Physics with Polarized Beams (II)

A tutorial for experimenters,

accelerator physicists, and students

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Brief summary of last lecture :

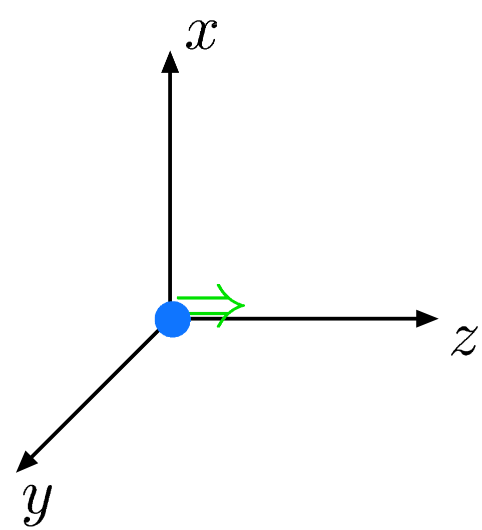
(1) Spin and Polarization

- a free spin$-1/2$ particle obeys Dirac equation

$$(\not{p} - m)\ u(p) = 0 \qquad \text{where} \quad \not{p} = \gamma_\mu\, p^\mu$$

- at rest, one has

$$u^+ = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad u^- = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$



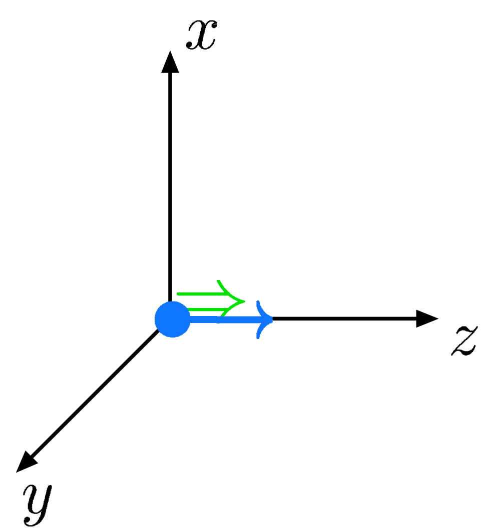
- they are eigenstates to the spin operator $\mathcal{S}_z$:

$$\mathcal{S}_z \, u^\pm \; = \; \pm \frac{1}{2} \, u^\pm$$

"polarized in z direction"

- now, we boost the particle to momentum $p \; = \; (E, \, 0, \, 0, \, p_z)$
- states become

$$u^+ \; = \; N \begin{pmatrix} 1 \\ 0 \\ \frac{p_z}{E+m} \\ 0 \end{pmatrix} \qquad u^- \; = \; N \begin{pmatrix} 0 \\ 1 \\ 0 \\ \frac{-p_z}{E+m} \end{pmatrix}$$



- they are eigenstates of the helicity operator :

$$\frac{\vec{S} \cdot \vec{p}}{|\vec{p}|} \, u^{\pm} \; = \; \pm \frac{1}{2} \, u^{\pm}$$

- they are also eigenstates of the Pauli-Lubanski (polarization) operator :

$$\frac{1}{2} \gamma_5 \, \not{n} \; u^{\pm} \; = \; \pm \frac{1}{2} \, u^{\pm}$$

where $n \; = \; (p_z, \, 0, \, 0, \, E)/m$

- at high energy, $E \approx p_z$ they also become eigenstates to chirality γ_5 :

$$\gamma_5 \; u^{\pm} \; = \; \pm \frac{1}{2} \, u^{\pm}$$

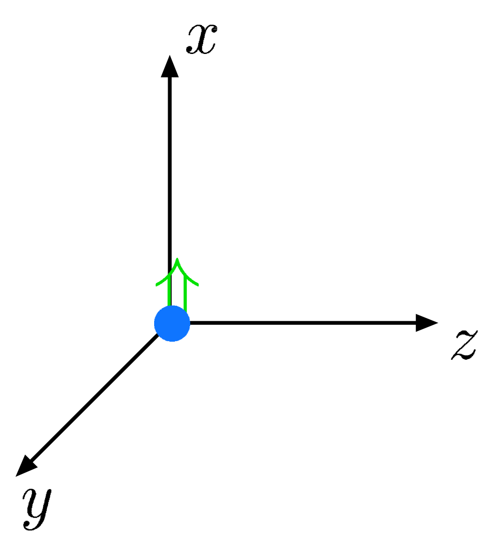
- back at rest : now let's construct

$$u^{\uparrow} = \frac{1}{\sqrt{2}} \left[u^{+} + u^{-} \right] \qquad u^{\downarrow} = \frac{1}{\sqrt{2}} \left[u^{+} - u^{-} \right]$$

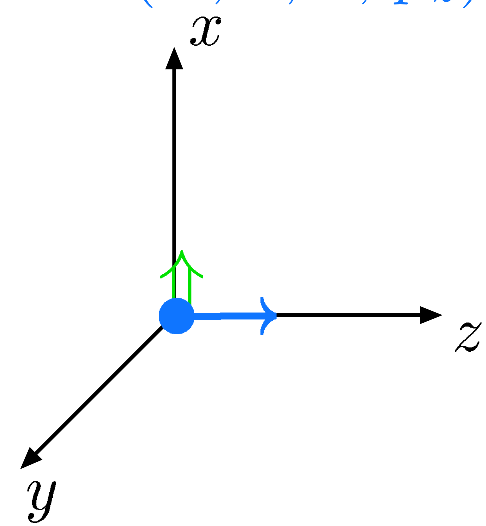
- they are eigenstates to the spin operator $\mathcal{S}_x$:

$$\mathcal{S}_x \, u^{\uparrow} = +\frac{1}{2} \, u^{\uparrow} \qquad \mathcal{S}_x \, u^{\downarrow} = -\frac{1}{2} \, u^{\downarrow}$$

"polarized in x direction"



- now, we again boost the particle to momentum $p = (E, 0, 0, p_z)$



- states become

$$u^{\uparrow} = \frac{N}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \\ \frac{p_z}{E+m} \\ \frac{-p_z}{E+m} \end{pmatrix} \qquad u^{\downarrow} = \frac{N}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \\ \frac{p_z}{E+m} \\ \frac{p_z}{E+m} \end{pmatrix}$$

- are still $u^{\uparrow} = (u^{+} + u^{-})/\sqrt{2}$ etc.

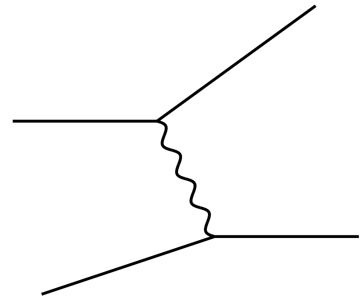
- they are eigenstates of the Pauli-Lubanski (polarization) operator :

$$\frac{1}{2}\gamma_5 \not{n}\, u^{\uparrow\downarrow} = \pm\frac{1}{2}\, u^{\uparrow\downarrow} \qquad \text{where} \quad n = (0, 1, 0, 0)$$

- they are no longer eigenstates of the transverse-spin operator :

$$\mathcal{S}_x\, u^{\uparrow} \neq +\frac{1}{2}\, u^{\uparrow}$$

(2) Polarized $e\,\mu \longrightarrow e\,\mu$ Scattering



- found angular dependence :

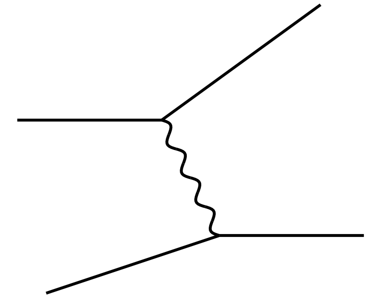
$$\frac{d\sigma}{d\Omega} \propto \left(1 + s_{\|}\, s'_{\|}\right) R_1 + \left(1 - s_{\|}\, s'_{\|}\right) R'_1 + \left(s_{\|} + s'_{\|}\right) R_2 + \left(s_{\|} - s'_{\|}\right) R'_2$$

$$+ s_{\perp} \left\{ \cos(\varphi)\, R_3 - \sin(\varphi)\, R_4 \right\} + s'_{\perp} \left\{ \cos(\varphi)\, R'_3 + \sin(\varphi)\, R'_4 \right\}$$

$$+ s'_{\|}\, s_{\perp} \left\{ \cos(\varphi)\, R_5 - \sin(\varphi)\, R_6 \right\} + s_{\|}\, s'_{\perp} \left\{ \cos(\varphi)\, R'_5 + \sin(\varphi)\, R'_6 \right\}$$

$$+ s_{\perp}\, s'_{\perp} \left\{ R_7 + \cos(2\varphi)\, R_8 - \sin(2\varphi)\, R_9 \right\}$$

(2) Polarized $e\,\mu \to e\,\mu$ Scattering



- found using parity and helicity conservation :

$$\frac{d\sigma}{d\Omega} \propto \left(1 + s_{\|}\, s'_{\|}\right) R_1 + \left(1 - s_{\|}\, s'_{\|}\right) R'_1 + \xcancel{\left(s_{\|} + s'_{\|}\right) R_2} + \xcancel{\left(s_{\|} - s'_{\|}\right) R'_2}$$

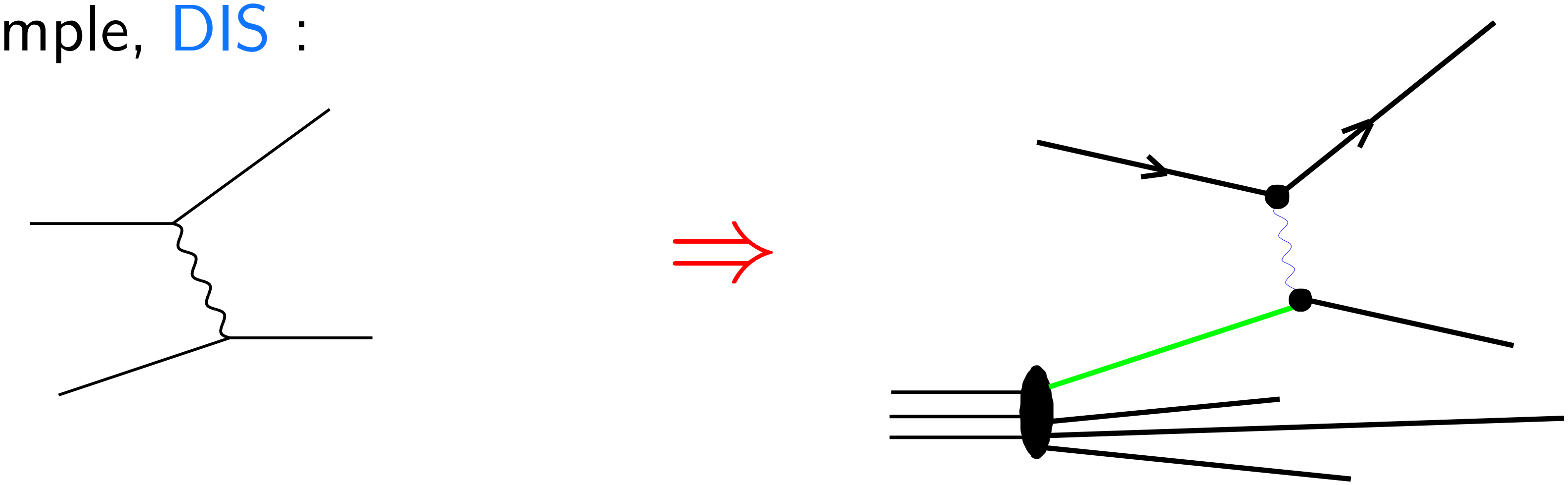
$$+ s_{\perp} \left\{ \xcancel{\cos(\varphi)\, R_3} - \xcancel{\sin(\varphi)\, R_4} \right\} + s'_{\perp} \left\{ \xcancel{\cos(\varphi)\, R'_3} + \xcancel{\sin(\varphi)\, R'_4} \right\}$$

$$+ s'_{\|}\, s_{\perp} \left\{ \xcancel{\cos(\varphi)\, R_5} - \xcancel{\sin(\varphi)\, R_6} \right\} + s_{\|}\, s'_{\perp} \left\{ \xcancel{\cos(\varphi)\, R'_5} + \xcancel{\sin(\varphi)\, R'_6} \right\}$$

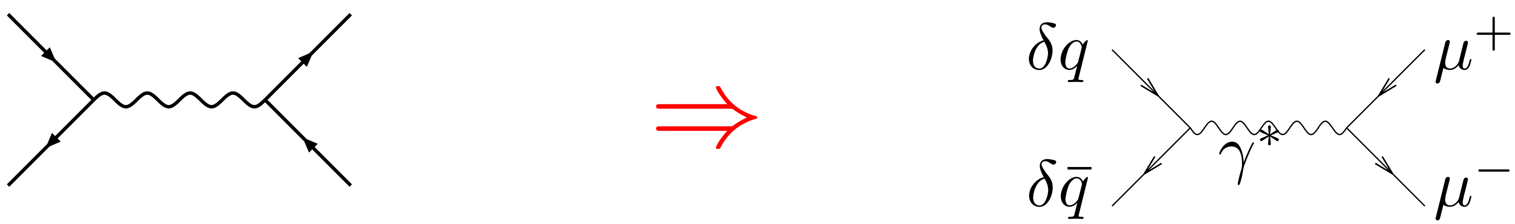
$$+ s_{\perp}\, s'_{\perp} \left\{ \xcancel{R_7} + \xcancel{\cos(2\varphi)\, R_8} - \xcancel{\sin(2\varphi)\, R_9} \right\}$$

Why interesting ?   We'll see :

- spin asymmetries in DIS and in inelastic hadronic scattering at RHIC may proceed via scattering off nucleon constituents – partons
- Example, DIS :



- Example, Drell-Yan dimuon production, $pp \to \mu^+\mu^- X$

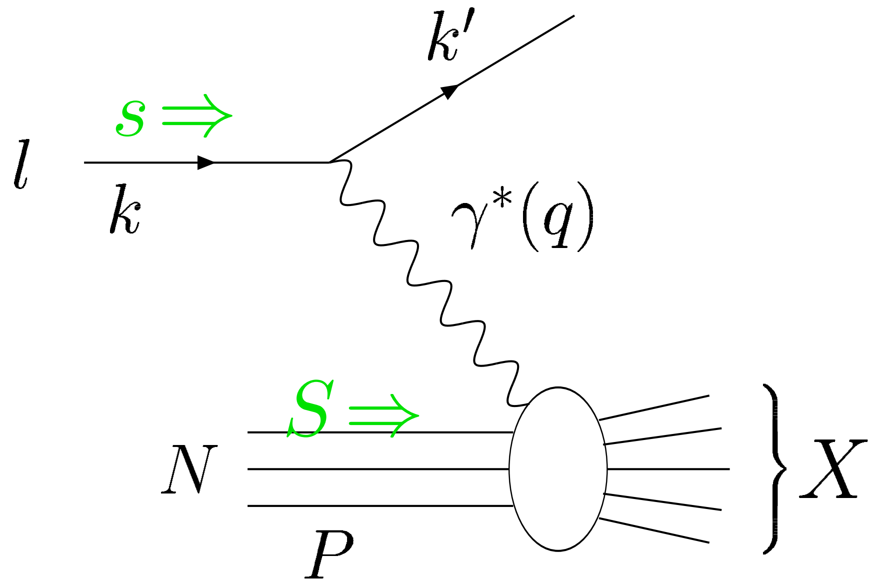


- therefore, understanding spin effects at elementary-particle level is crucial

Today :

- Deeply-inelastic Scattering
- Polarized Parton Distributions
- Scaling and its violation
- Factorized cross sections
- What do DIS data tell us about the nucleon ?

3.2 Deeply-inelastic lepton-nucleon scattering



$$Q^2 = -q^2 \gg m^2$$

$$x = \frac{Q^2}{2P\cdot q} = \frac{Q^2}{2m\nu}$$

- amplitude

$$\mathcal{M} = -e^2 \left(\frac{ig_{\mu\nu}}{Q^2}\right) \bar{u}(k')\,\gamma^\nu\, u(k,s)\ \ \langle\, X \mid J^\mu(0) \mid P,\, S\,\rangle$$

- cross section :

$$\text{cross section} \;\propto\; \left|\text{amplitude}\right|^2$$

- get

$$d\sigma \;=\; \frac{e^4}{Q^4} \sum\!\!\!\!\!\!\int_X \int \frac{d^3k'}{(2\pi)^3\, 2E'}\, (2\pi)^4 \delta^4(k + P - k' - p_X)$$

$$\times\; \langle P,\, S \mid J^{\mu}(0) \mid X \,\rangle \, \langle\, X \mid J^{\nu}(0) \mid P,\, S \,\rangle \, \Big[\bar{u}(k, s)\; \gamma_{\nu}\; u(k')\Big] \, \Big[\bar{u}(k')\; \gamma_{\mu}\; u(k, s)\Big]$$

- this can be written as

$$\frac{d\sigma}{dE'\, d\Omega} \;=\; \frac{\alpha^2}{Q^4}\, \frac{E'}{E}\; \underbrace{\mathcal{L}_{\mu\nu}(k, q, s)}_{\text{leptonic}} \cdot \underbrace{\mathcal{W}^{\mu\nu}(P, q, S)}_{\text{hadronic}}$$

$$\frac{d\sigma}{dE'\,d\Omega} \propto \underbrace{\mathcal{L}_{\mu\nu}(k,q,s)}_{\text{leptonic}} \cdot \underbrace{\mathcal{W}^{\mu\nu}(P,q,S)}_{\text{hadronic}}$$

$\mathcal{L}_{\mu\nu}(k,q,s) \;=\;$ calculable in QED

$$\mathcal{W}^{\mu\nu}(P,q,S) \;=\; \frac{1}{4\pi}\int d^4z\, \mathrm{e}^{iq\cdot z}\, \langle P,S|\, J_\mu(z)\, J_\nu(0)\,|P,S\rangle$$

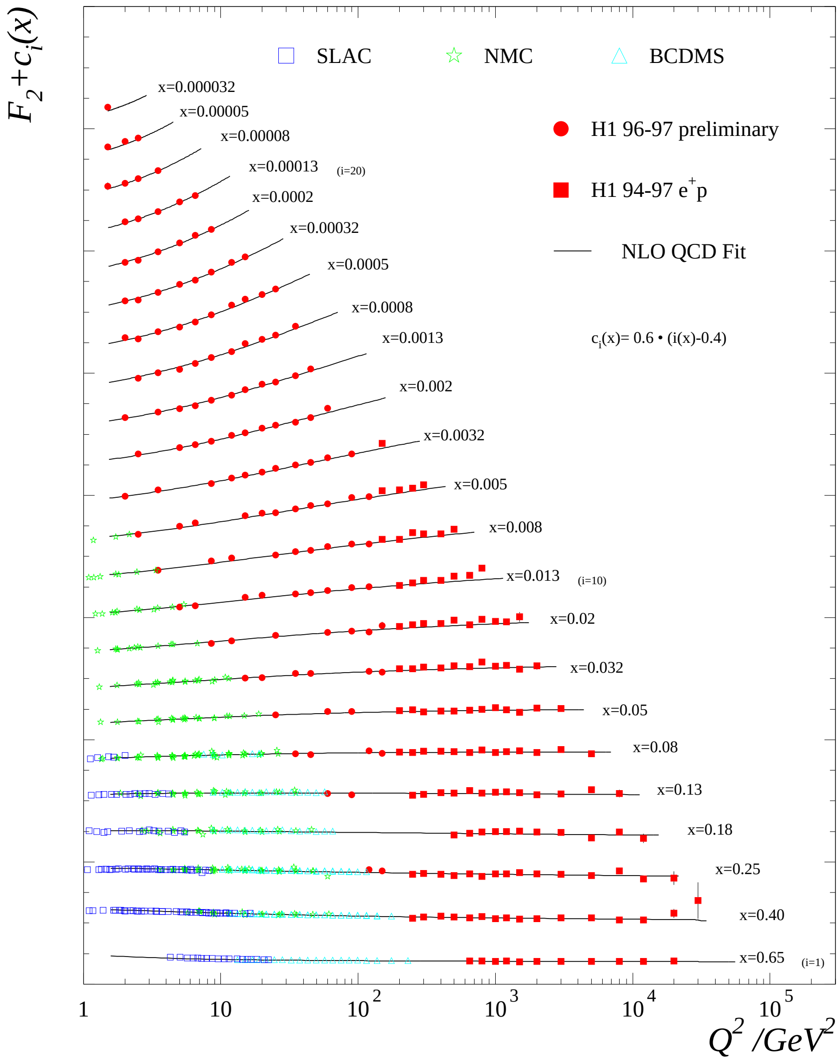
$$= \left(-\,g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) F_1(x,Q^2) + \left(P^\mu - \frac{P\cdot q}{q^2}q^\mu\right)\left(P^\nu - \frac{P\cdot q}{q^2}q^\nu\right) F_2(x,Q^2)$$

$$+\, i\, M\, \varepsilon^{\mu\nu\rho\sigma} q_\rho \left[\frac{S_\sigma}{P\cdot q}\, g_1(x,Q^2) + \frac{S_\sigma(P\cdot q) - P_\sigma(S\cdot q)}{(P\cdot q)^2}\, g_2(x,Q^2)\right]$$

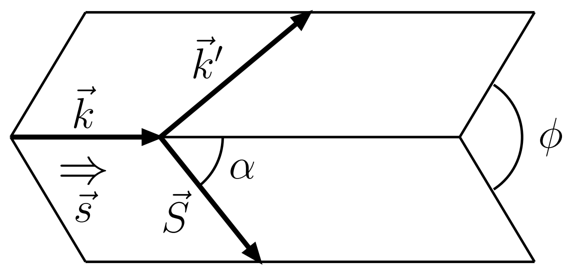
- F_i , g_i : nucleon structure functions

- spin-averaged cross section : $\quad (\, y = 1 - E'/E \,)$

$$\frac{d^2\sigma}{dxdy} = \frac{4\pi\alpha^2}{Q^2xy}\left[xy^2\,F_1(x,Q^2) + (1-y)\,F_2(x,Q^2)\right]$$



- for g_i : differences $\mathcal{W}^{\mu\nu}(P,q,S) - \mathcal{W}^{\mu\nu}(P,q,-S)$
- specialize to lepton with helicity λ and $\angle(\hat{k},\hat{S}) \equiv \alpha$:



- find

$$\frac{d\sigma^{(\alpha)}}{dx\,dy\,d\phi} - \frac{d\sigma^{(\alpha+\pi)}}{dx\,dy\,d\phi} = \frac{\lambda\, e^4}{4\pi^2 Q^2} \times$$

$$\times \Bigg\{ \cos\alpha \left\{ \left[1 - \frac{y}{2} - \frac{m^2x^2y^2}{Q^2} \right] g_1(x,Q^2) - \frac{2m^2x^2y}{Q^2}\, g_2(x,Q^2) \right\}$$

$$- \sin\alpha\cos\phi\, \frac{2mx}{Q} \sqrt{\left(1 - y - \frac{m^2x^2y^2}{Q^2} \right)} \left(\frac{y}{2}\, g_1(x,Q^2) + g_2(x,Q^2) \right) \Bigg\}$$

- $\alpha = 0 : \Rightarrow g_1$
- $\alpha = \pi/2 : \Rightarrow y\, g_1 + 2\, g_2$, suppressed m/Q

• experimentally: spin-*asymmetries*, e.g. case $\alpha = 0$:

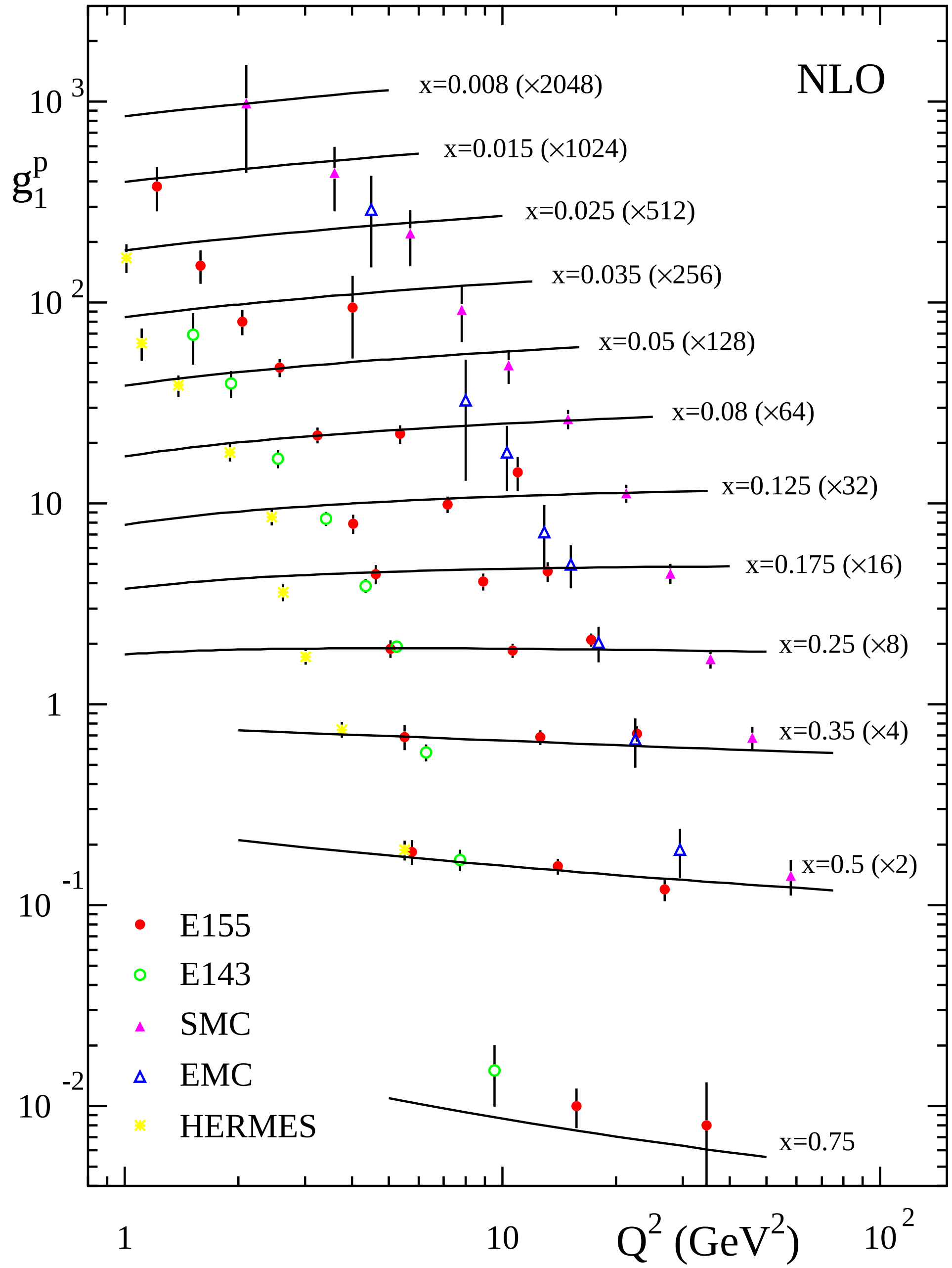
$$A_{\parallel} = \frac{d\sigma^{(\rightarrow\Leftarrow)} - d\sigma^{(\rightarrow\Rightarrow)}}{d\sigma^{(\rightarrow\Leftarrow)} + d\sigma^{(\rightarrow\Rightarrow)}} = D(y)\,\frac{g_1(x,Q^2)}{F_1(x,Q^2)} \equiv D(y)\,A_1(x,Q^2)$$

• so far only "fixed-target" experiments :

SLAC: E80,130 (p); E142 (n); E143 (p, d); E154 (n);
E155 (p, d)

CERN: EMC, SMC (p, d)

DESY: HERMES (p, d, n)



NLO
x=0.008 (×2048)
x=0.015 (×1024)
x=0.025 (×512)
x=0.035 (×256)
x=0.05 (×128)
x=0.08 (×64)
x=0.125 (×32)
x=0.175 (×16)
x=0.25 (×8)
x=0.35 (×4)
x=0.5 (×2)
x=0.75
E155
E143
SMC
EMC
HERMES
g_1^p
Q^2 (GeV^2)

3.3 **Heuristic Parton Model**

Feynman; Bjorken,Paschos

- target rest frame (recall, $x = Q^2/2m\nu$) :

$$P = (m, 0, 0, 0)$$

$$q = (\nu, 0, 0, \sqrt{\nu^2 + Q^2})$$

- boost to frame where nucleon has large momentum component p_3 ("infinite-momentum frame") : $(\beta = p_3/\sqrt{p_3^2 + m^2})$

$$P^{\mathrm{IMF}} \approx \left(p_3 + \frac{m^2}{2p_3}, 0, 0, p_3\right)$$

$$q^{\mathrm{IMF}} \approx \left(xp_3 - \frac{m\,\nu}{2p_3}, 0, 0, -xp_3 - \frac{m\,\nu}{2p_3}\right)$$

- time scales : Lorentz-dilated by $\gamma = \sqrt{p_3^2 + m^2}/m$!
  - internal interactions : $\Delta t \sim \gamma \times \frac{1}{m} = \frac{\sqrt{p_3^2 + m^2}}{m^2} \approx \frac{p_3}{m^2}$
  - DIS interaction :

phase $$q^{\mathrm{IMF}} \cdot z = \frac{1}{2}(q_0 - q_3)(t + z_3) + \frac{1}{2}(q_0 + q_3)(t - z_3)$$

$$\approx x p_3 (t + z_3) - \frac{m\,\nu}{2 p_3}(t - z_3)$$

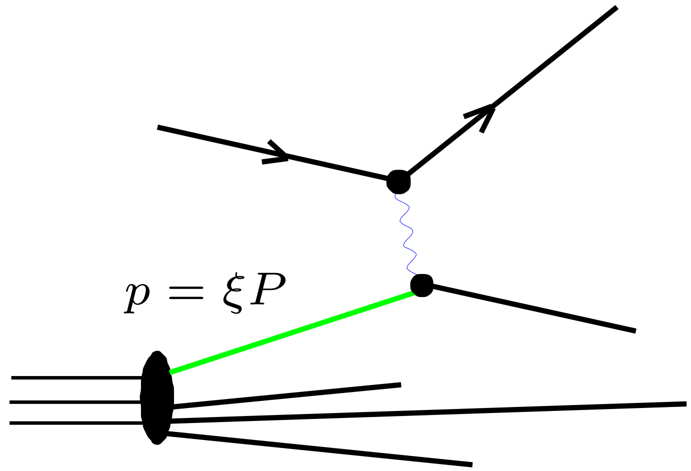
$p_3 \to \infty \Rightarrow z_3 \approx -t \Rightarrow \Delta t' \sim \frac{p_3}{m\nu}$

- therefore :

$$\frac{\Delta t'}{\Delta t} \sim \frac{m^2\, x}{Q^2} \ll 1$$

- $\longrightarrow$ lepton sees “snapshot” of nucleon in virtual parton state

- scatters incoherently off “free” quark-partons :



- elastic $e\,q$ scattering : partonic Bjorken-variable $= 1$

$$1 \;=\; x_{\text{parton}} \;=\; \frac{Q^2}{2\,p \cdot q} \;=\; \frac{Q^2}{2\,\xi\,P \cdot q} \;=\; \frac{x}{\xi} \qquad \Leftrightarrow \quad \boxed{\xi \;=\; x}$$

- ($x_{\text{parton}} \;\leq\; 1$ if scattering inelastic !)

- can calculate $e\,p$ cross section :

$$\frac{d\sigma^{ep}}{dxdy}(x) = \sum_f \int_x^1 d\xi \;\; f(\xi) \;\; \underbrace{\frac{d\sigma^{ef}}{dxdy}\left(x_{\text{parton}} = \frac{Q^2}{2p\cdot q} = \frac{x}{\xi}\right)}_{\propto\; \delta(x_{\text{parton}}-1)}$$

↑ # of partons of type $f = q, \bar{q}, g$

- in terms of structure functions, this becomes :

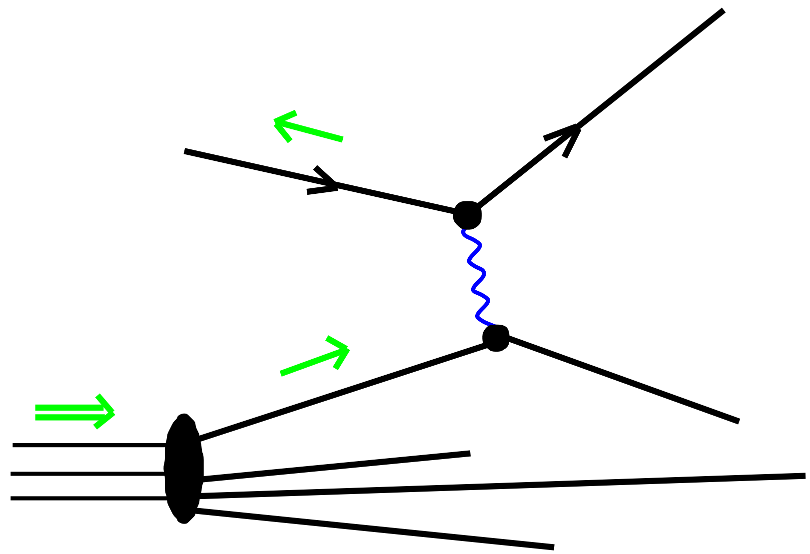
$$F_1(x) = \sum_f \int_x^1 \frac{d\xi}{\xi} \;\; f(\xi) \;\; \underbrace{\hat{F}_1^{\text{parton}}\left(x_{\text{parton}} = \frac{x}{\xi}\right)}_{\propto\; \delta(x_{\text{parton}}-1)}$$

- therefore, can calculate structure functions :

$$F_1(x) = \tfrac{1}{2}\textstyle\sum_q e_q^2 \; [\, q(x) + \bar{q}(x)\,] \qquad\qquad F_2(x) = 2\,x\,F_1(x)$$

- Bjorken scaling $\leftrightarrow$ structure of nucleon independent of resolution
- the physics : m has become irrelevant $\Rightarrow$ depend only on $Q^2/\nu \;\propto\; x$

Polarized scattering : g_1, too, can be interpreted in parton model !

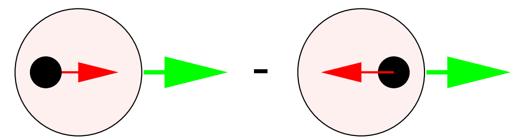


$\Rightarrow$ have to consider $e(\lambda_e)\, q(\lambda_q) \rightarrow eq$ etc., *and :*

- $f^{+}(\xi)$ # of partons with *same* helicity as nucleon
- $f^{-}(\xi)$ # of partons with *opposite* helicity

Define

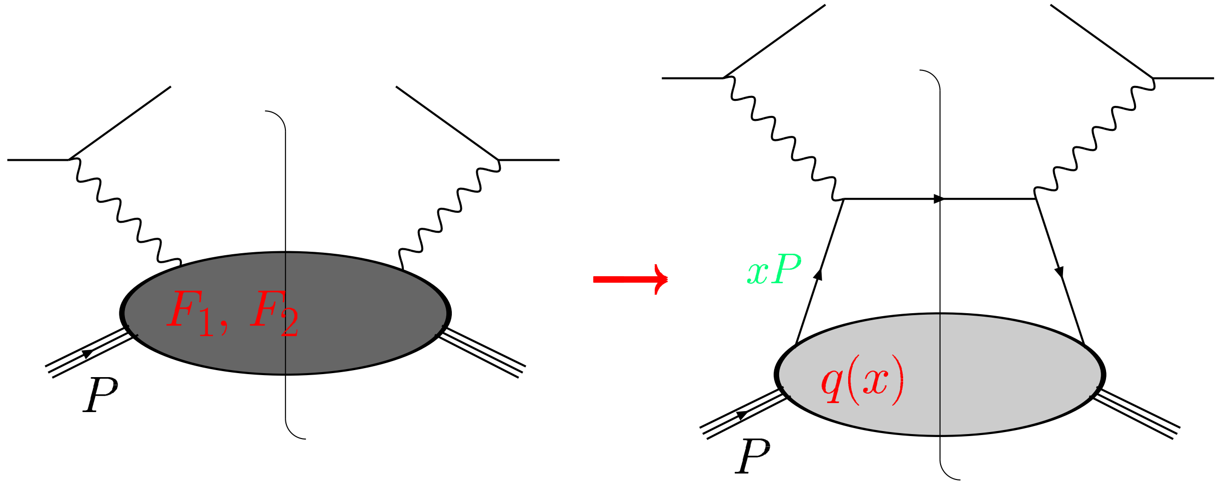
$$\Delta f(\xi) \equiv f^{+}(\xi) - f^{-}(\xi)$$



$$\Rightarrow \quad g_1(x) = \frac{1}{2} \sum_q e_q^2 \left[\Delta q(x) + \Delta \bar{q}(x) \right]$$

Δq, $\Delta \bar{q}$: information on nucleon spin structure

Executive summary :



$$F_1(x) = \frac{1}{2}\sum_q e_q^2 \left[q(x) + \bar{q}(x) \right]$$

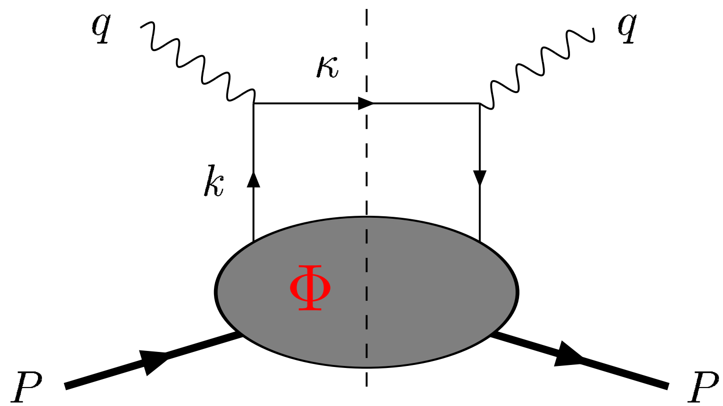
$$g_1(x) = \frac{1}{2}\sum_q e_q^2 \left[\Delta q(x) + \Delta\bar{q}(x) \right]$$

- write it out :

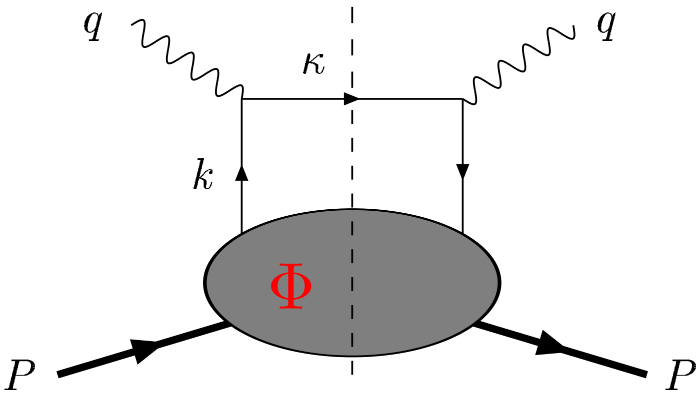
$$g_1 = \frac{1}{2}\left[\frac{4}{9}\left(\Delta u + \Delta\bar{u}\right) + \frac{1}{9}\left(\Delta d + \Delta\bar{d} + \Delta s + \Delta\bar{s}\right) \right]$$

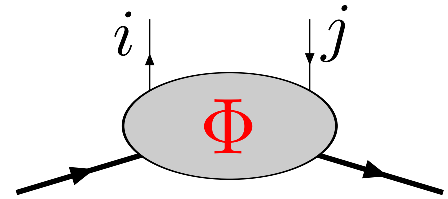
3.4 Systematics of polarized parton distributions

Parton model :



- Φ represents the structure of the nucleon !
- since the quark is described by a Dirac spinor, Φ is a 4×4 matrix components of the matrix related to polarization of quark





- find

$$\Phi_{ij}(k,P,S) \;=\; \sum_X \int \frac{d^3\mathbf{P}_X}{(2\pi)^3\, 2E_X}\,(2\pi)^4\,\delta^4(P-k-P_X)\,\langle PS|\bar{\psi}_j(0)|X\rangle\,\langle X|\psi_i(0)|PS\rangle$$

$$= \int d^4z\; e^{ik\cdot z}\,\langle\, PS\,|\,\bar{\psi}_j(0)\,\psi_i(z)\,|\,PS\,\rangle$$

- this gives (one flavor only !) :

$$\mathcal{W}^{\mu\nu} \;=\; e^2 \int \frac{d^4k}{(2\pi)^4}\,\delta\big((k+q)^2\big)\;\mathrm{Tr}\,[\,\Phi\,\gamma^\mu(\not{k}+\not{q})\gamma^\nu\,]$$

- let's choose frame as follows :
  - proton momentum : $P = (p, 0, 0, p)$,
  - parton : $k^\mu \sim \xi P^\mu$
  - virtual photon : $q^\mu = (P \cdot q)\, n^\mu - \xi P^\mu$

    where $n = (1, 0, 0, -1)$ $\quad (q^2 = -Q^2 \ \checkmark)$

- for convenience, let's for each 4-vector v introduce

$$v^+ = \tfrac{1}{2}(v_0 + v_3) \qquad v^- = \tfrac{1}{2}(v_0 - v_3)$$

- that is, $P^+ = p\,,\ k^+ = \xi\, p\,,\ P^- = k^- = 0\,,\ n^+ = 0\,,\ n^- = 1$

- this gives : $\delta\big((k+q)^2\big) = \frac{1}{2P\cdot q}\,\delta(x-\xi) = \frac{1}{2P\cdot q}\,\delta\left(x - \frac{k^+}{P^+}\right)$

therefore :

$$\mathcal{W}^{\mu\nu} = \frac{e^2}{2} \underbrace{\int \frac{d^4k}{(2\pi)^4}\,\delta\left(x - \frac{k^+}{P^+}\right) \mathrm{Tr}\Big[\,\Phi}_{\equiv\,\phi(x)}\ \gamma^\mu \not{n} \gamma^\nu\Big]$$

- ϕ must have general expansion in terms of $\not{P}$, $\not{n}$, $\not{s}$ etc.
- proton polarization vector $s^\mu = s_\parallel \frac{P^\mu}{m} + s_\perp^\mu$
- find leading contributions

$$\phi(x) = \frac{1}{2}\Big[\, q(x)\not{P} + s_\parallel \, \Delta q(x)\, \gamma_5 \not{P} + \delta q(x) \not{P}\, \gamma_5 \not{s}_\perp \Big]$$

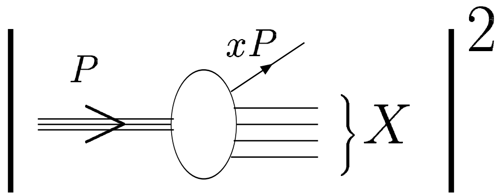
where we have three quark-parton densities

$$q(x) = \frac{1}{4\pi}\int dz^- \, e^{iz^- xP^+} \langle P,S|\bar{\psi}(0)\,\gamma^+\,\psi\left(0,z^-,\mathbf{0}_\perp\right)|P,S\rangle$$

$$\Delta q(x) = \frac{1}{4\pi}\int dz^- \, e^{iz^- xP^+} \langle P,S|\bar{\psi}(0)\,\gamma^+\,\gamma_5\,\psi\left(0,z^-,\mathbf{0}_\perp\right)|P,S\rangle$$

$$\delta q(x) = \frac{1}{4\pi}\int dz^- \, e^{iz^- xP^+} \langle P,S|\bar{\psi}(0)\,\gamma^+\,\gamma_\perp\,\gamma_5\,\psi\left(0,z^-,\mathbf{0}_\perp\right)|P,S\rangle$$

"unpolarized" – "longitudinally polarized" – "transversity"

Are these 

? Yes :

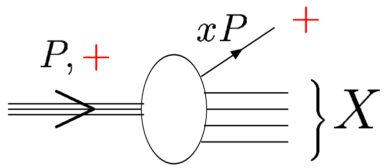
- Defining $\mathcal{P}^{\pm} \equiv \frac{1 \pm \gamma_5}{2}$ and $\mathcal{P}^{\uparrow\downarrow} \equiv \frac{1 \pm \gamma_\perp \gamma_5}{2}$ one can show

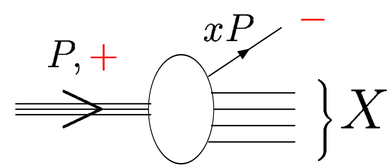
$$q(x) = \frac{1}{2} \sum_X \delta\left(P_X^+ - (1-x) P^+\right) \times \left[\left| \langle X | \mathcal{P}^+ \psi_+(0) | P, \lambda = \frac{1}{2} \rangle \right|^2 + \left| \langle X | \mathcal{P}^- \psi_+(0) | P, \lambda = \frac{1}{2} \rangle \right|^2 \right]$$

$$\Delta q(x) = \frac{1}{2} \sum_X \delta\left(P_X^+ - (1-x) P^+\right) \times \left[\left| \langle X | \mathcal{P}^+ \psi_+(0) | P, \lambda = \frac{1}{2} \rangle \right|^2 - \left| \langle X | \mathcal{P}^- \psi_+(0) | P, \lambda = \frac{1}{2} \rangle \right|^2 \right]$$

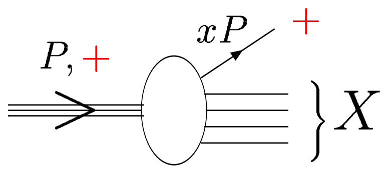
$$\delta q(x) = \frac{1}{2} \sum_X \delta(P_X^+ - (1-x) P^+) \times \left[\left| \langle X | \mathcal{P}^\uparrow \psi_+(0) | P, S_\perp = \frac{1}{2} \rangle \right|^2 - \left| \langle X | \mathcal{P}^\downarrow \psi_+(0) | P, S_\perp = \frac{1}{2} \rangle \right|^2 \right]$$

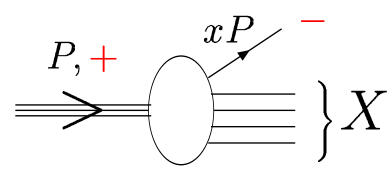
- Pictorially :

$q(x) = \Big|$ 

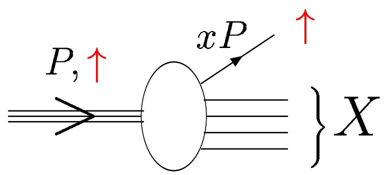
$\Big|^2 + \Big|$ 

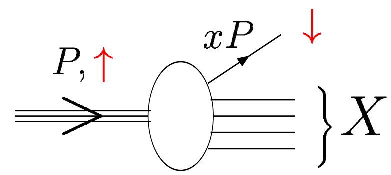
$\Big|^2$

$\Delta q(x) = \Big|$ 

$\Big|^2 - \Big|$ 

$\Big|^2$

$\delta q(x) = \Big|$ 

$\Big|^2 - \Big|$ 

$\Big|^2$

- recall

$$\phi(x) = \frac{1}{2}\left[\, q(x)\not{P} \;+\; s_{\parallel}\,\Delta q(x)\,\gamma_5\not{P} \;+\; \delta q(x)\not{P}\,\gamma_5\,\not{S}_{\perp} \right]$$

- previously we had for pointlike particle at high energy :

$$\frac{1}{2}\not{p}\left[\,\mathbb{1} \;-\; s_{\parallel}\,\gamma_5 \;+\; \gamma_5\,\not{s}_{\perp}\,\right]$$

with density matrix :

$$\rho = \frac{1}{2}\begin{pmatrix} 1+s_{\parallel} & s_x - i s_y \\ s_x + i s_y & 1-s_{\parallel} \end{pmatrix}$$

- $\longrightarrow$ density matrix of a quark in the nucleon :

$$\rho_q = \frac{1}{2\,q(x)}\begin{pmatrix} q(x) \;+\; s_{\parallel}\,\Delta q(x) & s_{\perp}\delta q(x) \\ s_{\perp}\delta q(x) & q(x) \;-\; s_{\parallel}\,\Delta q(x) \end{pmatrix}$$

Important :

- partonic structure of $\phi(x)$ doesn't mean that $q(x)$, $\Delta q(x)$, $\delta q(x)$ will all contribute to a process with arbitrary polarization !

$$\mathcal{W}^{\mu\nu} = \frac{e^2}{2} \underbrace{\int \frac{\mathrm{d}^4k}{(2\pi)^4}\, \delta\left(x - \frac{k^+}{P^+}\right)\ \mathrm{Tr}\Big[\Phi}_{\equiv\, \phi(x)} \gamma^\mu \not{n} \gamma^\nu\Big] = \frac{e^2}{2}\mathrm{Tr}\Big[\phi(x)\, \gamma^\mu \not{n} \gamma^\nu\Big]$$

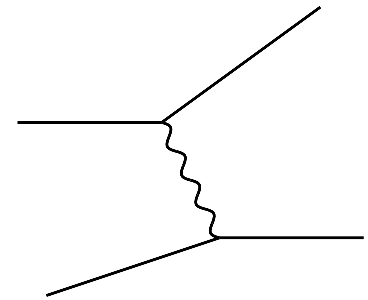
$$\phi(x) = \frac{1}{2}\left[q(x)\not{P} + s_{\|}\, \Delta q(x)\, \gamma_5 \not{P} + \delta q(x) \not{P}\, \gamma_5\, \not{S}_\perp \right]$$

- gives parton model expressions for F_1, g_1 . . .

  . . . but no contribution from transversity !

- in particular, g_2 does not measure transversity

Was expected : $\vec{e}\,\vec{q}$ scattering $\leftrightarrow$ $\vec{e}\,\vec{\mu}$ scattering !



- recall we found using chirality conservation :

$$\frac{d\sigma}{d\Omega} \propto$$

$$\left(1 + s_{\|}\, s'_{\|}\right) R_1 + \left(1 - s_{\|}\, s'_{\|}\right) R'_1 + \xcancel{\left(s_{\|} + s'_{\|}\right) R_2} + \xcancel{\left(s_{\|} - s'_{\|}\right) R'_2}$$

$$+\, s_{\perp} \left\{ \xcancel{\cos(\varphi)\, R_3} - \xcancel{\sin(\varphi)\, R_4} \right\} + s'_{\perp} \left\{ \xcancel{\cos(\varphi)\, R'_3} + \xcancel{\sin(\varphi)\, R'_4} \right\}$$

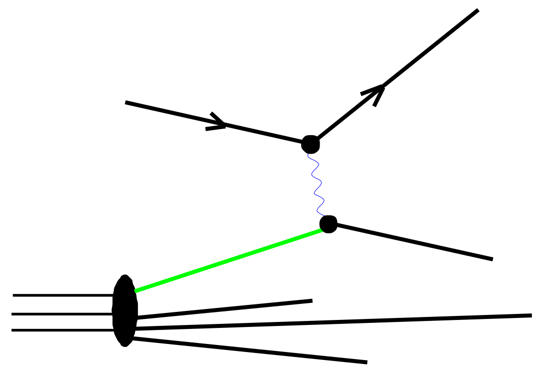
$$+\, s'_{\|}\, s_{\perp} \left\{ \xcancel{\cos(\varphi)\, R_5} - \xcancel{\sin(\varphi)\, R_6} \right\} + s_{\|}\, s'_{\perp} \left\{ \xcancel{\cos(\varphi)\, R'_5} + \xcancel{\sin(\varphi)\, R'_6} \right\}$$

$$+\, s_{\perp}\, s'_{\perp} \left\{ \xcancel{R_7} + \xcancel{\cos(2\varphi)\, R_8} - \xcancel{\sin(2\varphi)\, R_9} \right\}$$

- no transverse-spin effect !

3.5 Scaling is violated !

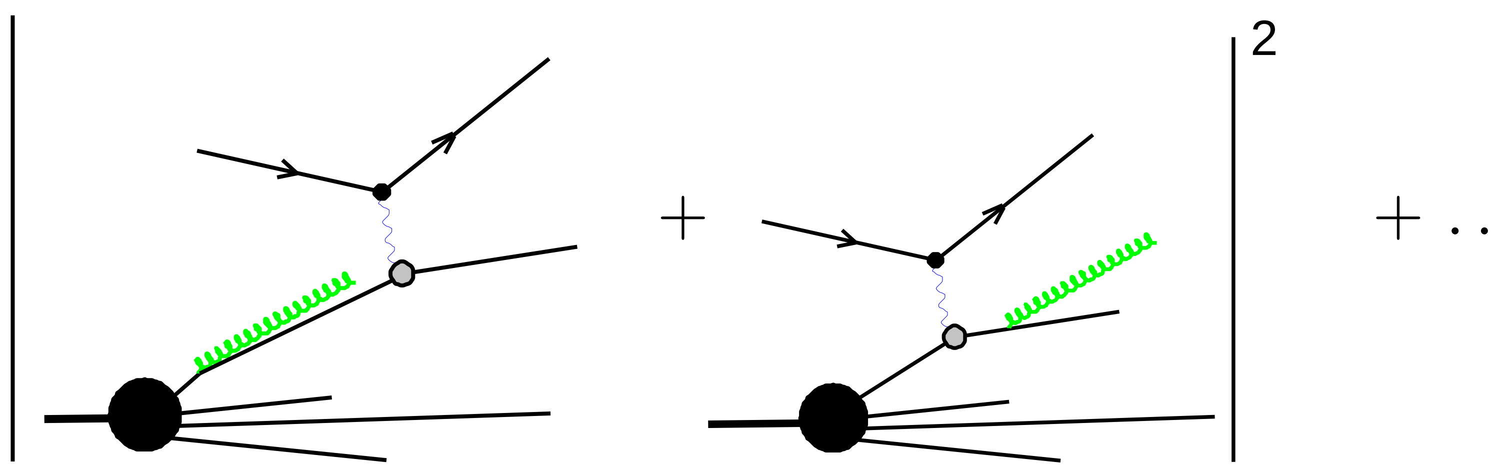
- parton model neglects interactions :

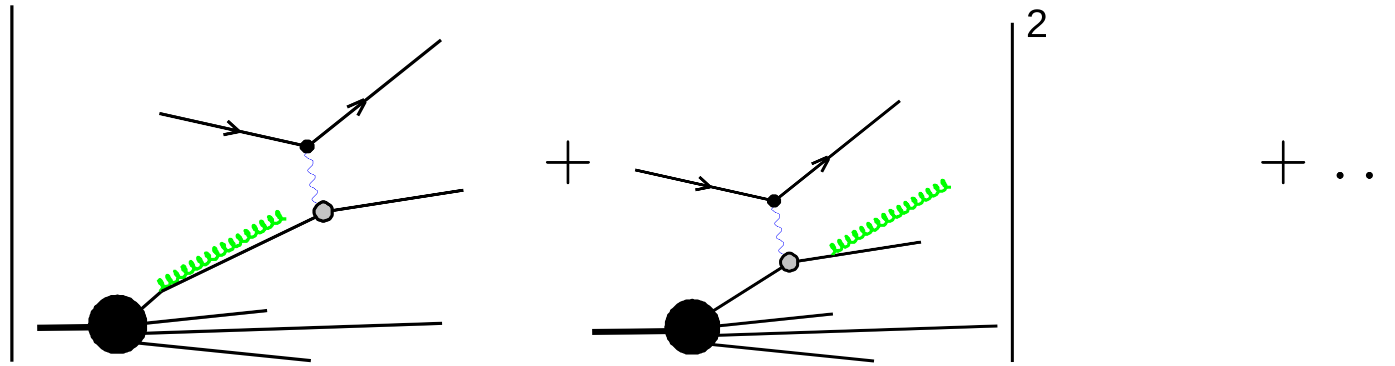


- parton states not truly frozen. Some states fluctuate on scales $\sim 1/Q$

  $\longrightarrow$ expect dependence on Q^2

a typical interaction :





- try to calculate radiative correction – without spin for now.

- recall, parton model expression for structure function (one quark) :

$$F_1(x) \;=\; \int_x^1 \frac{d\xi}{\xi}\; q(\xi)\; \hat{F}_1^{\text{parton}}\left(\frac{x}{\xi}\right)$$

- a convenient, equivalent, way of handling is to take Mellin moments :

$$F_1^{\,n} \;\equiv\; \int_0^1 dx\; x^{n-1}\, F_1(x)$$

- this gives :

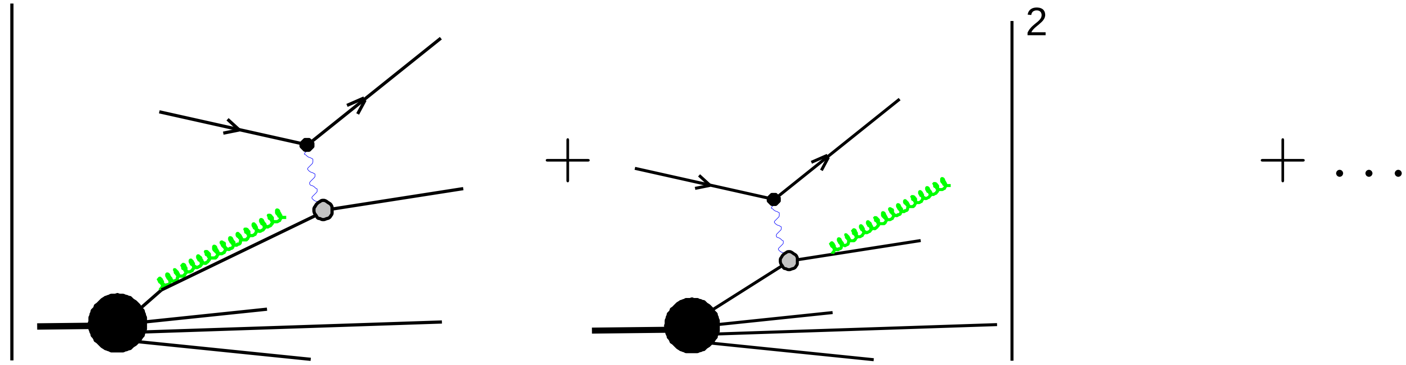
$$F_1^{\,n} \;=\; \int_0^1 dx\; x^{n-1} \int_x^1 \frac{d\xi}{\xi}\; q(\xi)\;\; \hat{F}_1 \underbrace{\left(\frac{x}{\xi}\right)}_{\equiv\, x_{\mathsf{p}}}$$

$$\;=\; \int_0^1 d\xi\; \xi^{n-1}\, q(\xi) \int_0^1 dx_{\mathsf{p}}\; x_{\mathsf{p}}^{n-1}\, \hat{F}_1(x_{\mathsf{p}})$$

$$\;=\; q^{\,n} \cdot \hat{F}_1^{\,n}$$

- convolution integral $\longrightarrow$ simple product
- Mellin-inverse :

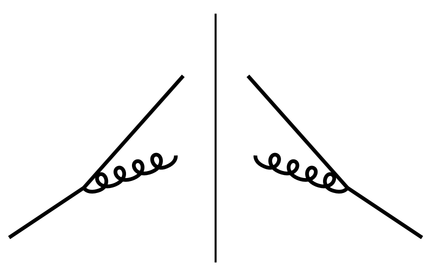
$$f(x) = \frac{1}{2\pi i} \int_{\mathcal{C}_n} dn\; x^{-n}\, f^{\,n}$$



- Need to integrate over gluon phase space. Find :

$$F_1^{\,n} \;\propto\; \left[\, 1 \;+\; \frac{\alpha_s}{2\pi}\left(P_{qq}^{\,n} \;\; \underbrace{\int_0^Q \frac{dk_T}{k_T}}_{\text{log. divergent !}} \;+\; \underbrace{r^{\,n}}_{\text{finite}} \right)\right] q^{\,n}$$

- logarithmic divergence occurs when gluon is emitted collinearly by initial-state quark. $P_{qq}^{\,n}$ is the residue of the singularity



$P_{qq}^{\,n}$ = "splitting function"

- let's "tame" the singularity ! Give quark a mass $m \neq 0$:

$$F_1^n \propto \left[1 + \frac{\alpha_s}{2\pi} \left(P_{qq}^n \log \frac{Q}{m} + r^n \right) \right] q^n$$

$$= \left[1 + \frac{\alpha_s}{2\pi} \left(P_{qq}^n \left(\log \frac{Q}{\mu} + \log \frac{\mu}{m} \right) + r^n \right) \right] q^n$$

$$\approx \left[1 + \frac{\alpha_s}{2\pi} \left(P_{qq}^n \log \frac{Q}{\mu} + r^n \right) \right] \left[1 + \frac{\alpha_s}{2\pi} P_{qq}^n \log \frac{\mu}{m} \right] q^n$$

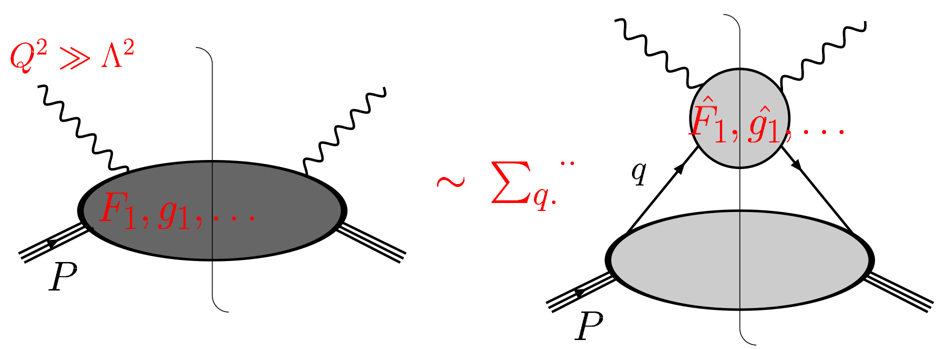
$$\equiv \left[1 + \frac{\alpha_s}{2\pi} \left(P_{qq}^n \log \frac{Q}{\mu} + r^n \right) \right] \tilde{q}^n \left(\frac{\mu}{m} \right)$$

- all dependence on long-distance scales in "new" parton distributions
- all dependence on short-distance scale Q in $[\ldots]$

• this procedure can be proven to really work : "Factorized DIS"

$$F_1^n(Q^2) \sim \sum_f \underbrace{f^n\left(\frac{\mu}{m}, \alpha_s(\mu)\right)}_{\text{pdf}} \; \underbrace{\hat{F}_1^n\left(\frac{Q}{\mu}, \alpha_s(\mu)\right)}_{\text{perturbative}}$$

• rescues – and generalizes – the parton model !



(Gross,Wilczek; Georgi,Politzer; Christ,Hasslacher,Mueller; Sterman,Libby; Amati et al.; Ellis et al.; Curci,Furmanski,Petronzio; Collins,Soper,Sterman; Collins; . . .)