Pion Production at RHIC and eRHIC

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- Motivation
- Calculations & Results
  - Hadroproduction
  - Photoproduction
- Conclusions & Outlook

. . . barely known . . .

$\Delta f^{\gamma}(x,\mu^2)$

– need $\vec{e}\vec{e}$ or $\vec{e}\vec{p}$ collider
– accessible in one-jet incl., di-jet or hadron photoproduction

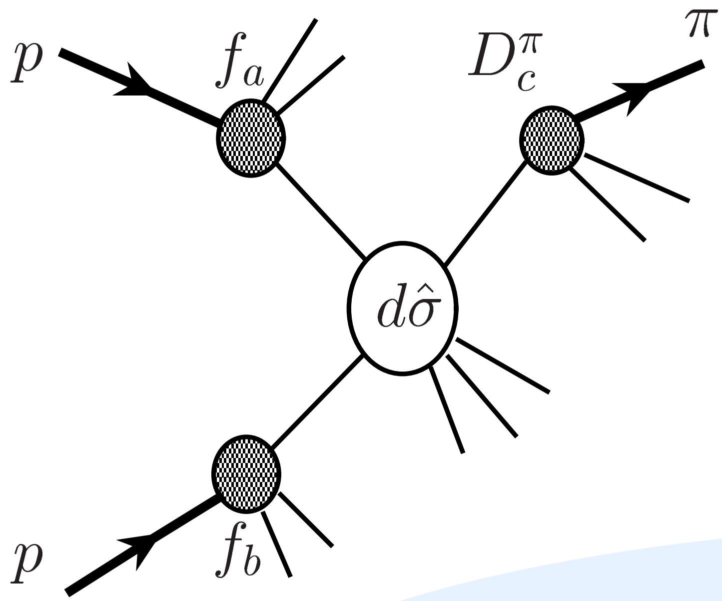
$\Delta g(x,\mu^2)$

need processes sensitive to polarized gluon content of proton

examples:
$\vec{p}\,\vec{p} \to \gamma X$
$\vec{p}\,\vec{p} \to \mathrm{jet} X$
$\vec{p}\,\vec{p} \to H X$

investigate:
$\vec{p}\,\vec{p} \to \pi X$
$\vec{e}\,\vec{p} \to \vec{e}\,'\, \pi X$

Hadronic Cross Section $d\sigma$



parton distribution functions

parton-to-pion fragmentation function

$$d\sigma^{pp\to\pi X} = \sum_{a,b,c} \int dx_a dx_b dz_c f_a(x_a,\mu_f) f_b(x_b,\mu_f) D_c^\pi(z_c,\mu'_f) \times d\hat{\sigma}^{ab->cX'}(x_a P_A, x_b P_B, P_\pi/z_c, \mu_f, \mu'_f, \mu_r)$$

partonic cross section

Factorization

bound state dynamics
of physical particles

hard scattering of
hadronic constituents

factorization theorems …

… applicable if scattering at short distances decouples from nonperturbative hadronic structure

… require a hard scale, e.g., pion with high transverse momentum p_T

… foundation for predictive power of QCD

physical motivation – hadron collisions at high energy:

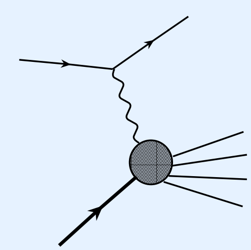
- one parton per colliding hadron takes part in hard scattering
- time scale of hadronization $\gg$ hard scattering

Parton Distribution Functions

- PDFs $f_a^H(x,\mu)$ describe bound state dynamics of hadronic constituents
- at LO: probability density for finding parton of type a (quark, gluon) in hadron H at a scale μ, carrying a longitudinal fraction x of the hadron momentum
- extracted from experiment at a scale μ_0, e.g.:

  f_q^N …DIS: $e^- N \to e^- X$

  (MRST, CTEQ …)

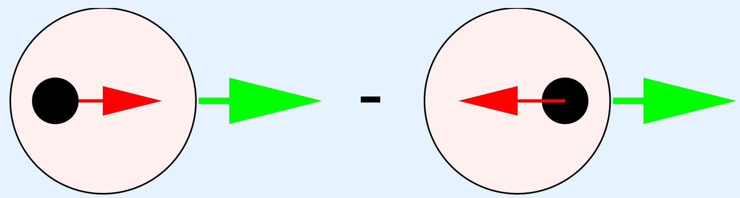


- μ dependence predicted by perturbative QCD:

$$\mu \frac{d}{d\mu} \begin{pmatrix} f_q(x,\mu) \\ f_g(x,\mu) \end{pmatrix} = \int_x^1 \frac{dz}{z} \begin{pmatrix} \mathcal{P}_{qq} & \mathcal{P}_{qg} \\ \mathcal{P}_{gq} & \mathcal{P}_{gg} \end{pmatrix}_{(z,\alpha_s(\mu))} \cdot \begin{pmatrix} f_q \\ f_g \end{pmatrix} \left(\frac{x}{z},\mu\right)$$

Spin Dependent Parton Densities

$$\Delta f^N(x,\mu) \equiv f_+^{N_+}(x,\mu) - f_-^{N_+}(x,\mu)$$



$f_+^{N_+}$ ... probability density for finding spins aligned

$f_-^{N_+}$ ... probability density for finding spins anti-aligned

μ-evolution known up to NLO

Mertig, van Neerven; Vogelsang

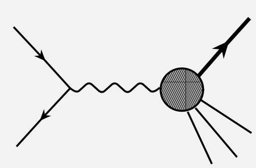
Fragmentation Functions

- FFs $D_i^H(z,\mu)$ ... probability for a parton of type i (quark, gluon) to fragment into a hadron H at a scale μ

z ... fraction of the parton momentum taken by the hadron.

- extracted from e^+e^- data (LEP):

$e^- e^+ \to \pi X$
(Kretzer, KKP)



- problems:
  - $e^+ e^-$ data do not provide flavor separation, but only inclusive sum D^π_{u+d+s}
  - gluon FF D^π_g rather poorly constrained

...ongoing work

- PDFs and FFs describe internal structure of hadrons
- independent of process

$\to$ universal
(factorization theorem)

Partonic Cross Section $d\hat{\sigma}$

calculate $d\hat{\sigma}$ in terms of a perturbation series

$$d\hat{\sigma} = d\hat{\sigma}^{(0)} + \frac{\alpha_S}{\pi} d\hat{\sigma}^{(1)} + \left(\frac{\alpha_S}{\pi}\right)^2 d\hat{\sigma}^{(2)} + \ldots$$

LO-cross section

merely captures main features of process

Need for Higher Order Corrections

Scale Dependence

factorization of perturbative and non-perturbative phenomena introduces arbitrary scale μ

physical cross section must not depend on μ:

$$\mu \frac{d}{d\mu} d\sigma = 0$$

LO approximation:

no control on scales at all

enhancing N

$\updownarrow$

reducing scale dependence

N-th order calculation:
residual μ-dependence
of order $\alpha_S^{(N+1)}$

Need for Higher Order Corrections

More Reliable Information

- higher order corrections often large, e.g.:
  - prompt photon production
  - heavy flavors
- closer to experiment (more realistic final state)
- test of perturbative QCD

Beyond QCD

thorough understanding of QCD background

$\updownarrow$

open up ways to search for signatures of new physics

Partonic Cross Section $d\hat{\sigma}$

…need for higher order corrections

NLO-cross section

calculate $d\hat{\sigma}$ up to NLO

$$d\hat{\sigma} = d\hat{\sigma}^{(0)} + \frac{\alpha_S}{\pi} d\hat{\sigma}^{(1)} + \mathcal{O}\left(\frac{\alpha_S^2}{\pi^2}\right)$$

$\vec{p}\vec{p} \to \pi X$

Outline of the Calculation

$$E_\pi \frac{d\Delta\sigma^{\vec{p}\vec{p}\to\pi X}}{d^3p_\pi} = \sum_{a,b,c} \Delta f_a(\mu_f) \otimes \Delta f_b(\mu_f) \otimes D_c^\pi(\mu_f')$$

$$\otimes \quad E_c \frac{d\Delta\hat{\sigma}^{\vec{a}\vec{b}->cX'}}{d^3p_c}(\mu_f, \mu_f', \mu_r)$$

partonic matrix
elements at
LO and NLO

– dim. regularization
$(n = 4 - 2\varepsilon)$
–$\overline{\mathrm{MS}}$-renormalization

numerical
convolution with
PDFs and FFs

factorization of
initial/final state
singularities

phase space
integration

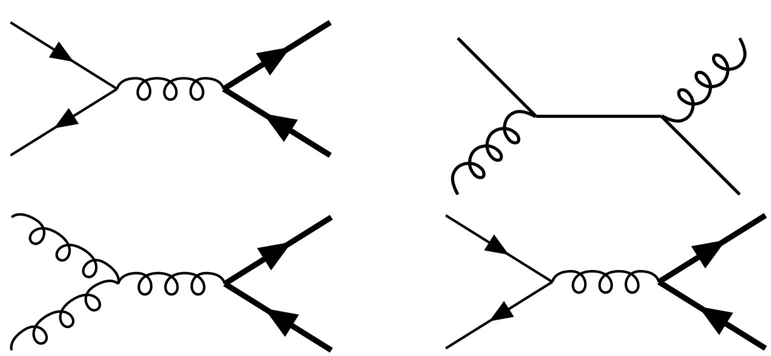
Partonic Cross Section

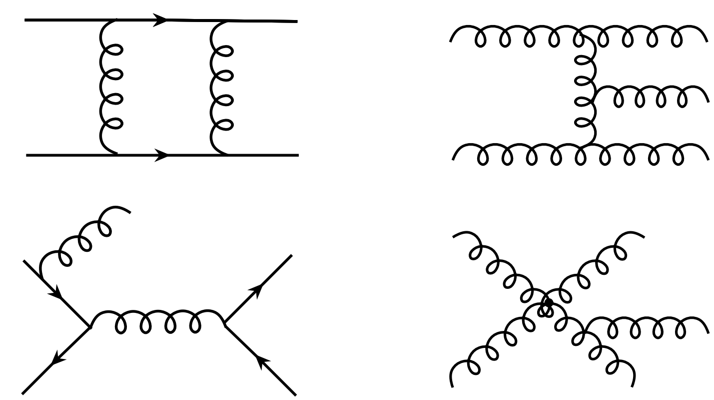
LO and NLO contributions

$LO - \mathcal{O}(\alpha_s^2)$

- all possible tree diagrams for 10 elementary $2 \to 2$ processes:

$$
\begin{array}{llllll}
qq' & \to & qX, & q\bar{q}' & \to & qX, \\
q\bar{q} & \to & q'X, & qq & \to & qX, \\
q\bar{q} & \to & qX, & q\bar{q} & \to & gX, \\
qg & \to & qX, & qg & \to & gX, \\
gg & \to & gX, & gg & \to & qX.
\end{array}
$$





$NLO - \mathcal{O}(\alpha_s^3)$

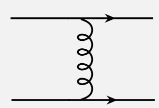
- virtual corrections to all $2 \to 2$ diagrams
- $2 \to 3$ diagrams for these and 6 additional processes:

$$
\begin{array}{llllll}
qq' & \to & gX, & q\bar{q}' & \to & gX, \\
qq & \to & gX, & qg & \to & q'X, \\
qg & \to & \bar{q}'X, & qg & \to & \bar{q}X.
\end{array}
$$

Sample Process: $qq' \to qX$

LO Matrix Elements

- evaluate all contributing Feynman diagrams (tree level only)

$$|M|^2 = \Big|$$  $$\Big|^2$$

$$p_1 + p_2 \to p_3 + p_4$$

- Mandelstam variables:

$$s = (p_1 + p_2)^2$$
$$t = (p_1 - p_3)^2$$
$$u = (p_1 - p_4)^2$$

$$s + t + u = 0$$

Implementation of Polarization

polarized quarks / gluons

$\updownarrow$

$\gamma_5, \varepsilon^{\mu\nu\rho\sigma}$

. . . apply HVBM scheme:

n-dim space

4dim	$(n-4)$dim
$\{\gamma_5, \gamma^\mu\} = 0$	$[\gamma_5, \gamma^\mu] = 0$
$\varepsilon^{\mu\nu\rho\sigma}$	$\hat{\varepsilon}^{\mu\nu\rho\sigma} = 0$
$(k_0, k_1, k_2, k_3, \vec{0})$	$(0, 0, 0, 0, \hat{k})$

$\to$ complicated phase space

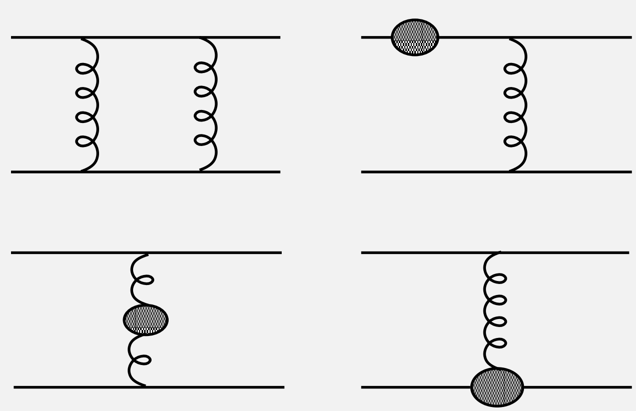
Virtual Corrections

Virtual Corrections

…same kinematics as LO

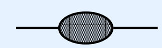
$\mathcal{O}(\alpha_S^3)$: interference of Born diagrams with

– box diagrams
– vertex corrections
– selfenergy corrections

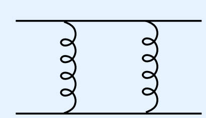


Calculation


: IR + UV divergencies

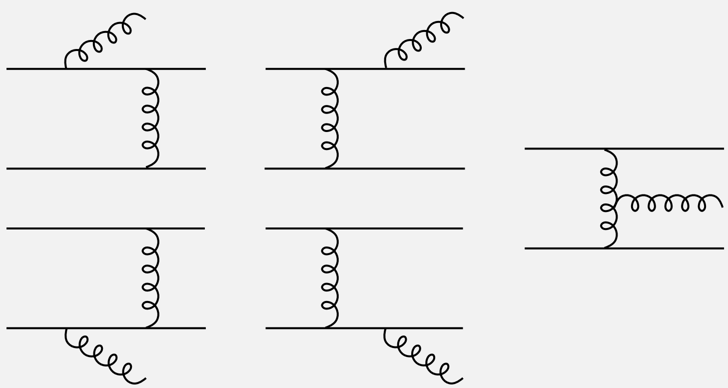
use  tabulated by *Nowak, Praszalowicz, Slominski*

…UV-divergencies subtracted in n dimensions at arbitrary renormalization scale μ_r


: UV – finite calculate from scratch in n dimensions

$2 \to 3$ *processes*

number of diagrams and interference terms significantly increases



$\updownarrow$

$$|M|^2 = |M_1 + M_2 + M_3 + M_4 + M_5|^2$$

Phase Space Integration

calculated up to now: $qq' \to qq'g$

needed $qq' \to qX$

$\updownarrow$

integrate out unobserved particles (parametrized by two angles $\theta_{1,2}$)

$$d\hat{\sigma}_{2\to3} \sim \int d\theta_1 d\theta_2 \sin^{1-2\epsilon}\theta_1 \times \sin^{-2\epsilon}\theta_2 |M_{2\to3}|^2$$

…additional integral $\tilde{I}(\hat{k}^2)$ for $\hat{k}$-terms of polarized contributions

each term can be cast into (extensive partial fractioning)

$$I^{(k,l)} = \int \frac{d\theta_1 \sin^{1-2\epsilon}\theta_1 d\theta_2 \sin^{-2\epsilon}\theta_2}{(1+\cos\theta_1)^k (1 + A\cos\theta_1 + B\sin\theta_1\cos\theta_2)^l}$$

$\to$ can be done analytically

Cancelation of Singularities

UV divergencies

$\updownarrow$

renormalization of α_S at scale μ_r

collinear $1/\varepsilon$-singularities

$\updownarrow$

factorization

sum of all real and virtual contributions to one observed process: finite for $\varepsilon \to 0$

IR $1/\varepsilon$-singularities

$\updownarrow$

cancel in sum of 1-loop and $2 \to 3$ contributions

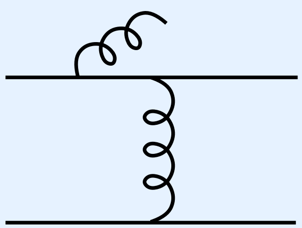
$1/\varepsilon^2$-singularities

$\updownarrow$

IR and collinear singularities coincide : cancel in sum

Factorization of Collinear Singularities

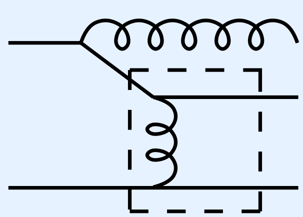
unobserved particle radiated off collinearly from initial/final state parton:



$2 \to 3$ diagrams become singular ($1/\varepsilon$-poles)

remove by factorization:

subtraction of poles



$$\sim \frac{1}{\varepsilon} \int dx \, \Delta P_{qq}(x) \, \Delta\hat{\sigma}_{qq' \to qq'}$$

arbitrary initial/final state factorization scales in PDFs and FFs: $\Delta f(x, \mu_f)$, $D_i^{\pi}(x, \mu'_f)$

finite results for partonic matrix elements

$\vec{p}\vec{p} \to \pi X$

Numerics

WANTED!! ☞

$$E_\pi \frac{d\Delta\sigma^{\vec{p}\vec{p}\to\pi X}}{d^3p_\pi} = \sum_{a,b,c} \Delta f_a(\mu_f) \otimes \Delta f_b(\mu_f) \otimes D_c^\pi(\mu'_f) \otimes E_c \frac{d\Delta\hat{\sigma}^{\vec{a}\vec{b}->cX'}}{d^3p_c}(\mu_f, \mu'_f, \mu_r)$$

convolutions: Monte Carlo integration

FFs: *Kniehl, Kramer, Pötter*

unpolarized PDFs: CTEQ5

polarized PDFs: GRSV

$\vec{p}\vec{p} \to \pi X$

Single Pion Inclusive Cross Section

input at $\sqrt{S}$ =200 GeV
(RHIC c.m. energy):
scales: $\mu_r = \mu_f = \mu'_f = p_T$

	unp.	pol.
LO :	CTEQ5L	GRSVstd.(LO)

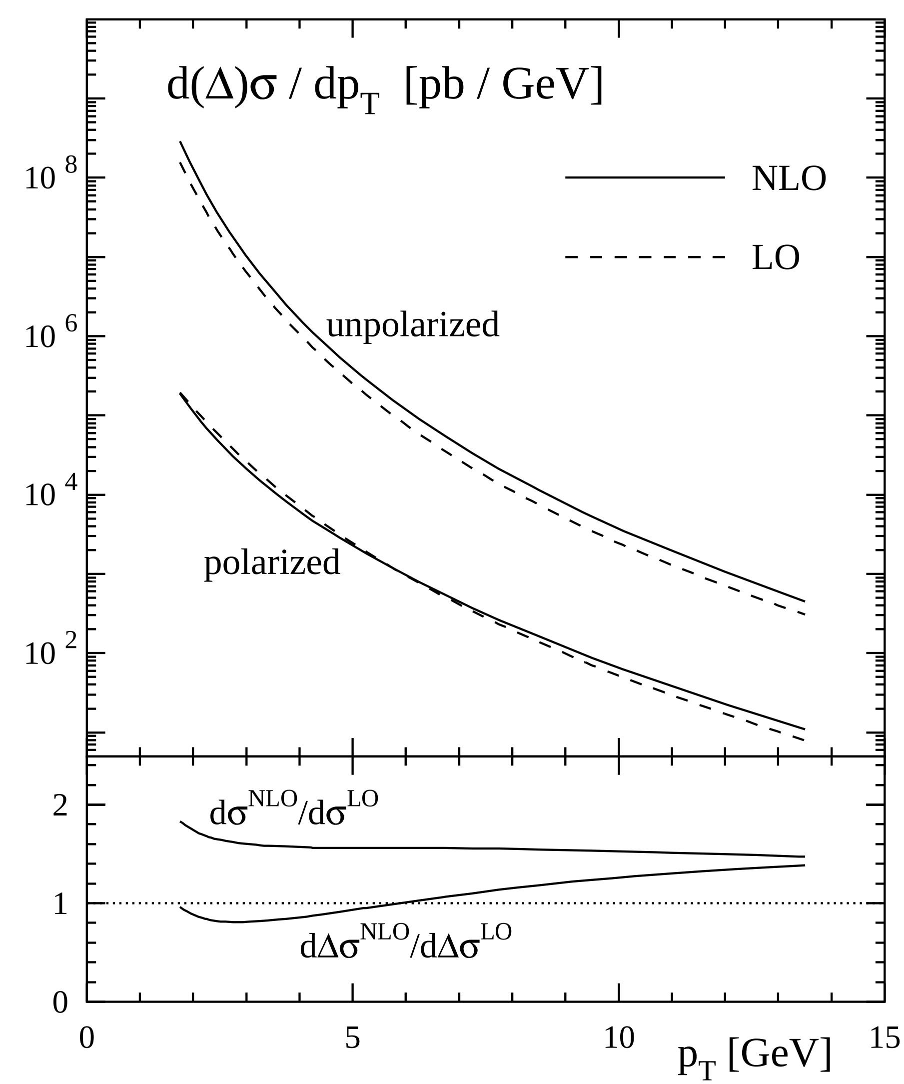
KKP(LO), α_S at one loop

NLO :	CTEQ5M	GRSVstd.(NLO)

KKP(NLO), α_S at two loops

"K-factor": $K = \frac{d(\Delta)\sigma^{NLO}}{d(\Delta)\sigma^{LO}}$

"measure" for importance of NLO corrections

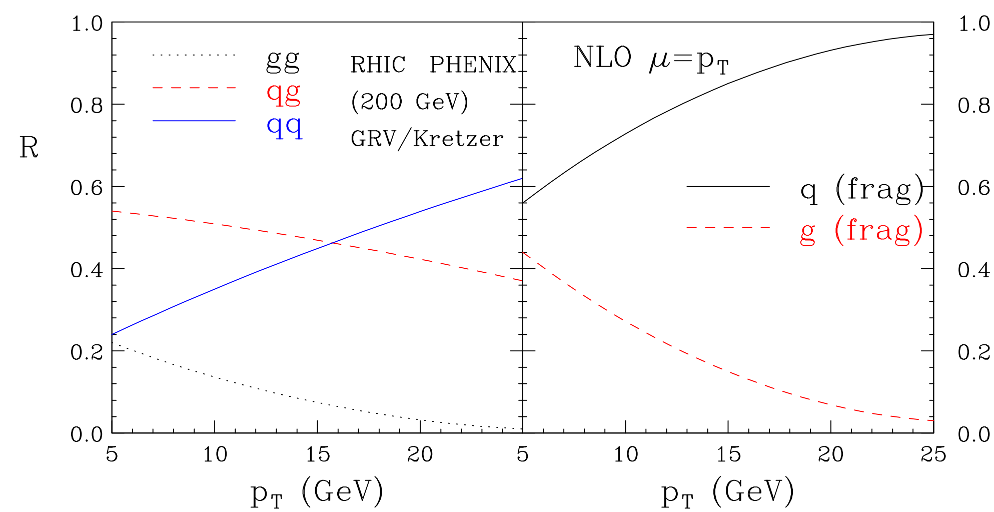


Related Works

– *Aversa et al.*,
NP B327,1989:
unp. pion production
…used for comparison with our unp. results

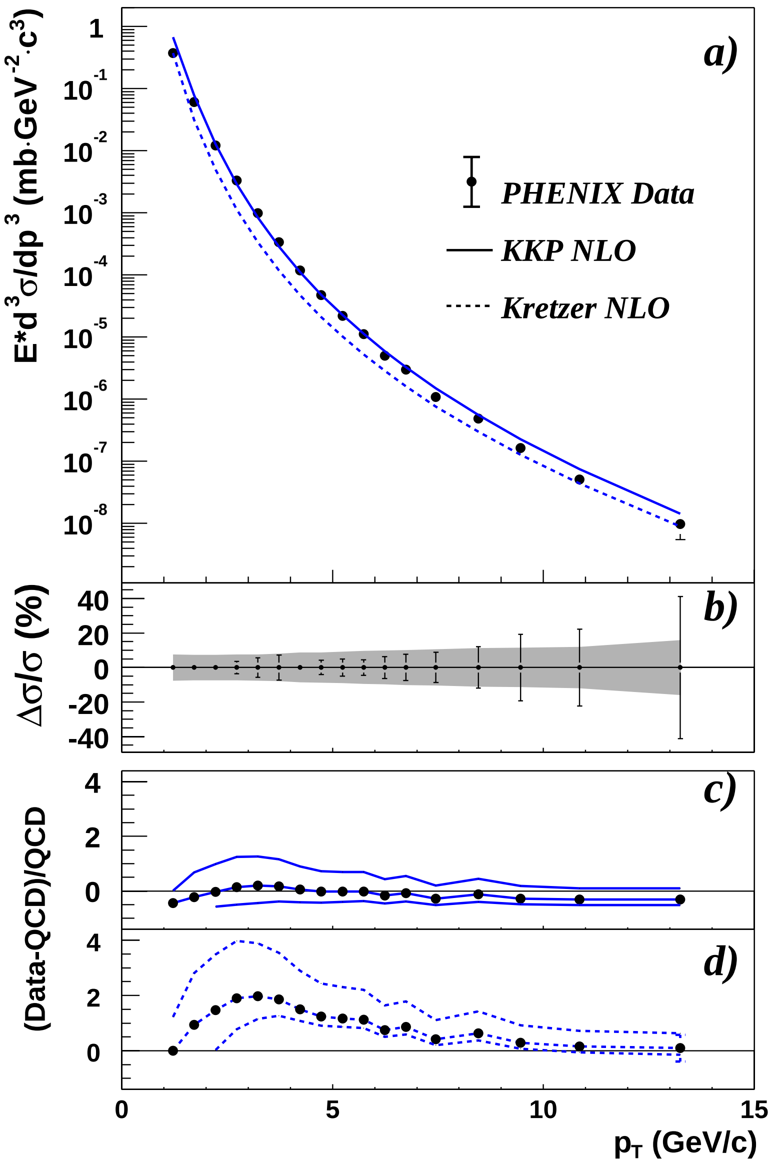
– *D. de Florian*,
PR D67, 2003:
$\vec{p}\vec{p} \to \pi X$
in Monte Carlo approach
…results agree fairly well with our predictions (detailed comparison in progress)

ratios of the unpolarized cross sections for different combinations of initial (left) and final (right) partonic states, taken from *de Florian*



$pp \to \pi X$

Recent Results from PHENIX



data taken from $p + p$ collisions during run-02 *(hep-ex/0304038)*:

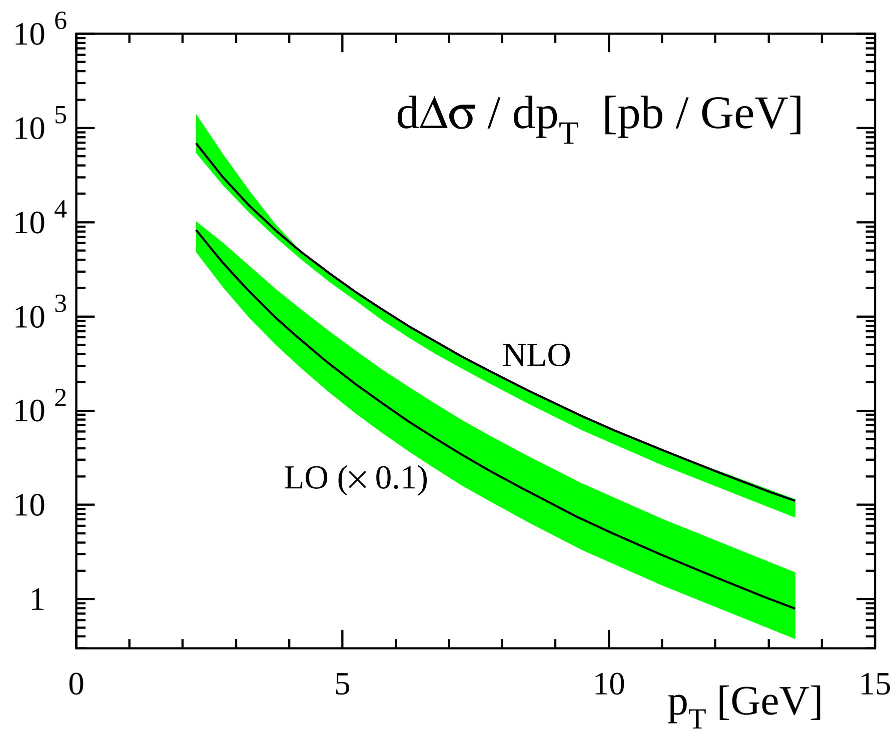
a) data points and NLO predictions for unpolarized differential cross section $E^{\pi}(d^3\sigma/dp_{\pi}^3)$

b) relative statistical errors (points) and systematic errors (bands) of data

c, d) relative difference between data and theoretical predictions

$\vec{p}\vec{p} \to \pi X$

Scale Dependence



recall: motivation

NLO corrections expected to

reduce dependence on

unphysical scales

↓

study variation of scales

in typical range

$$p_T/2 \leq \mu_r = \mu_f = \mu'_f \leq 2p_T.$$

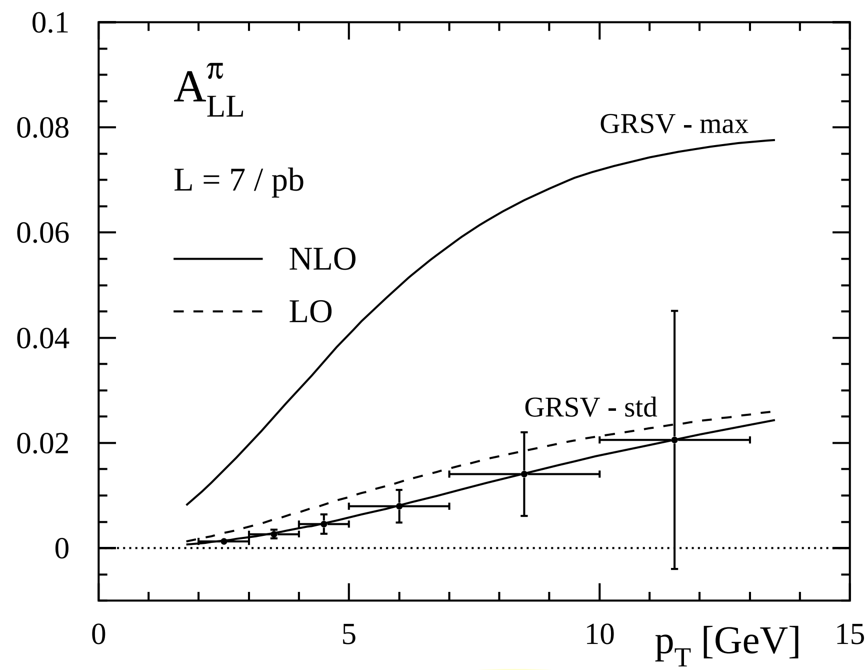
$\vec{p}\vec{p} \to \pi X$

Double Spin Asymmetry $A_{LL}^{\pi^0}$

… defined by

$$A_{LL}^{\pi} = \frac{d\Delta\sigma}{d\sigma} = \frac{d\sigma^{++} - d\sigma^{--}}{d\sigma^{++} + d\sigma^{--}}$$

- very sensitive to Δg through polarized qg and gg scattering
- some difference between LO and NLO
- expected experimental errors at RHIC smaller than difference between sets of PDFs with different gluon densities

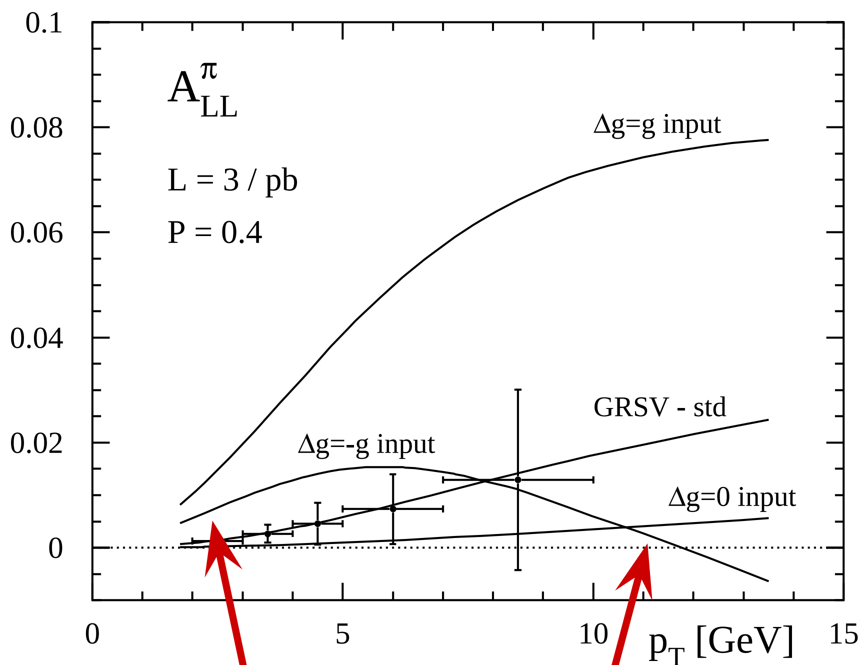


data should allow for determination of Δg even at rather low luminosities (design $\mathcal{L}$: 320 pb^{-1})

$\vec{p}\vec{p} \to \pi^0 X$

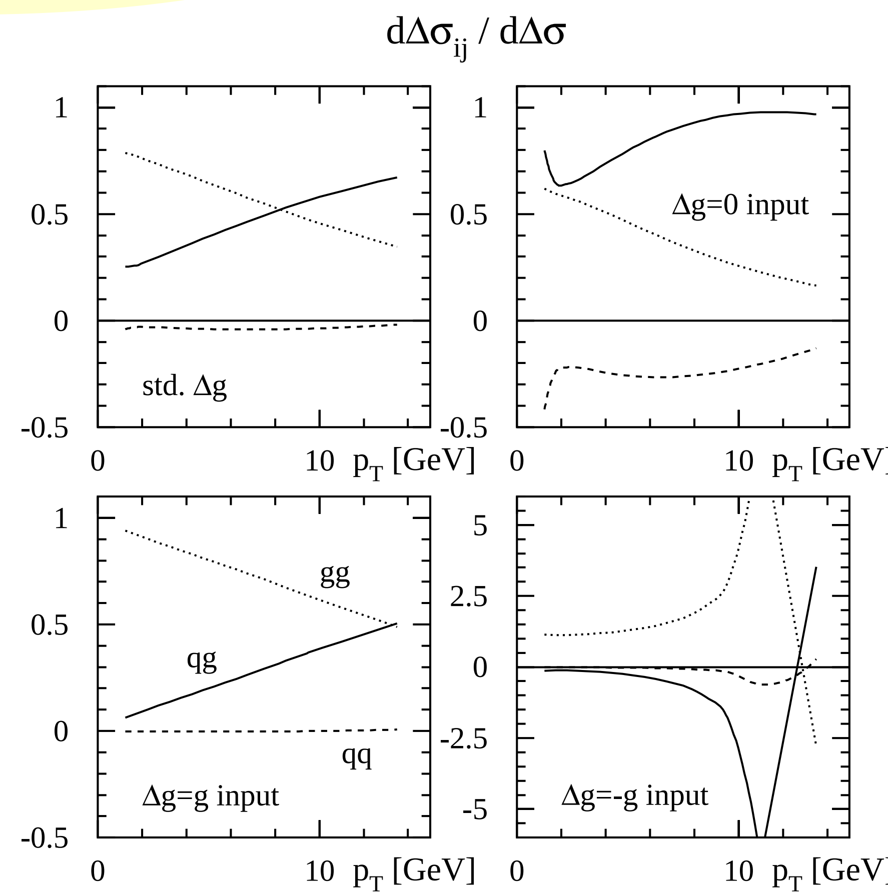
$A_{LL}^{\pi^0}$ ... *surprise?!*

Naohito: "What happens for negative Δg?"



gg dominates

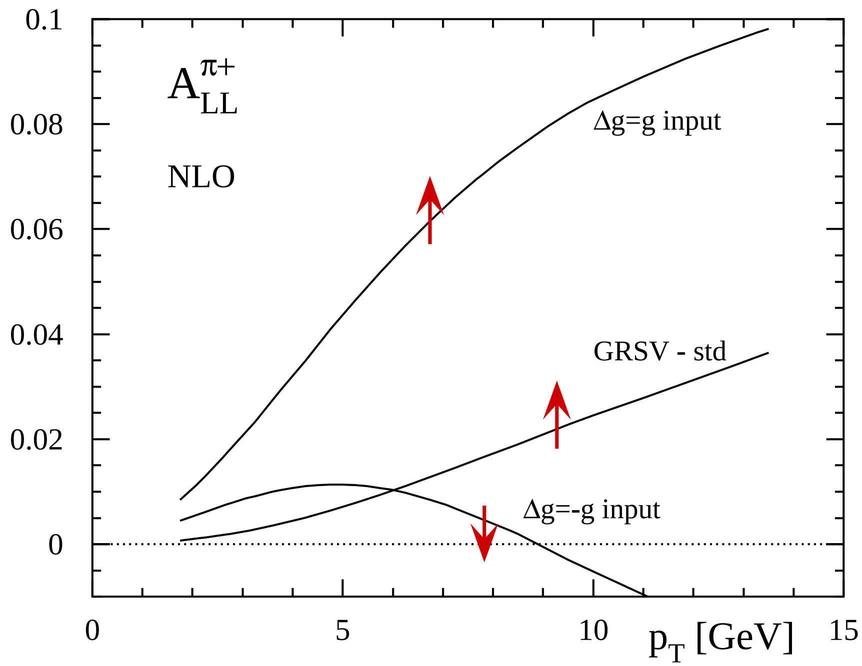
qg takes over

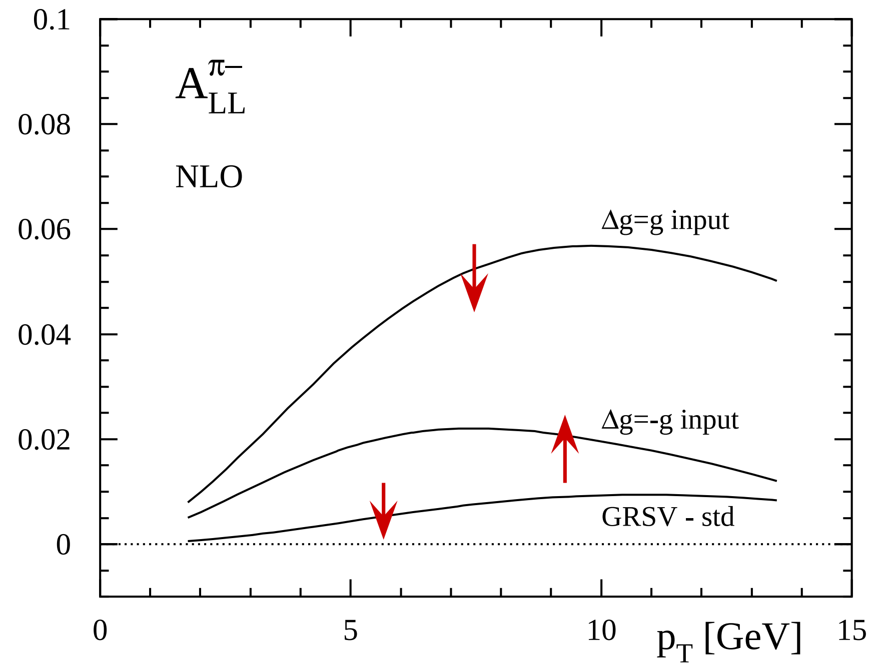


way out: study π^+ *and* π^-

positive Δg: $A_{LL}^{\pi^+} > A_{LL}^{\pi^0}$
negative Δg: $A_{LL}^{\pi^+} < A_{LL}^{\pi^0}$

positive Δg: $A_{LL}^{\pi^-} < A_{LL}^{\pi^0}$
negative Δg: $A_{LL}^{\pi^-} > A_{LL}^{\pi^0}$

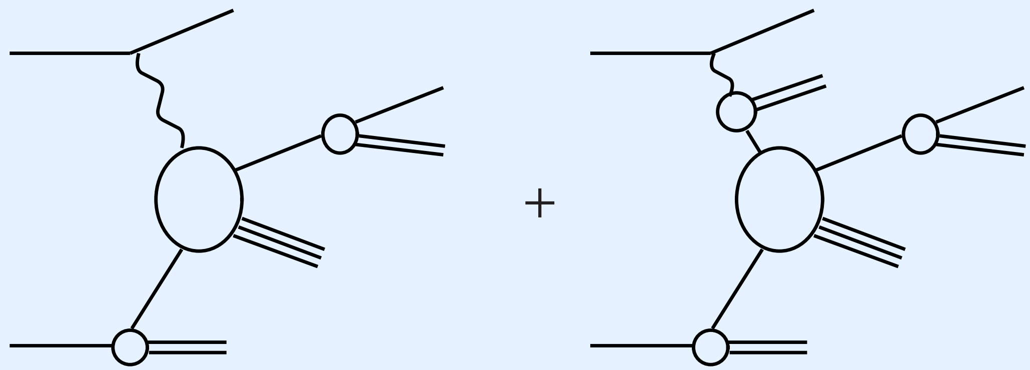




…only at $p_T > 5$ GeV, good statistics required

Photoproduction of Inclusive Pions

$$\vec{\gamma}\vec{p} \to \pi X$$



- predictions for eRHIC

…gaining info on parton
content of the polarized photon
(completely unmeasured so far)

- LO results promising

(*Stratmann*, *Vogelsang*)

Photon Distribution Functions Δf^γ

$$\Delta f^\gamma = f_+^{\gamma+} - f_-^{\gamma+}$$
$$f^\gamma = f_+^{\gamma+} + f_-^{\gamma+}$$

…spin dependent parton densities in the photon completely unmeasured so far

⇓

need models to describe processes involving Δf^γ

constraint: positivity

$$|\Delta f^\gamma(x,\mu^2)| \le f^\gamma(x,\mu^2)$$

…construct two extreme models:

’maximal’ input

$$\Delta f^\gamma(x,\mu_0^2) = f^\gamma_{GRV}(x,\mu_0^2)$$

pure VMD input at μ_0

’minimal’ input

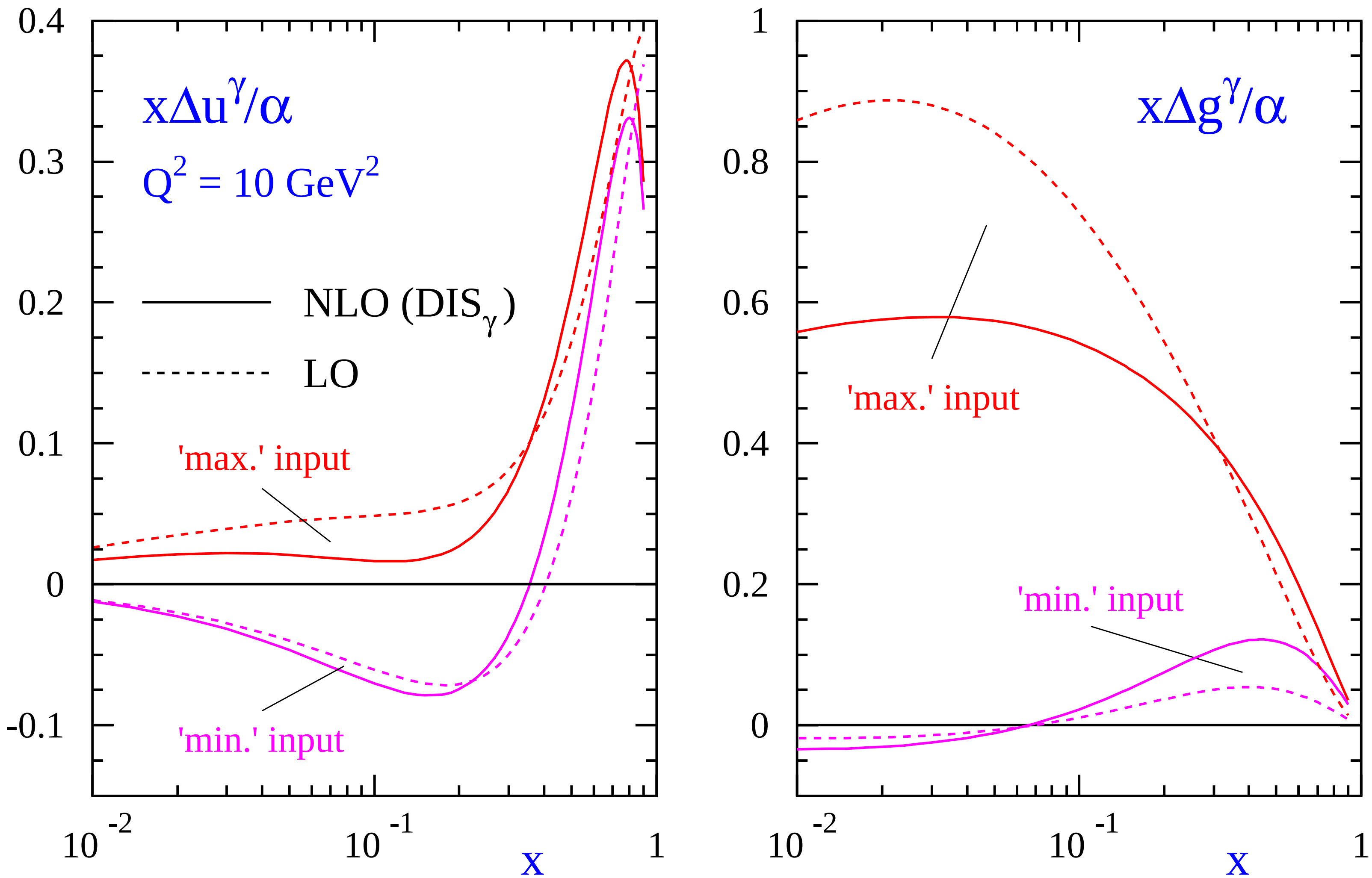
$$\Delta f^\gamma(x,\mu_0^2) = 0$$

pointlike for all μ

---

at an input scale for QCD evolution $\mu_0 \sim 0.6$ GeV

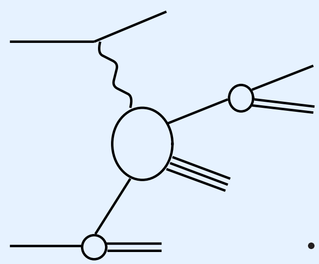
Photon Scenarios



$\vec{\gamma}\vec{p} \to \pi X$

Direct & Resolved Contributions

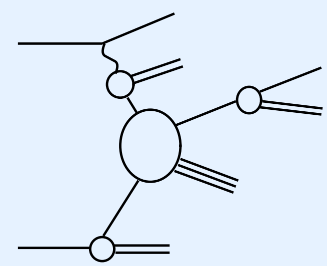
$$E_\pi \frac{d\Delta\sigma^{\vec{\gamma}\vec{p}\to\pi X}_{dir/res}}{d^3p_\pi} = \sum_{a,b,c} \Delta f_a^l(x_l,\mu_f) \otimes \Delta f_b^p(x_p,\mu_f) \otimes D_c^\pi(z,\mu_f') \otimes E_c \frac{d\Delta\hat{\sigma}^{\vec{a}\vec{b}->cX'}_{dir/res}}{d^3p_c}(\mu_f,\mu_f',\mu_r)$$



direct part: $\Delta f_a^l = \Delta f_\gamma^l$

$\ldots$ Weizsäcker-Williams spectrum

$$\Delta f_\gamma^l(y) = \frac{\alpha_{em}}{2\pi}\left\{2m_l^2y^2\left(\frac{1}{Q^2_{max}} - \frac{1-y}{m_l^2y^2}\right) + \left[\frac{1-(1-y)^2}{y}\right]\ln\frac{Q^2_{max}(1-y)}{m_l^2y^2}\right\}$$



resolved part:

$\Delta f_a^l \ldots$ effective density for finding parton a in longitudinally polarized lepton l,

$$\Delta f_a^l(x_l) = \int_{x_l}^1 \frac{dy}{y} \Delta f_\gamma^l(y) \Delta f_a^\gamma\left(x_\gamma = \frac{x_l}{y}\right)$$

$\vec{\gamma}\vec{p} \to \pi X$

π^0-*Photoproduction Cross Section*

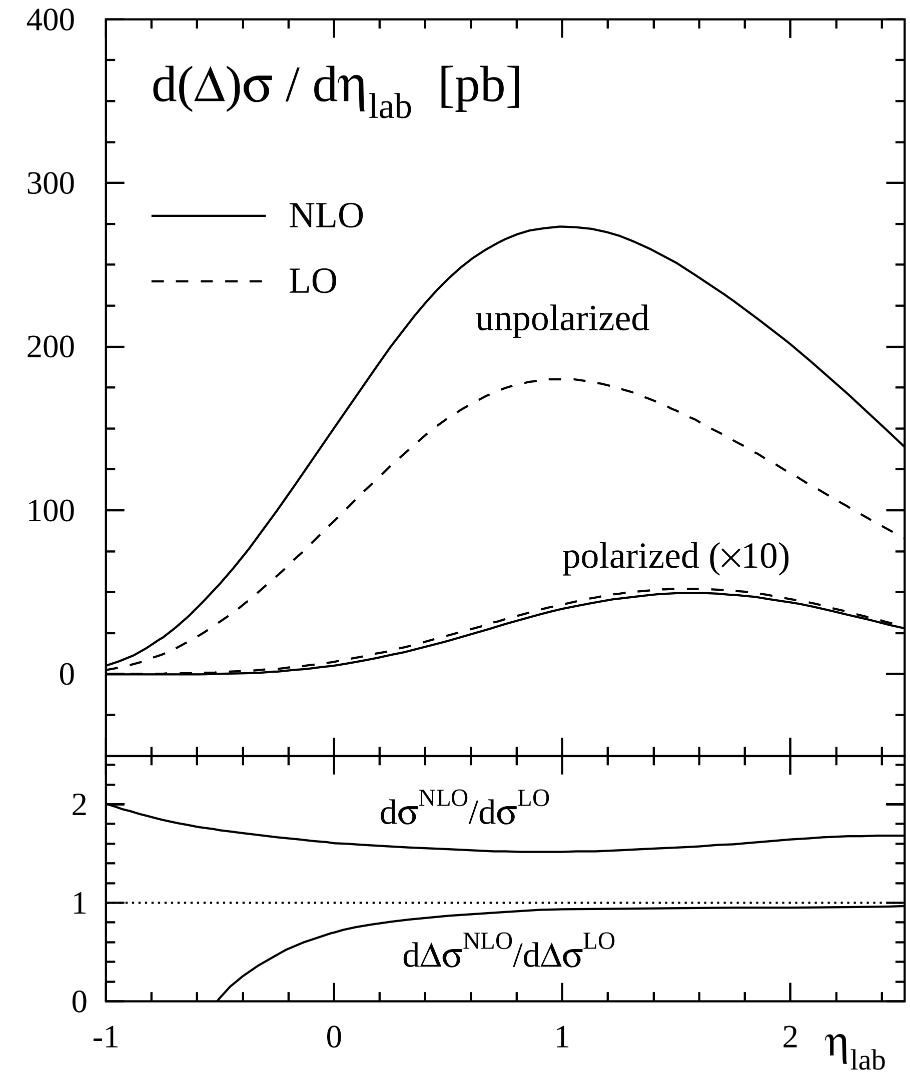
input at $\sqrt{S}$ =100 GeV
(proposed c.m.energy for eRHIC):

scales: $\mu_r = \mu_f = \mu'_f = p_T$

	unp.	pol.
proton :	CTEQ5	GRSVstd.
pion :	KKP – fragmentation functions	

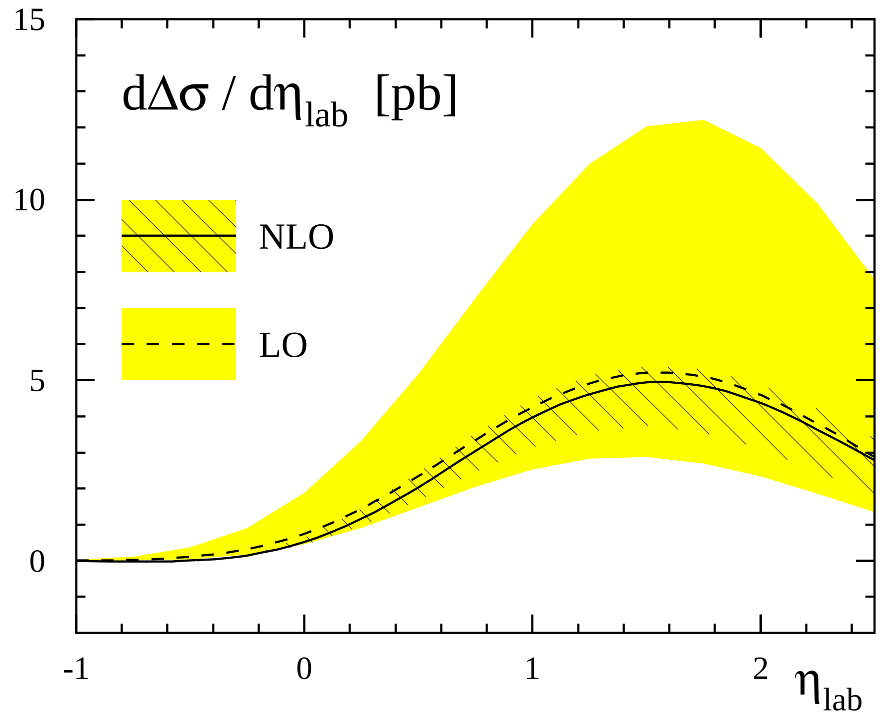
photon: WW-equivalent photon spectrum, parameters resemble H1 and ZEUS at HERA:

$$Q^2_{max} = 1\mathrm{GeV}^2,$$
$$0.2 \leq y \leq 0.85.$$



$\vec{\gamma}\vec{p} \to \pi X$

Scale Dependence



study variation of scales

in typical range

$$p_T/2 \leq \mu_r = \mu_f = \mu'_f \leq 2p_T$$

↓

NLO corrections

reduce dependence on

unphysical scales

$\vec{\gamma}\vec{p} \to \pi X$

Double Spin Asymmetry $A_{LL}^{\pi^0}$

…defined by

$$A_{LL}^{\pi} = \frac{d\Delta\sigma}{d\sigma} = \frac{d\sigma^{++} - d\sigma^{--}}{d\sigma^{++} + d\sigma^{--}}$$

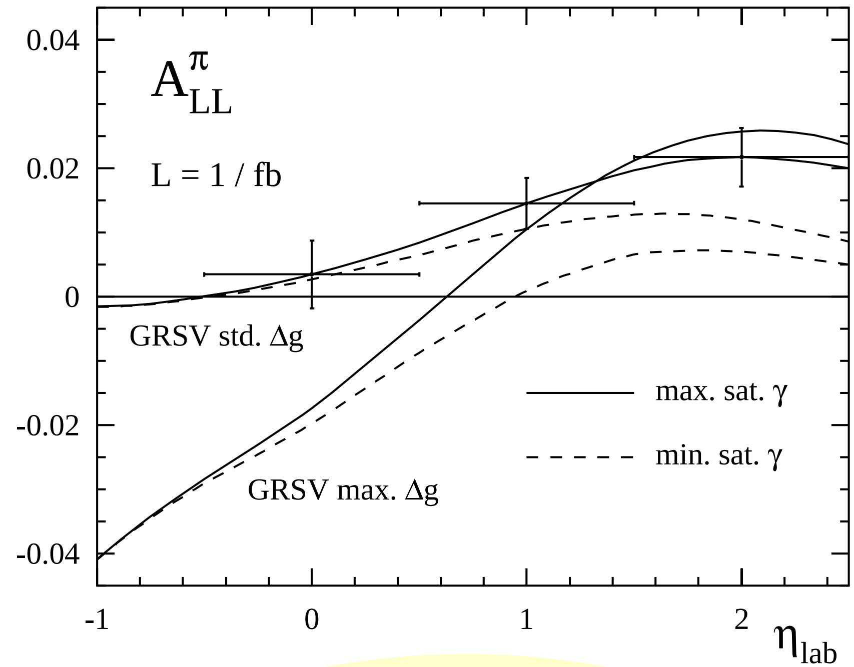
- assume Δg sufficiently known

↓

sensitive to Δf^{γ} at large positive rapidities

- expected experimental errors at eRHIC ($\mathcal{P}_{p,e} = 0.7$):

$$\delta A_{LL}^{\pi} \simeq \frac{1}{\mathcal{P}_e \mathcal{P}_p \sqrt{\mathcal{L}\,\sigma_{\rm bin}}}$$



data should yield information on Δf^{γ} even at rather low luminosities (estimate for $\mathcal{L} = 1\ \text{fb}^{-1}$)

NLO Calculations in Progress

Single Inclusive Λ Production

$$\vec{p}\,p \to \vec{\Lambda} X$$

$$\hookrightarrow p\,\pi^-$$

- polarization experimentally accessible through selfanalyzing weak decay
- helicity transfer $\to$ info on polarized Λ-fragmentation function ΔD_c^{Λ}
- LO results available by *de Florian, Stratmann, Vogelsang, PRL 81 (1998)*

Single Inclusive Jet Production

$$\vec{p}\,\vec{p} \to \text{jet } X$$

- calculation proceeds along similar ways as for π production
  - modifications:
    - treatment of final state (singularities)
    - phase space integration
  - measurements at RHIC expected soon (STAR)
  - source for Δg

Conclusions

computed NLO QCD corrections to

$$\vec{p}\vec{p} \to \pi X \quad \& \quad \vec{\gamma}\vec{p} \to \pi X$$

↓

- reduce dependence on unphysical scales
- open up ways for first determination of Δg
  and, afterwards, Δf^{γ}

☞ hope for data from RHIC & eRHIC

- QCD calculations can now be challenged by experiment
  - for many years to come main information on spin
    will be provided by BNL