.

$$\Delta q + \Delta G + L_z = 1/2$$

Naturalness?

if ΔG is very large, there must be a large negative L_z to cancel this---(fine tuning!)

Theory difficulty

- In gauge invariant form, $\Delta g(x)$ is a non-local operator which cannot be calculated in lattice QCD.

$$\Delta g(x) = -\frac{i}{x}\int \frac{d\lambda}{2\pi} e^{-i\lambda x} \langle P | F^{+\alpha}(\lambda n) W \tilde{F}^{+}_{\alpha}(0) | P \rangle$$

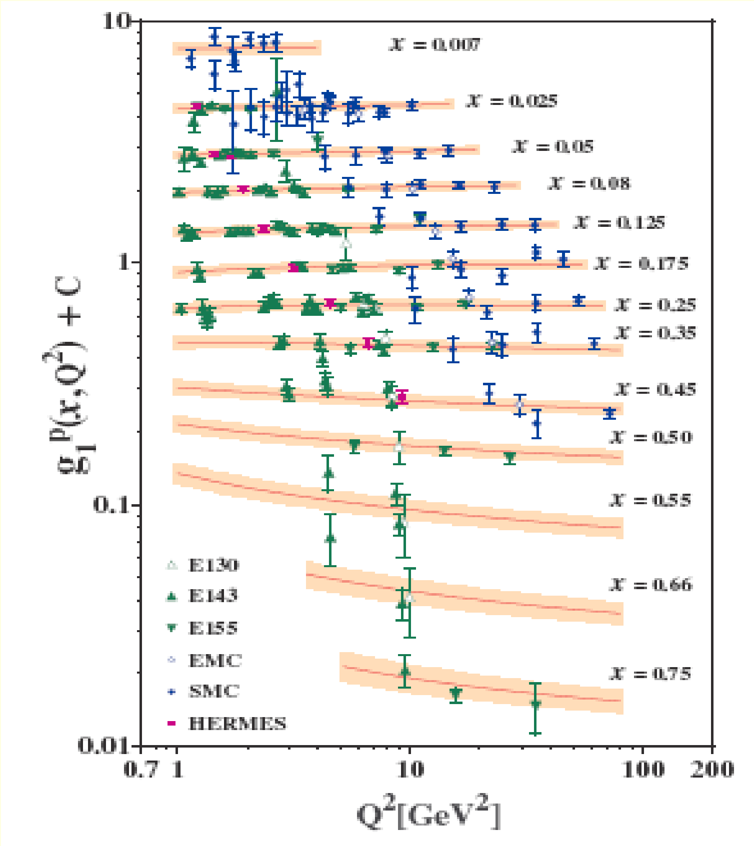
- Only in light-cone gauge, ΔG reduces to a local operator.

$$\Delta G = \left\langle PS \left| \vec{E} \times \vec{A} \right| PS \right\rangle$$

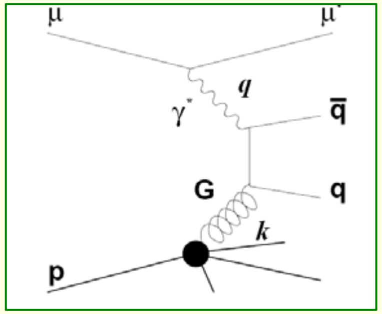
  - However, light-cone gauge cannot be implemented in lattice QCD calculation!

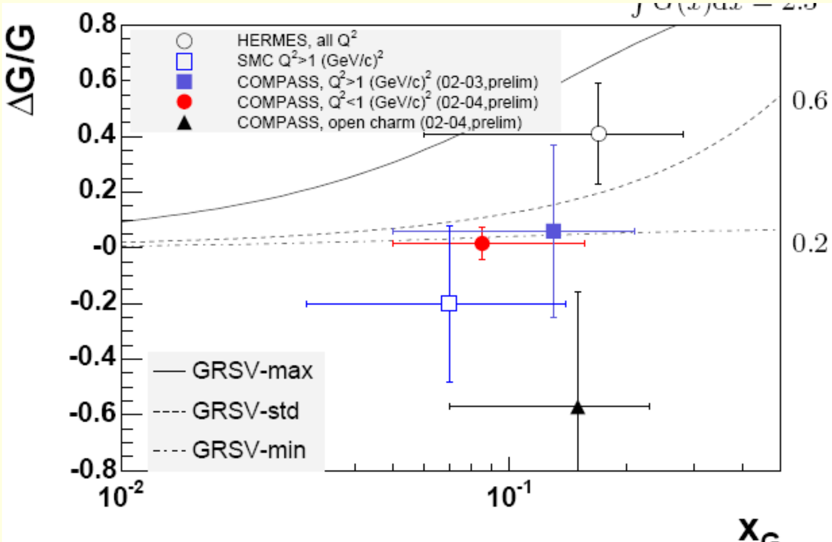
Experimental measurements (I)

- Q-evolution in inclusive spin structure function $g_1(x)$



- Two leading-hadron production in semi-inclusive DIS

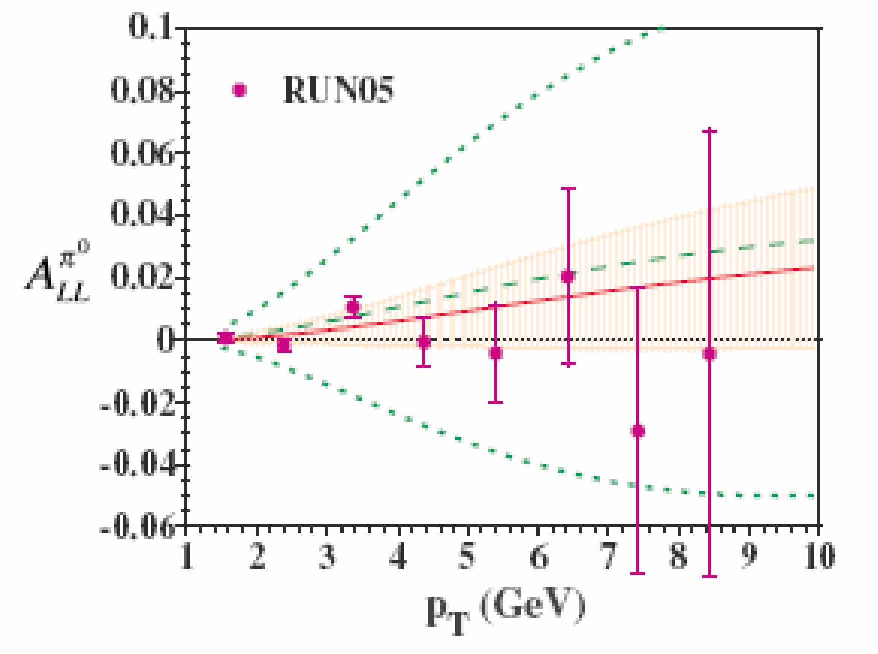




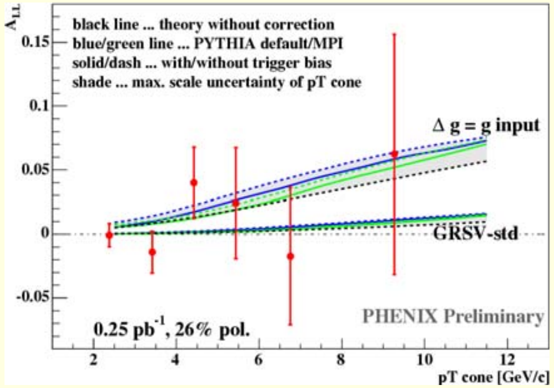
$\Delta G/G$ is small or has a node around $x_g \approx 0.1$

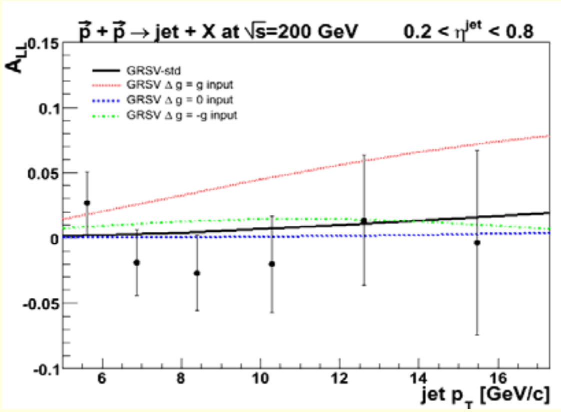
Experimental measurements (II)

- ◻ production in polarized PP collision at RHIC



- Two jet production in polarized PP collision at RHIC

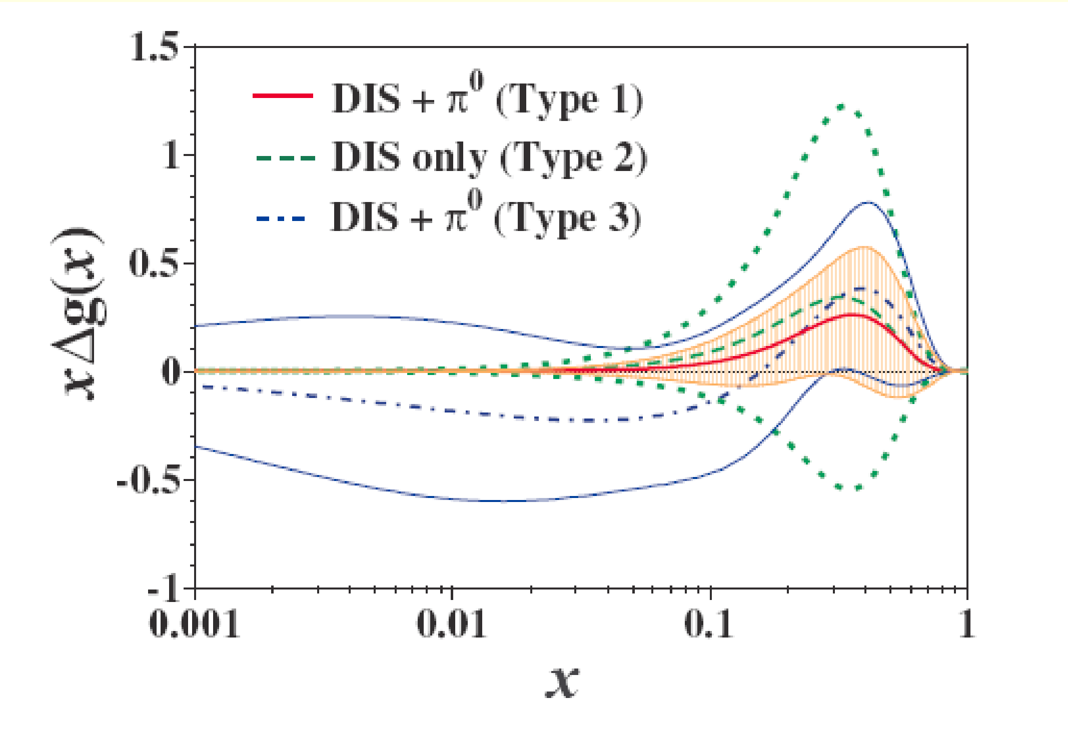




Fit to data

- Generally depend on the function forms assumed.

Hirai, Kumano, Saito, hep-ph/003213 (ACC)



$\Delta g = 0.31 \pm 0.32$ type-1

$= 0.47 \pm 1.0$ type-2

$= -0.56 \pm 2.16$ type-3

Type-3 fit assumes gluon polarization is negative at small x.

Large N_c limit

- The polarized gluon matrix elements go like N_c^0 ” 1 in the large limit. Thus the polarized part of the gluon field is
  - $1/N_c$ suppressed relative to the measurable gluon field in the proton.
  - $1/N_c^2$ suppressed relative to the gluon fields in the vacuum.
  - $\Delta g(x) = N_c\, h(N_c x)$
- The polarized gluon field represents a weak response of the gluon system to the proton polarization.

Calculating $\Delta g(x)$ in Models

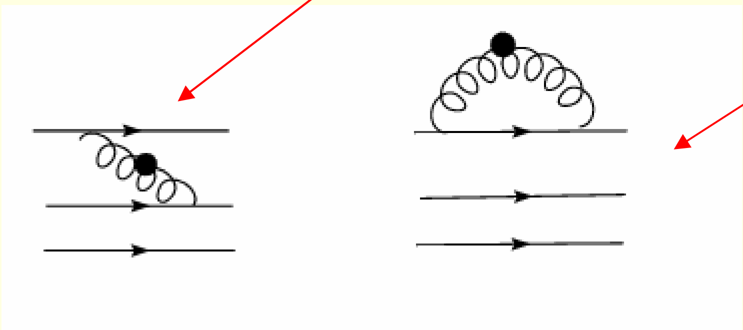
- Since the quarks are the primary constituents of a proton, and $\Delta g(x)$ effect is small, the polarized gluons may be calculated from

$$D_\mu G^{\mu\nu} = j^\nu$$

  where color current is generated by valence quarks
  - The dominant gluon responsible for the motions of the quarks have no pol. effect.
- This is very much like small-x gluons whose sources are mostly from the valence quarks.

ΔG: positive? negative?

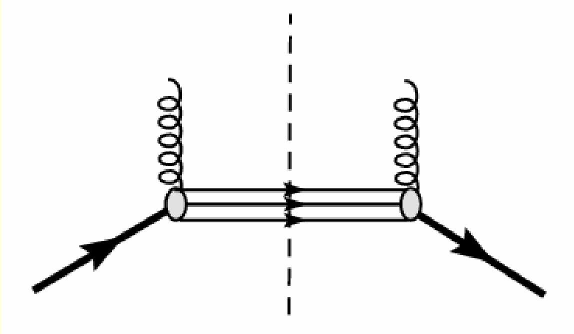
- There was a calculation by Jaffe (PRB365, 1996), showing a negative result for ΔG in NR quark and MIT bag models (two-body contribution)
- However, there is also the one-body contribution



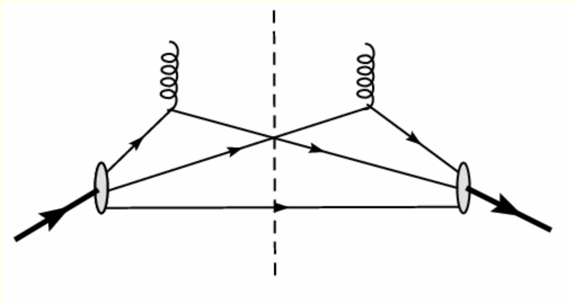
Barone et al., PRB431,1998

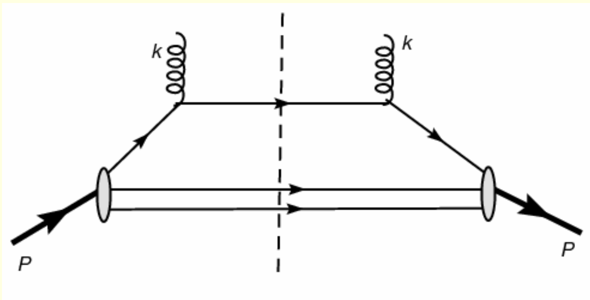
- Part of the one-body contribution cancels the two-body one, a positive residue remains.

x-dependence

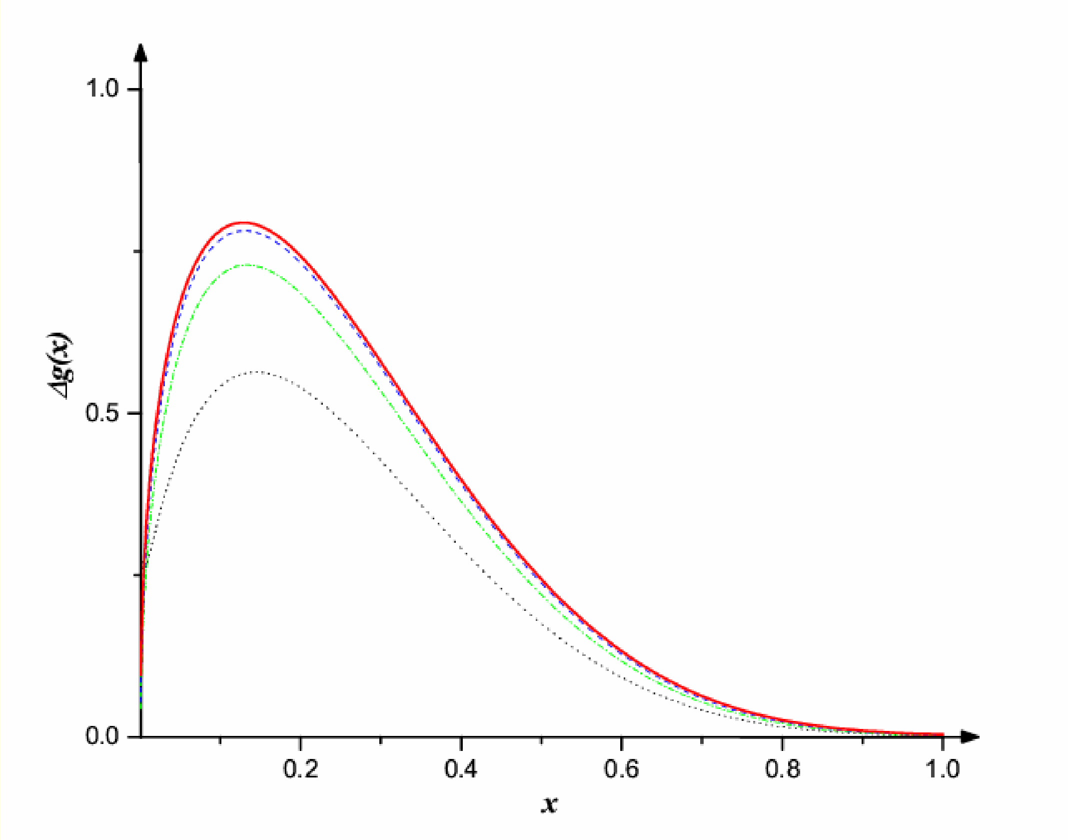


- No model calculation for x-dependent $\Delta g(x)$ has ever reported in the literature so far.
- A calculation has recently been made in MIT bag model (P. Chen, X. Ji)





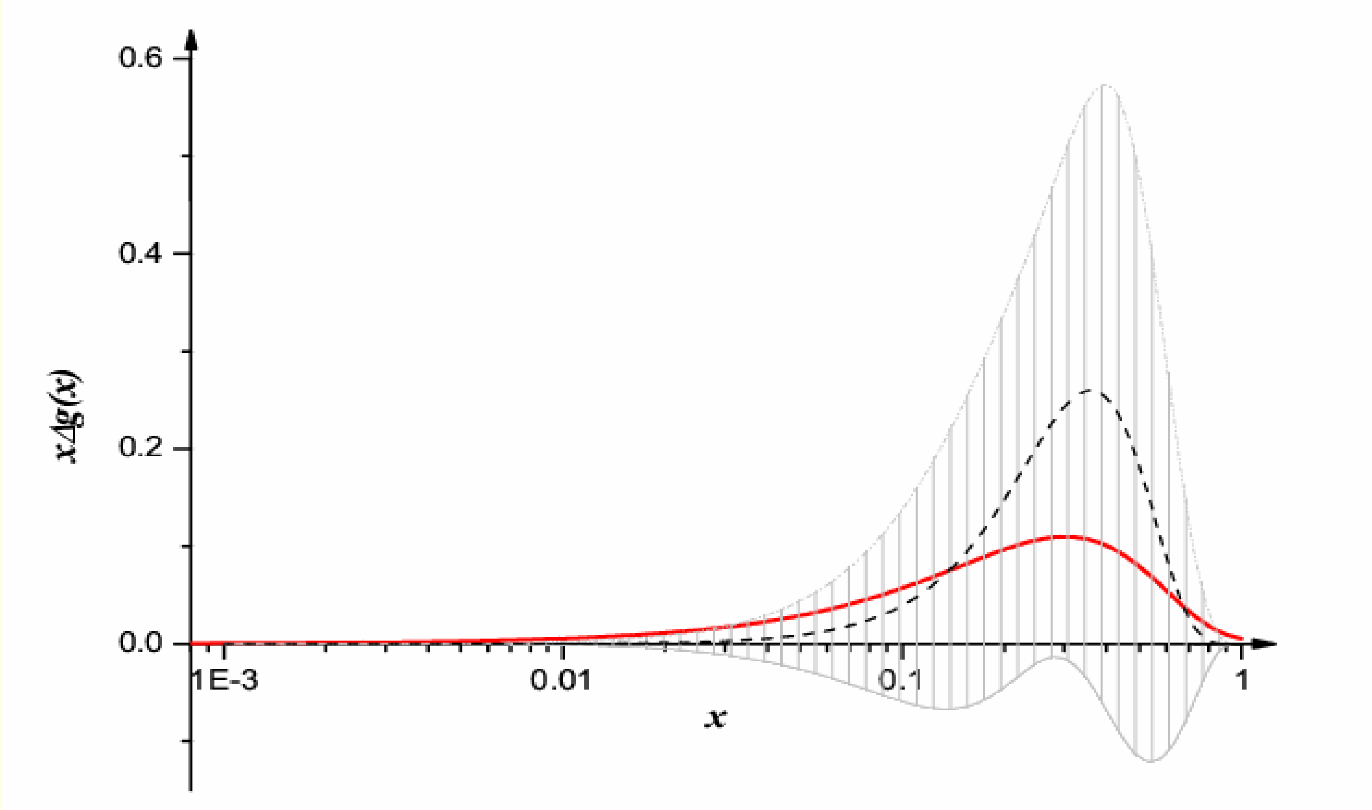
A bag model $\Delta g(x)$



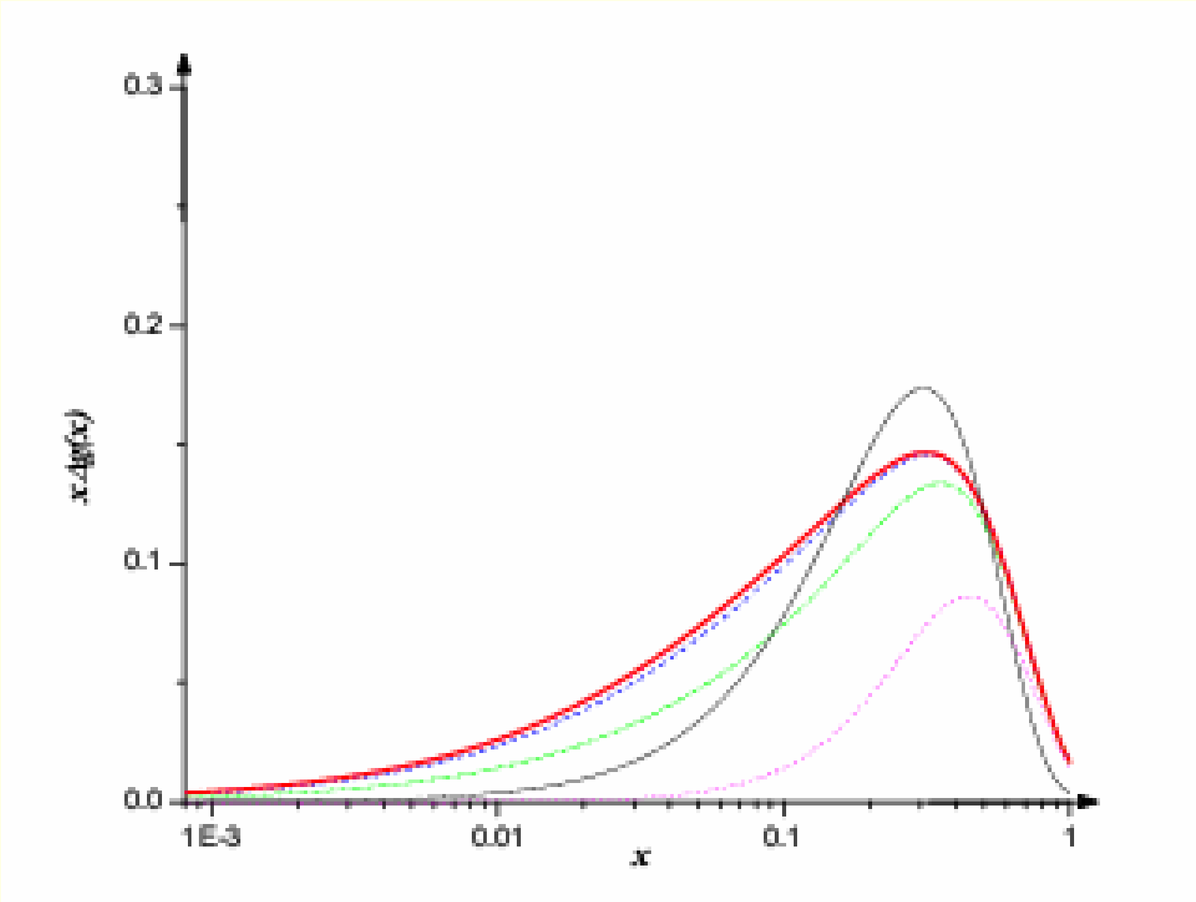
It is positive at all x! Similar to the correlation between the angular momentum and magnetic moment.

Compare with the fit

- Compared with the AAC fit with positivity constraint.



Non-relativistic quark model vs. the bag model



Gluons and strange quark contribution to proton's magnetic moment

Magnetic moment of the proton

- Magnetic moment of the proton was measured in early 1930s, a first indication that proton has a nontrivial internal structure

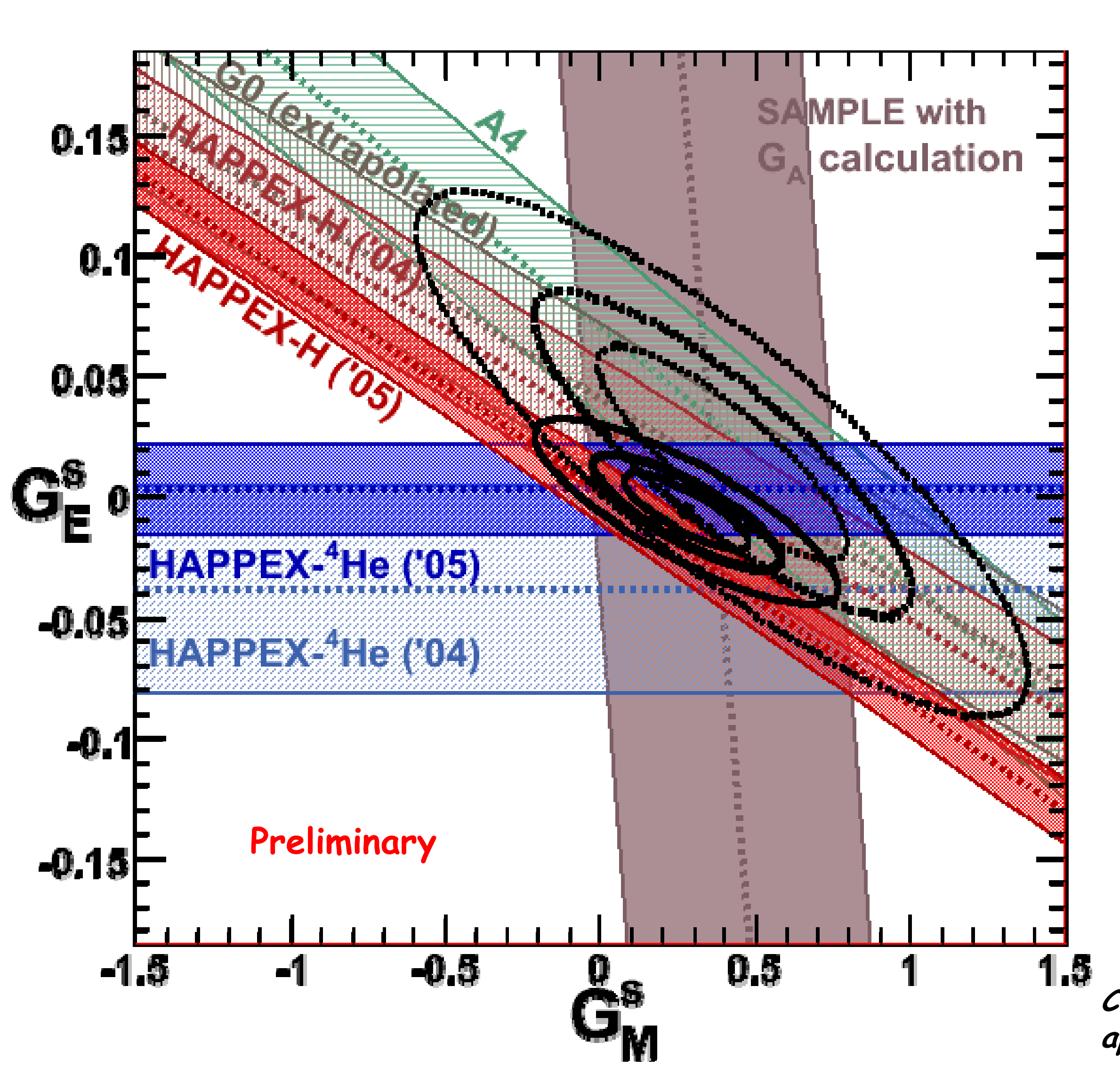
$$\mu_p = 2.7\,\mu_N$$

- Individual quark-flavor contributions add

$$\mu_p = \frac{2}{3}\mu_u - \frac{1}{3}\mu_d - \frac{1}{3}\mu_s + \ldots$$

- Different contributions can be obtained from isospin symmetry and parity-violating electron scattering (SAMPLE + HAPPEX + G0 + ...)

World Data near Q^2 ~0.1 GeV^2



G_M^s = 0.28 +/- 0.20

G_E^s = -0.006 +/- 0.016

~3% +/- 2.3% of proton magnetic moment

~0.2 +/- 0.5% of electric distribution

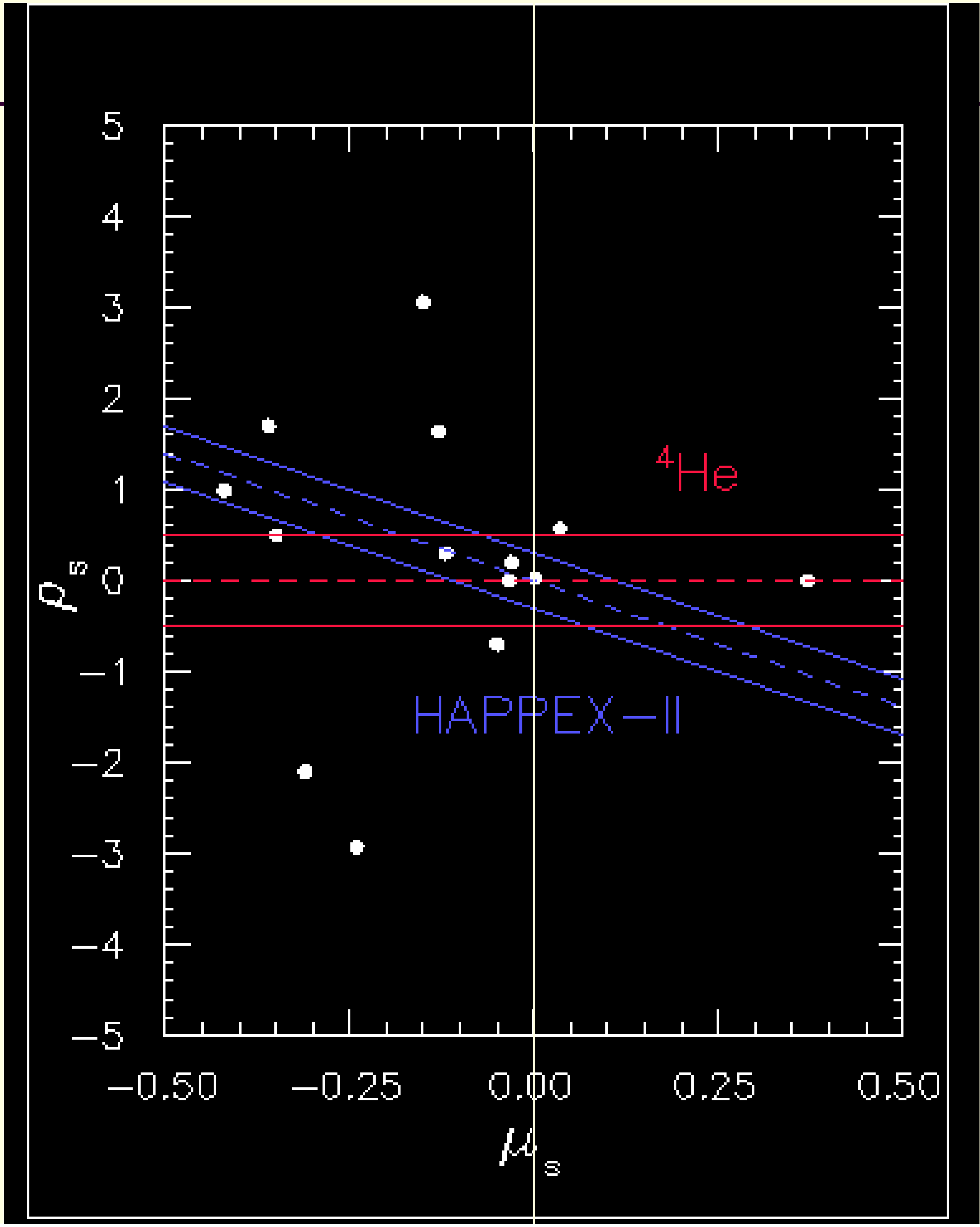
HAPPEX-only fit suggests something even smaller:

G_M^s = 0.12 +/- 0.24

G_E^s = -0.002 +/- 0.017

Caution: the combined fit is approximate. Correlated errors and assumptions not taken into account

Strangeness Models (as/of 2000)



Leading moments of form factors:

$\mu_s = G_M^s\,(Q^2{=}0)$

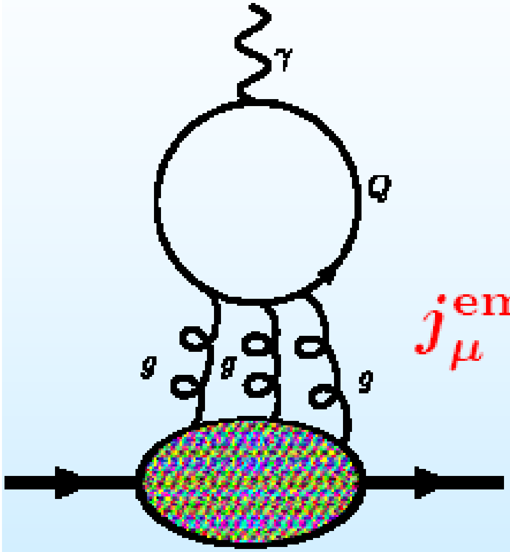
$\rho_s = \partial G_E^s/\partial\tau$ $(Q^2{=}0)$

A light strange quark

- Strange quark mass is about 100 MeV, neither light nor heavy.
- When a strange quark is considered light, QCD has an approximate $SU_L(3)XSU_R(3)$ chiral symmetry. The spontaneous breaking of the symmetry leads to 8 massless Goldstone bosons.
- If so, the proton may sometimes be dissociated into a ☹ and K^+.
- Simple calculation shows that in this picture, the strange quark contribution to the proton's magnetic moment is always negative!

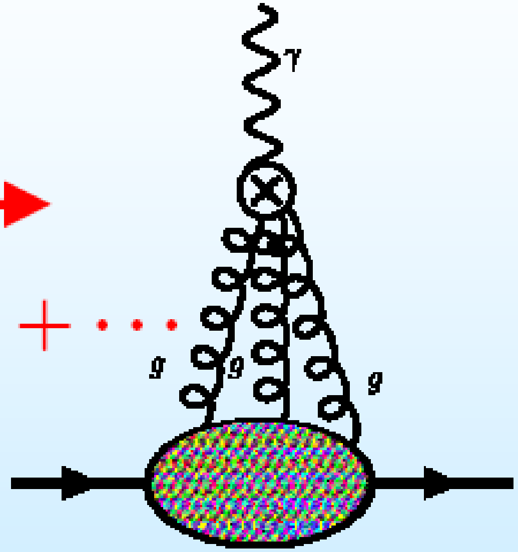
A heavy strange quark (an EFT calculation)

- **Heavy quark limit $\Longrightarrow$ effective operator**



$$\xrightarrow{m_Q \gg \Lambda_{\rm QCD}}$$

$$j_\mu^{\rm em} = C(m_Q)\partial^\alpha T_{\mu\alpha} + \cdots$$



  - $C(m_Q) = \kappa\, g^3(m_Q)/m_Q^4$

Light-by-light scattering

  - $T_{\mu\alpha} = 14\, G_{\mu\sigma}\{G^{\sigma\tau}, G_{\tau\alpha}\} - 5\, G_{\sigma\tau}\{G^{\sigma\tau}, G_{\alpha\mu}\}$

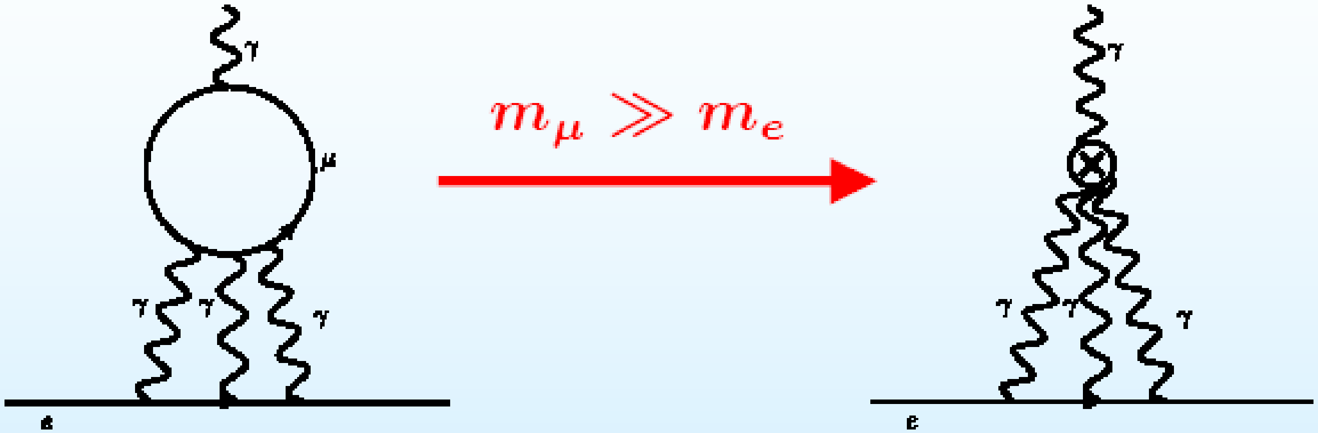
Kaplan and Manohar

- **Use effective $j_\mu^{\rm em}$ in def. of μ_p**

Gluon matrix element

Muon contribution to electron's MM

- **Heavy muon limit**



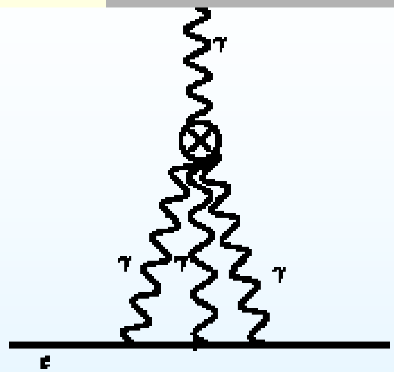
  - $\delta\mu_e^\mu = (C_e/m_\mu^4)\ \langle e|\ T_{yx}^\gamma\ |e\rangle + \cdots$

- **Exact calculation:**
  - $\delta\mu_e^\mu/\mu_e = K_e\ \alpha_{\rm em}^3\ m_e^2/m_\mu^2 + \ldots\ > 0$

Laporta and Remiddi

  - **Therefore:** $\langle e|\ T_{yx}^\gamma\ |e\rangle \sim m_\mu^2\ m_e$

Understanding the EFT calculation



- **How can we understand this?**
  - ➢**Power counting: k^3 divergence**
  - ➢**Two scales: Mom. flow (m_μ), m_e**
  - ➢**Factorization, dim. 3:**

$$\langle e|\ T^{\gamma}_{yx}\ |e\rangle = a\ \cancel{m_\mu^3} + b\ m_\mu^2 m_e + c\ \cancel{m_\mu} m_e^2 + d\ m_e^3$$

$\longrightarrow$ Lorentz invariance

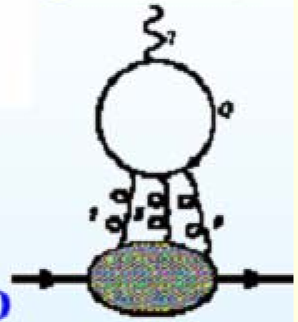
- **Symmetry** $\Rightarrow T^{\gamma}_{\mu\nu} = m_\mu^2 \left(\kappa\ m_e\ \bar{\psi}\boldsymbol{\sigma}_{\mu\nu}\psi\right) + \cdots$

$$\Rightarrow \boxed{\delta\mu_e^\mu = (C_e/m_\mu^4)(\kappa\ m_\mu^2 m_e \langle e|\bar{\psi}\boldsymbol{\sigma}_{yx}\psi|e\rangle) > 0}$$

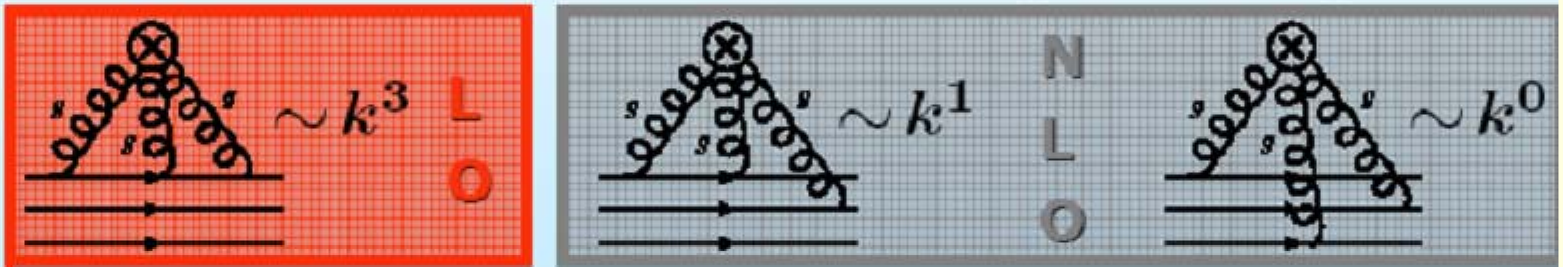
  - ➢**Same result as exact calculation at LO**

Proton matrix element

$$\langle p\uparrow|\, T_{yx}\, |p\uparrow\rangle = a\,\cancel{m_Q^3} + b\, m_Q^2 \Lambda_{\rm QCD} + \cancel{c\, m_Q \Lambda^2_{\rm QCD}} + d\, \Lambda^3_{\rm QCD}$$



Contributions from 1, 2, and 3 quarks in proton:



Similar to muon contribution to μ_e

$$\langle p\uparrow|\, T_{yx}\, |p\uparrow\rangle_{\rm single\ quark} \sim m_Q^2 \Lambda_{\rm QCD} + \cdots$$

Contribution is positive at large M

- **Heavy quark limit:** $\delta\mu_p^Q = C(m_Q)\ \langle p\uparrow|\ T_{yx}\ |p\Uparrow\rangle$
  - $\langle p\uparrow|T_{yx}|p\Uparrow\rangle = K m_Q^2\ \underbrace{\sum_f m_f\ \langle p\uparrow|\bar{\psi}_f \sigma_{yx}\psi_f|p\Uparrow\rangle}$

Proton tens. charge $\longrightarrow\ m_u\ \delta_u + m_d\ \delta_d\ > 0$

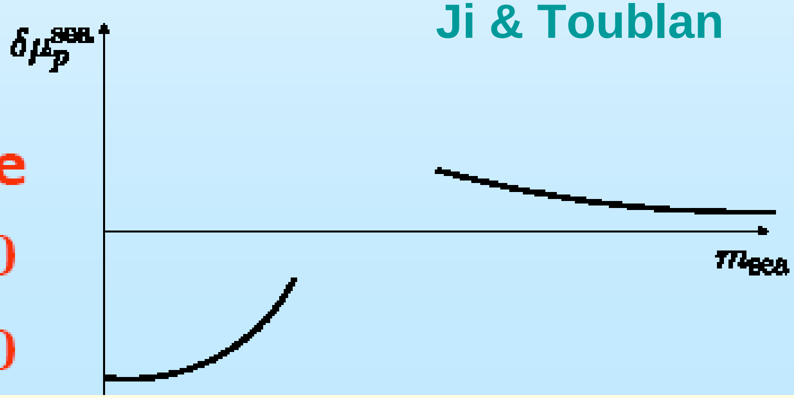
$$\Rightarrow \boxed{\delta\mu_p^Q = A(m_u\ \delta_u + m_d\ \delta_d)/m_Q^2 > 0}$$

- **Therefore:**

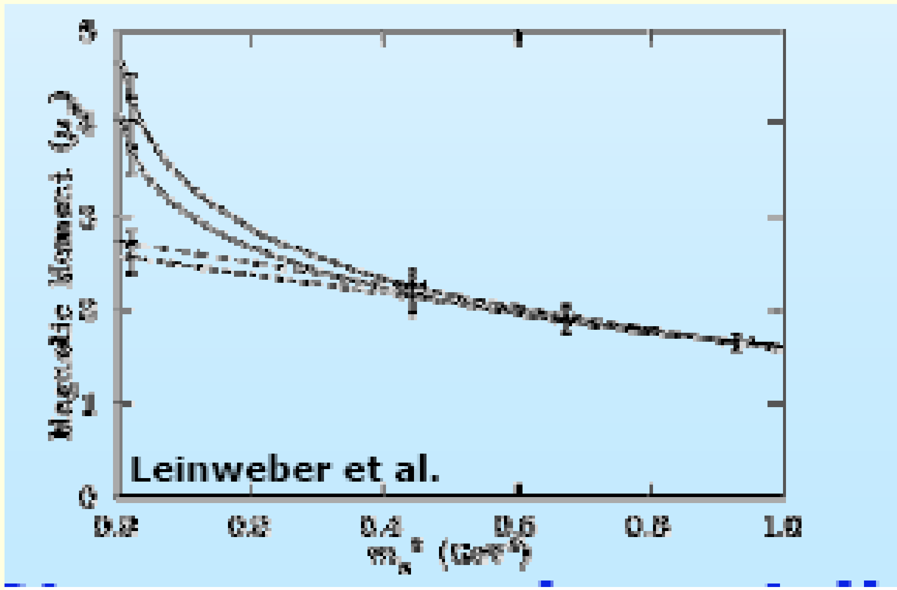
Non-trivial dependence

- **Light sea:** $\delta\mu_p^{\mathrm{sea}} < 0$
- **Heavy sea:** $\delta\mu_p^{\mathrm{sea}} > 0$

Ji & Toublan



Where does the transition happen?

- It is difficult to estimate where the transition happen in QCD
- However, lattice calculation seems to indicate that sharp transition occurs at relatively small quark mass.
- 

Contribution is positive at large M

- **Heavy quark limit:** $\delta\mu_p^Q = C(m_Q)\ \langle p\uparrow|\ T_{yx}\ |p\uparrow\rangle$
  - $\langle p\uparrow|T_{yx}|p\uparrow\rangle = K m_Q^2\ \underbrace{\sum_f m_f\ \langle p\uparrow|\bar{\psi}_f\sigma_{yx}\psi_f|p\uparrow\rangle}$

Proton tens. charge $\longrightarrow$ $m_u\ \delta_u + m_d\ \delta_d\ > 0$

$$\Rightarrow \boxed{\delta\mu_p^Q = A(m_u\ \delta_u + m_d\ \delta_d)/m_Q^2 > 0}$$

Ji & Toublan

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  Non-trivial dependence
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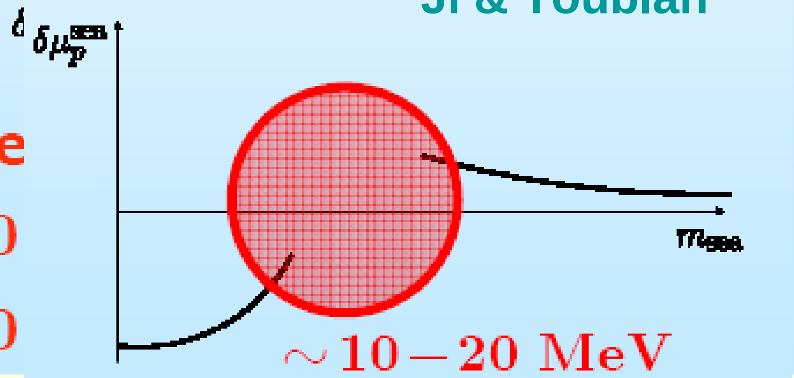

Conclusion

- Gluons play a critical role in QCD. They dominate in the vacuum and high-energy.
- Although they are less visible in hadron physics at low-energy, their contribution to the mass and spin of the proton is as important as the quarks.
- The polarized gluons effects in the proton are small. However, precision experimental data allow us to learn their effects through QCD analysis.