Nucleon Matrix Elements in lattice QCD

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- What Lattice QCD can do?
- Domain Wall Fermions.
- Nucleon Structure functions.
- What is our status and our plan.
- Discussion.

What Lattice QCD can do?

The expectation value of any observable $\mathcal{O}$ is:

$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int dU \; \mathcal{O} \; det(\not{D})^{n_f} \; e^{-S_g[U_\mu(x)]} \tag{1}$$

Quenching: $det(\not{D})^{n_f} = 1$

Two point function:

$$C_{2pt}(t) = \frac{1}{\mathcal{Z}} \int dU \; S^\dagger(t) \, S(0) \;\; e^{-S_g[U_\mu(x)]} \tag{2}$$

Three point function:

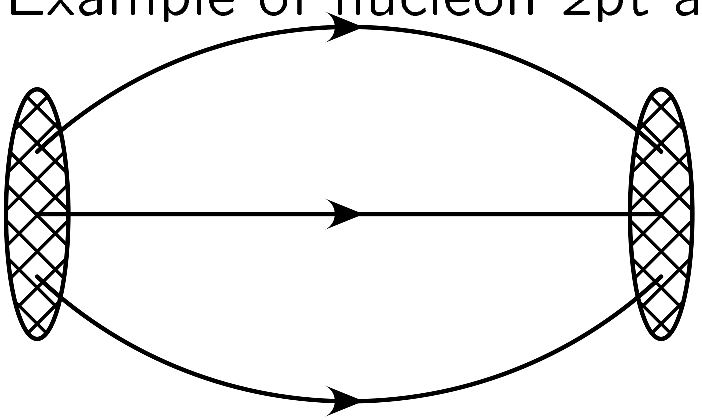
$$C_{3pt}(t, t_i) = \frac{1}{\mathcal{Z}} \int dU \; S^\dagger(t) \, \mathcal{O}(t_i) S(0) \;\; e^{-S_g[U_\mu(x)]} \tag{3}$$

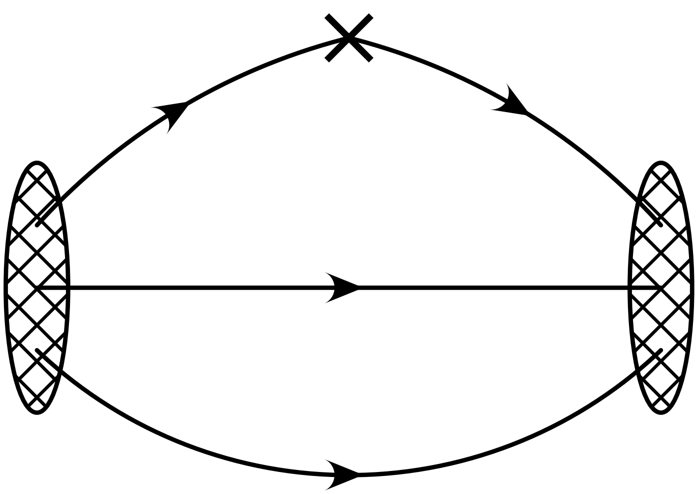
Matrix Element:

$$\langle S | \mathcal{O} | S \rangle = C_{3pt}(t, t_i) / C_{2pt}(t) \tag{4}$$

t,t_i and $t - t_i$ is chosen to be large enough so that the ground state $|S\rangle$ is dominating.

Example of nucleon 2pt and 3pt functions:

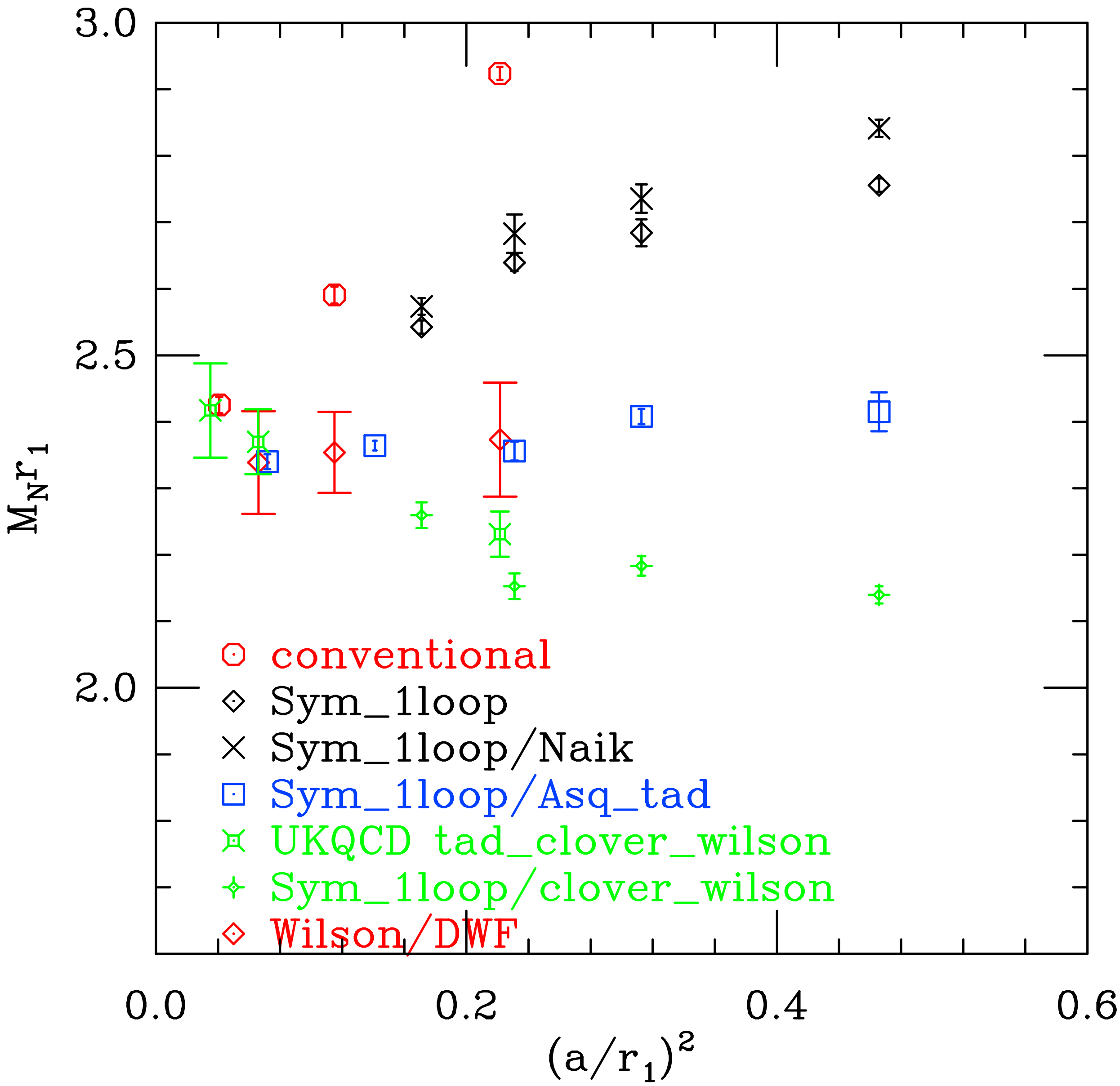


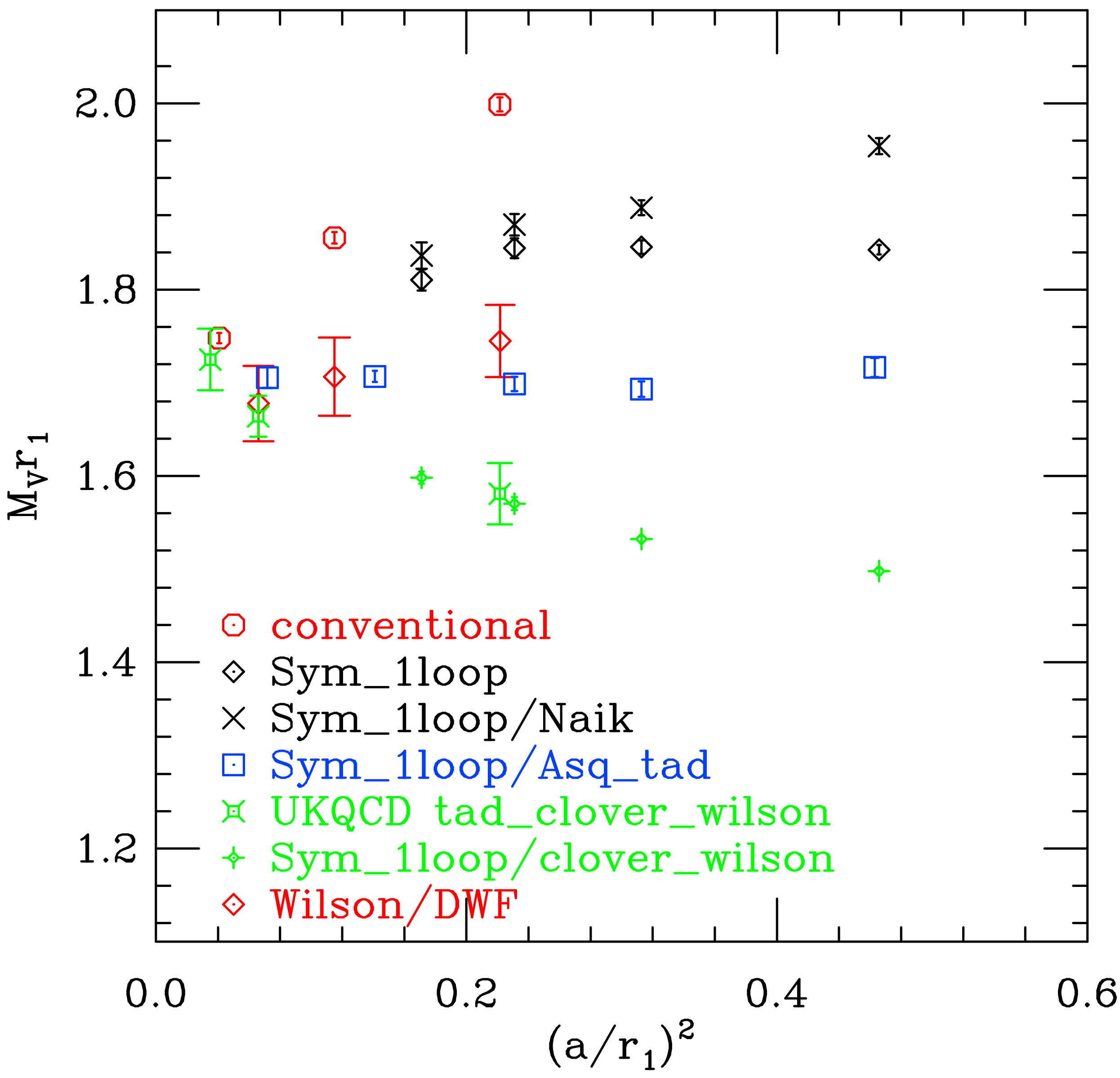


DW Fermions

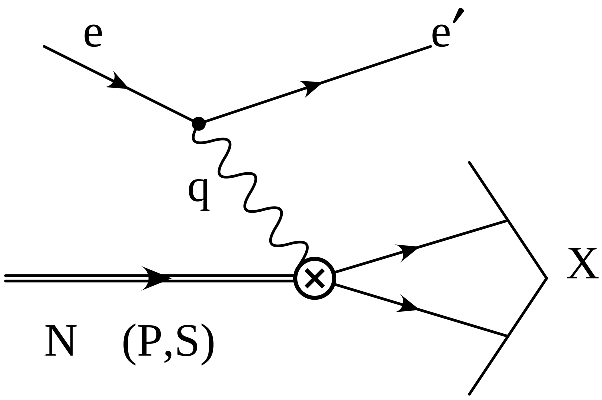
The Domain Wall Fermions are a particular lattice descritization of fermions.

- Have very good chiral properties
- Have $O(a^2)$ errors
- Should have very good scaling properties
- Are particularly useful in studding phenomena where chiral symmetry is important.
- Used by the RBC group!


3.0
2.5
2.0
$M_N r_1$
0.0
0.2
0.4
0.6
$(a/r_1)^2$
conventional
Sym_1loop
Sym_1loop/Naik
Sym_1loop/Asq_tad
UKQCD tad_clover_wilson
Sym_1loop/clover_wilson
Wilson/DWF


2.0
1.8
1.6
1.4
1.2
$M_V r_1$
0.0
0.2
0.4
0.6
$(a/r_1)^2$
conventional
Sym_1loop
Sym_1loop/Naik
Sym_1loop/Asq_tad
UKQCD tad_clover_wilson
Sym_1loop/clover_wilson
Wilson/DWF

Nucleon Structure Functions



$$\left|\frac{\mathcal{A}}{4\pi}\right|^2 = \frac{\alpha^2}{Q^4} l^{\mu\nu} W_{\mu\nu}$$

$$W^{\mu\nu} = W^{[\mu\nu]} + W^{\{\mu\nu\}}$$

$$\begin{aligned} W^{\{\mu\nu\}}(x,Q^2) &= \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) F_1(x,Q^2) \\ &+ \left(P^\mu - \frac{\nu}{q^2} q^\mu\right)\left(P^\nu - \frac{\nu}{q^2} q^\nu\right) \frac{F_2(x,Q^2)}{\nu} \\ W^{[\mu\nu]}(x,Q^2) &= i\epsilon^{\mu\nu\rho\sigma} q_\rho \left(\frac{S_\sigma}{\nu}(g_1(x,Q^2) + g_2(x,Q^2)) - \right. \\ &- \left. \frac{q \cdot S P_\sigma}{\nu^2} g_2(x,Q^2)\right) \end{aligned} \tag{5}$$

with $\nu = q \cdot P$, $S^2 = -M^2$, $x = Q^2/2\nu$

Moments of Structure Functions

$$
\begin{aligned}
2\int_0^1 dx x^{n-1} F_1(x,Q^2) &= \sum_{f=u,d} c_{1,n}^{(f)}(\mu^2/Q^2, g(\mu))\, v_n^{(f)}(\mu), \\
\int_0^1 dx x^{n-2} F_2(x,Q^2) &= \sum_{f=u,d} c_{2,n}^{(f)}(\mu^2/Q^2, g(\mu))\, v_n^{(f)}(\mu), \\
2\int_0^1 dx x^{n} g_1(x,Q^2) &= \frac{1}{2}\sum_{f=u,d} e_{1,n}^{(f)}(\mu^2/Q^2, g(\mu))\, a_n^{(f)}(\mu), \\
2\int_0^1 dx x^{n} g_2(x,Q^2) &= \frac{1}{2}\frac{n}{n+1}\sum_{f=u,d} [e_{2,n}^{(f)}(\mu^2/Q^2, g(\mu))\, d_n^{(f)}(\mu) \\
&- e_{1,n}^{(f)}(\mu^2/Q^2, g(\mu))\, a_n^{(f)}(\mu)],
\end{aligned}
$$

where c_1, c_2 and e_1, e_2 are the Wilson coefficients, and v_n, a_n and d_n are forward nucleon matrix elements of certain local operators $\mathcal{O}$.

$$\frac{1}{2}\sum_s \langle P,S|\mathcal{O}^f_{\{\mu_1\mu_2\cdots\mu_n\}}|P,S\rangle = 2v_n^{(f)}(\mu)[P_{\mu_1}P_{\mu_2}\cdots P_{\mu_n} - traces]$$

$$\mathcal{O}_{\mu_1\mu_2\cdots\mu_n} = \left(\frac{i}{2}\right)^{n-1} \bar{\Psi}\gamma_{\mu_1} \overleftrightarrow{D}_{\mu_2} \cdots \overleftrightarrow{D}_{\mu_n} \Psi - traces$$

$$\frac{1}{2}\sum_s \langle P,S|\mathcal{O}^{5f}_{\{\sigma\mu_1\mu_2\cdots\mu_n\}}|P,S\rangle = \frac{1}{n+1}a_n^{(f)}(\mu)[S_\sigma P_{\mu_1}P_{\mu_2}\cdots P_{\mu_n} - traces]$$

$$\frac{1}{2}\sum_s \langle P,S|\mathcal{O}^{5f}_{[\sigma\{\mu_1]\mu_2\cdots\mu_n\}}|P,S\rangle = \frac{1}{n+1}d_n^{(f)}(\mu)[S_\sigma P_{\mu_1}P_{\mu_2}\cdots P_{\mu_n} - traces]$$

$$\mathcal{O}^{5f}_{\sigma\mu_1\mu_2\cdots\mu_n} = \left(\frac{i}{2}\right)^{n} \bar{\Psi}\gamma_\sigma\gamma_5 \overleftrightarrow{D}_{\mu_2} \cdots \overleftrightarrow{D}_{\mu_n} \Psi - traces$$

	H(4)	mix	$\vec{p}$	lattice operator
$xq_c^{(a)}$	$\mathbf{6}_3^+$	no	$\neq 0$	$\bar{q}\gamma_{\{1}\overleftrightarrow{D}_{4\}}q$
$xq_c^{(b)}$	$\mathbf{3}_1^+$	no	0	$\bar{q}\gamma_4\overleftrightarrow{D}_4 q$
				$-\frac{1}{3}\sum_{i=1}^{3}\bar{q}\gamma_i\overleftrightarrow{D}_i q$
x^2q_c	$\mathbf{8}_1^-$	yes	$\neq 0$	$\bar{q}\gamma_{\{1}\overleftrightarrow{D}_1\overleftrightarrow{D}_{4\}}q$
				$-\frac{1}{2}\sum_{i=2}^{3}\bar{q}\gamma_{\{i}\overleftrightarrow{D}_i\overleftrightarrow{D}_{4\}}q$
x^3q_c	$\mathbf{2}_1^+$	no*	$\neq 0$	$\bar{q}\gamma_{\{1}\overleftrightarrow{D}_1\overleftrightarrow{D}_4\overleftrightarrow{D}_{4\}}q$
				$+\bar{q}\gamma_{\{2}\overleftrightarrow{D}_2\overleftrightarrow{D}_3\overleftrightarrow{D}_{3\}}q$
				$-(3 \leftrightarrow 4)$
Δq_c	$\mathbf{4}_4^+$	no	0	$\bar{q}\gamma^5\gamma_3 q$
$x\Delta q_c^{(a)}$	$\mathbf{6}_3^-$	no	$\neq 0$	$\bar{q}\gamma^5\gamma_{\{1}\overleftrightarrow{D}_{3\}}q$
$x\Delta q_c^{(b)}$	$\mathbf{6}_3^-$	no	0	$\bar{q}\gamma^5\gamma_{\{3}\overleftrightarrow{D}_{4\}}q$
$x^2\Delta q_c$	$\mathbf{4}_2^+$	no	$\neq 0$	$\bar{q}\gamma^5\gamma_{\{1}\overleftrightarrow{D}_3\overleftrightarrow{D}_{4\}}q$
δq_c	$\mathbf{6}_1^+$	no	0	$\bar{q}\gamma^5\sigma_{34}q$
$x\delta q_c$	$\mathbf{8}_1^-$	no	$\neq 0$	$\bar{q}\gamma^5\sigma_{3\{4}\overleftrightarrow{D}_{1\}}q$
d_1	$\mathbf{6}_1^+$	no**	0	$\bar{q}\gamma^5\gamma_{[3}\overleftrightarrow{D}_{4]}q$
d_2	$\mathbf{8}_1^-$	no**	$\neq 0$	$\bar{q}\gamma^5\gamma_{[1}\overleftrightarrow{D}_{\{3]}\overleftrightarrow{D}_{4\}}q$

Status and plans

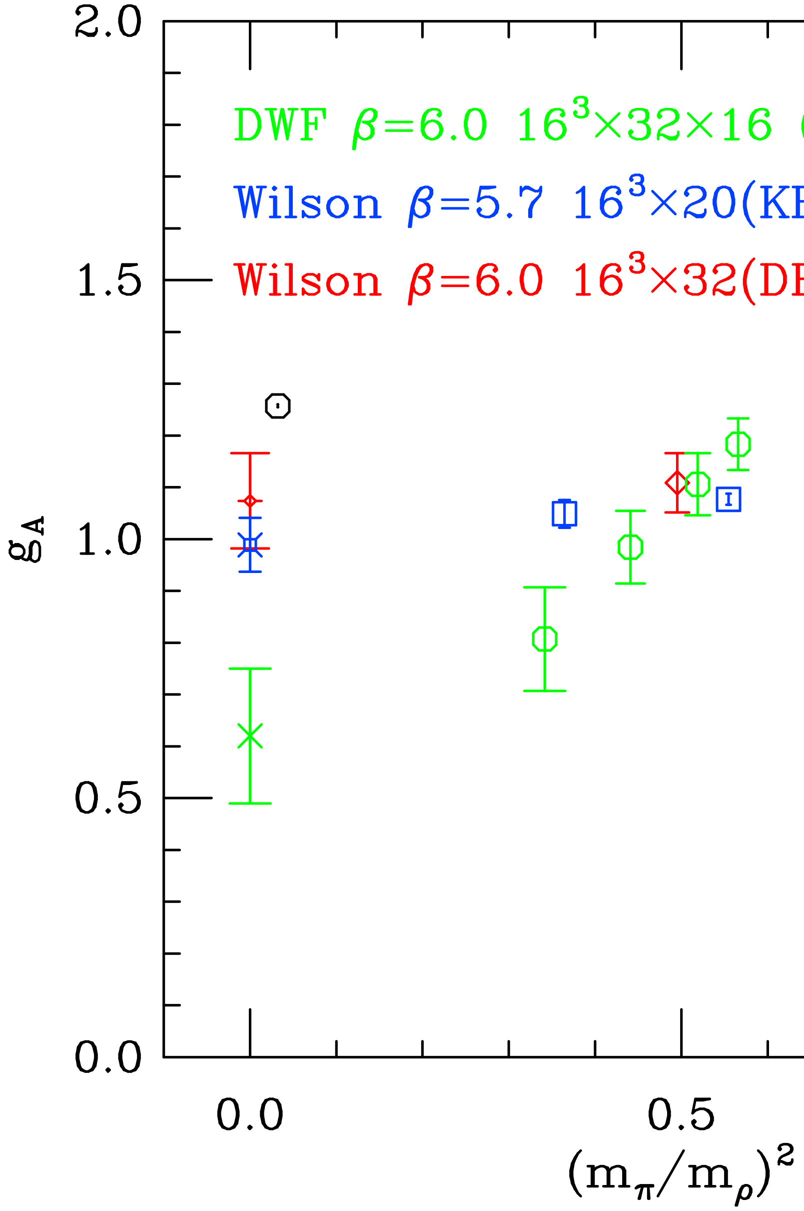
- Plan: Calculate the above matrix elements
  In Quenched approximation with DW fermions

- Code for doing the all the structure function matrix elements has been written.

- Use Non Perturbative Renormalization
  Works better with DWF. Wilson calculations used perturbative renormalization

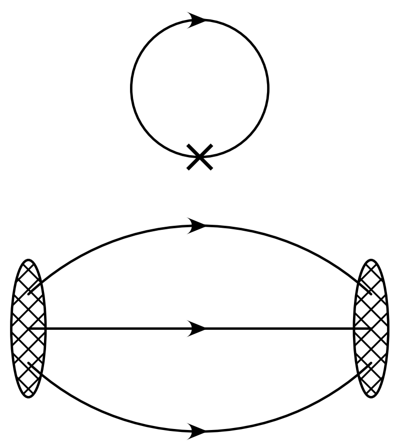
- $g_A^3 = \Delta u - \Delta d$ has been calculated (Sasaki Blum Ohta)
  Result may suffer from finite volume or quenching errors. Further study needed.

- Compute $g_A^1 = \Delta u + \Delta d + \Delta s$ which involves disconnected diagrams


2.0
1.5
1.0
0.5
0.0
DWF β=6.0 16³×32×16 (RBC)
Wilson β=5.7 16³×20(KEK)
Wilson β=6.0 16³×32(DESY)
g_A
0.0
0.5
1.0
(m_π/m_ρ)²

Experiment: $\Delta\Sigma = .27(4)$

Lattice results $\Delta\Sigma = .18 - .25$ with $\Delta s \sim -.15$



with the insertion of $O_\mu^5 = \gamma_5\gamma_\mu$ (code for this calculation done.)

Sum rule: $\frac{1}{2} = \frac{1}{2}\Delta\Sigma + ?$

better: $\frac{1}{2} = J^q + J^g$

J^q, J^g can be related to matrix elements of $T_{\mu\nu}^{g;q}$.

K.F. Liu computed J^q. It is only 60% of the proton spin.

SF lattice results

	QCDSF	Full QCD(MIT)	Phenomenology
xu_c	0.452(26)	0.459(29)	0.284
xd_c	0.189(12)	0.190(17)	0.104
$xu_c - xd_c$	0.263(17)	0.269(23)	0.180
x^2u_c	0.104(20)	0.176(63)	0.083
x^2d_c	0.037(10)	0.03(3)	0.025
x^3u_c	0.022(11)	0.07(4)	0.032
x^3d_c	−0.001(7)	−0.010(15)	0.008
Δu_c	0.830(70)	0.860(69)	0.918
Δd_c	−0.244(22)	−0.171(43)	−0.339
$\Delta u_c - \Delta d_c$	1.074(90)	1.031(81)	1.257
$x\Delta u_c$	0.198(8)	0.242(22)	0.150
$x\Delta d_c$	−0.048(3)	−0.029(13)	−0.055
$x^2\Delta u_c$	0.087(14)	0.116(42)	0.050
$x^2\Delta d_c$	−0.025(6)	0.001(25)	0.016
δu_c	0.93(3)	0.963(59)	
δd_c	−0.20(2)	−0.202(36)	
d_2^u	−0.206(18)	−0.228(81)	
d_2^d	−0.035(6)	0.077(31)	

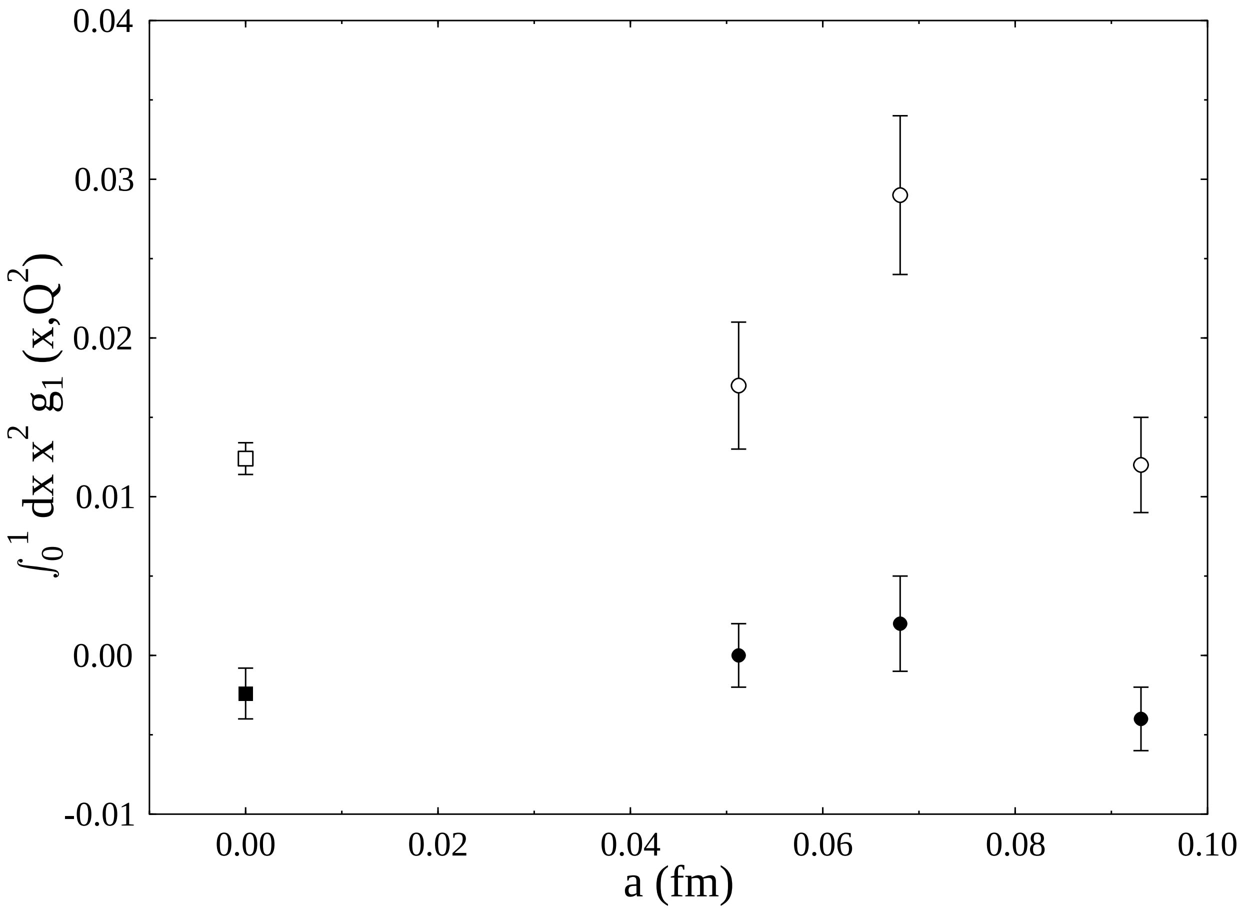
Comments on the results

- Quenched and dynamical results are not so different

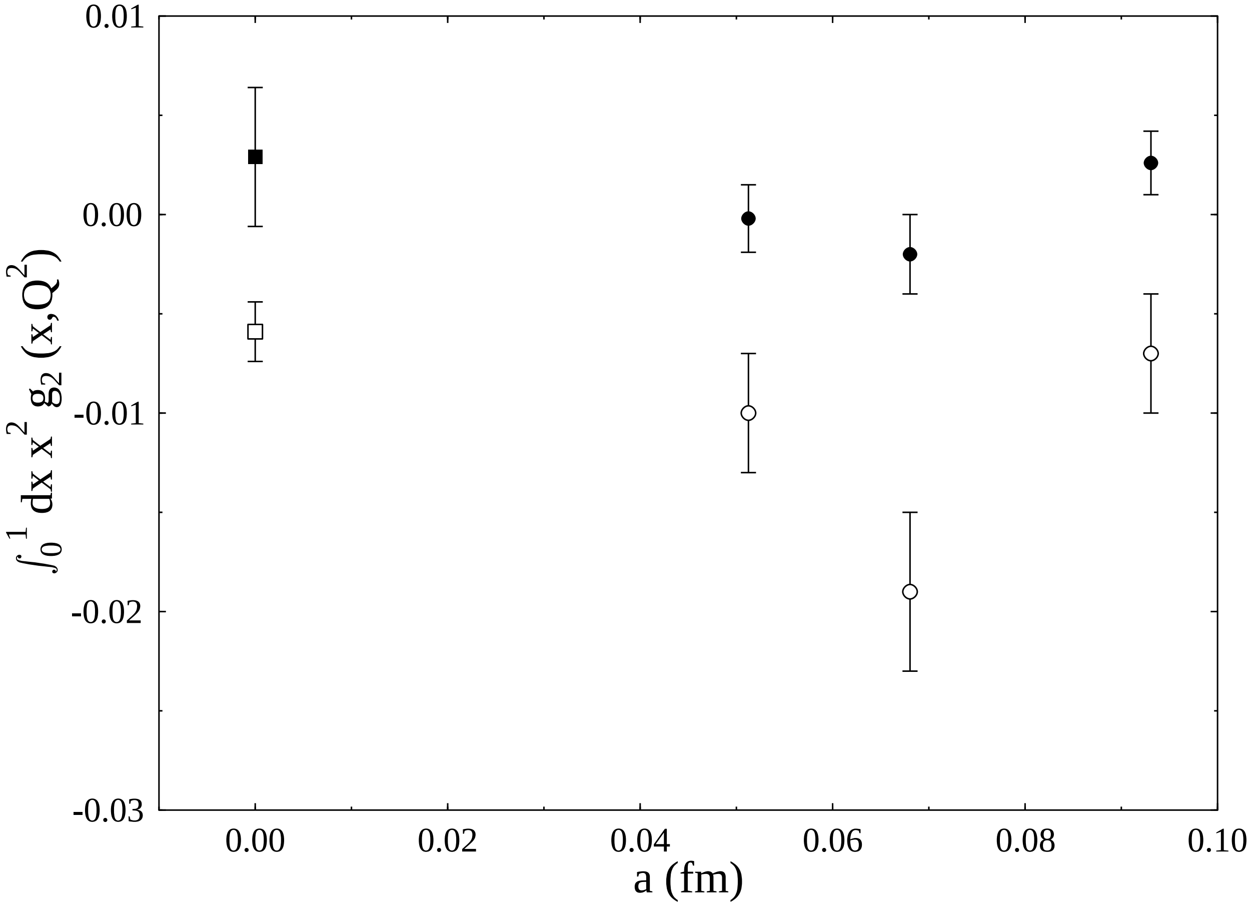
- Maybe dynamical quark masses are not light enough

- Finite volume effects have to be studied

- Continuum extrapolation is needed
  there are some continuum extrapolations done by QCDSF group for the g_2 moments



The moment $\int_0^1 \mathrm{d}x x^2 g_1(x, Q^2)$ at $Q^2 = 5\,\mathrm{GeV}^2$ for the proton (open symbols) and the neutron (filled symbols) plotted versus the lattice spacing a. The squares at $a = 0$ indicate the experimental values



The moment $\int_0^1 \mathrm{d}x x^2 g_2(x,Q^2)$ at $Q^2 = 5\,\mathrm{GeV}^2$ for the proton (open symbols) and the neutron (filled symbols) plotted versus the lattice spacing a. The squares at $a = 0$ indicate the experimental values

Conclusions

- Code for computing any quark bilinear Euclidean matrix element ready.

- Give us your favorite matrix element and we will computed!