Physics with Polarized Beams (III)

A tutorial for experimenters,

accelerator physicists, and students

Werner Vogelsang

RIKEN-BNL Research Center / BNL Nuclear Theory

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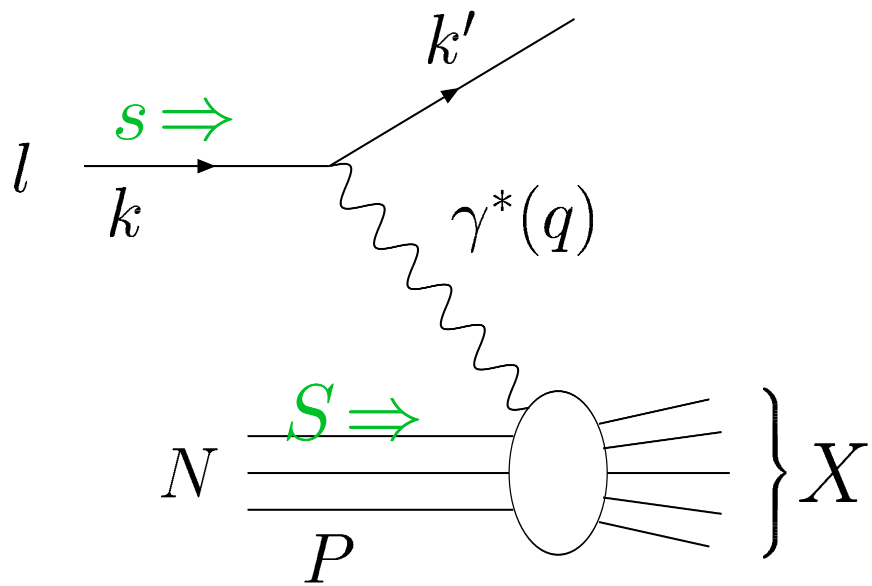
Brief summary of last lecture :

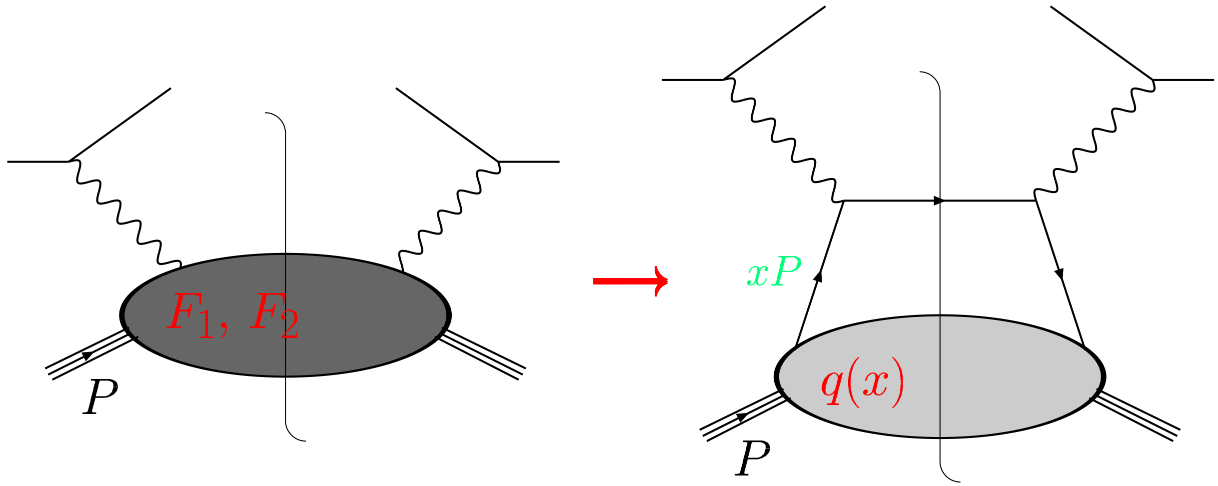
Deeply-inelastic lepton-nucleon scattering



$$Q^2 = -q^2 \gg m^2$$

$$x = \frac{Q^2}{2P\cdot q} = \frac{Q^2}{2m\nu}$$

- described in terms of structure functions $F_1,\ F_2,\ g_1,\ g_2$



$$F_1(x) = \frac{1}{2} \sum_q e_q^2 \left[q(x) + \bar{q}(x) \right]$$

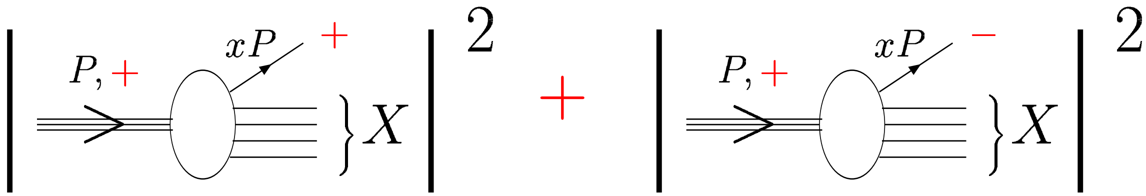
$$g_1(x) = \frac{1}{2} \sum_q e_q^2 \left[\Delta q(x) + \Delta \bar{q}(x) \right]$$

- general parton structure

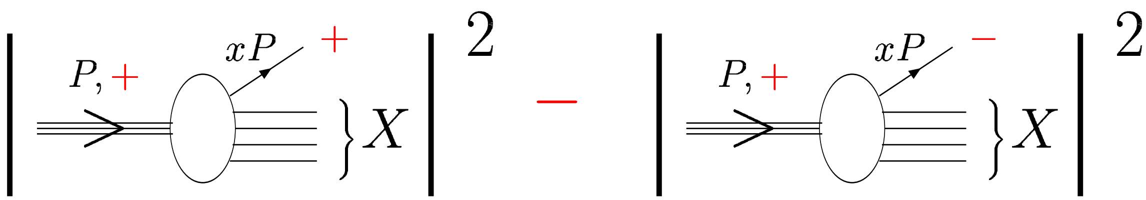
$$\phi(x) = \tfrac{1}{2} \Big[q(x) \not{P} + s_{\parallel} \, \Delta q(x) \, \gamma_5 \not{P} + \delta q(x) \not{P} \gamma_5 \not{S}_{\perp} \Big]$$

- quark-parton distributions of the nucleon :

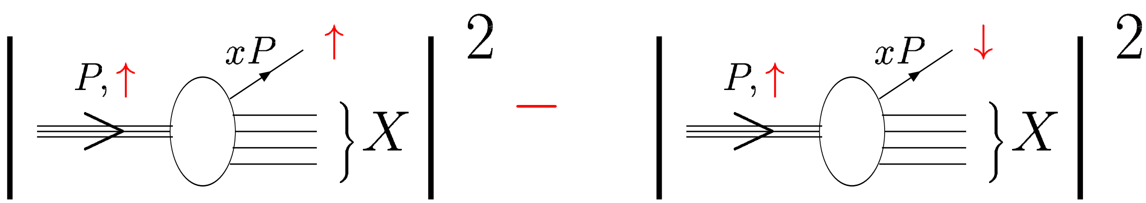
$q(x) =$



$\Delta q(x) =$

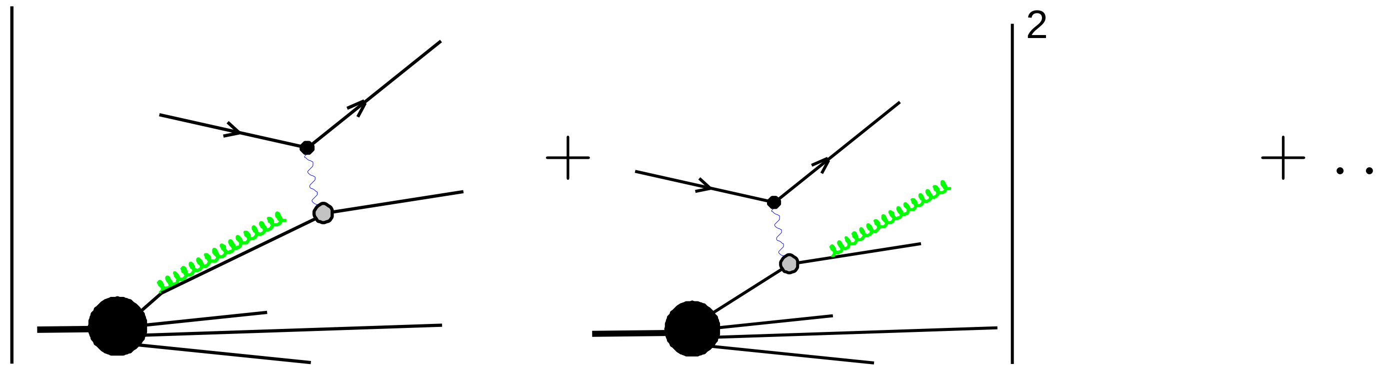


$\delta q(x) =$



Today :

- Scaling violations (cont‘d.)
- Factorization and Evolution
- What DIS data tell us about the nucleon
- What RHIC will tell us !



- had to integrate over gluon phase space. Found :

$$F_1^{\,n} \;\propto\; \left[\,1 \,+\, \frac{\alpha_s}{2\pi}\,\left(P_{qq}^{\,n}\,\log\frac{Q}{m} \,+\, r^{\,n}\right)\right]\; q^{\,n}$$

$$\equiv\; \left[\,1 \,+\, \frac{\alpha_s}{2\pi}\,\left(P_{qq}^{\,n}\,\log\frac{Q}{\mu} \,+\, r^{\,n}\right)\right]\;\; \tilde{q}^{\,n}\left(\frac{\mu}{m}\right)$$

- all dependence on long-distance scales in “new” parton distributions
- all dependence on short-distance scale Q in $[\ldots]$
- μ separates the two :
  - $k_T \le \mu \;\; \rightarrow$ parton distribution
  - $k_T \ge \mu \;\; \rightarrow$ $\mathcal{O}(\alpha_s)$ cross section

$$F_1^{\,n} \;=\; \left[\, 1 \;+\; \frac{\alpha_s}{2\pi} \left(P_{qq}^{\,n} \, \log \frac{Q}{\mu} \;+\; r^{\,n} \right) \right] \; q^{\,n}\left(\frac{\mu}{m}\right)$$

- how to interpret so far ?
  - F_1 clearly contains non-perturbative contributions
  - we have encountered them in a $\mathcal{O}(\alpha_s)$ calculation
  - we have introduced a scale $\mu \approx 1$ GeV, below which we have absorbed such contributions into pdf
- already now, we have gained something :
  - suppose, a measurement gives data for F_1^n at some Q_1^2
  - we could choose a value for $\mu \approx 1$ GeV, determine $q^{\,n}$ and predict F_1^n at another Q_2^2
- problem : Q_1, Q_2 could be very different from each other, and from μ

- procedure can be proven to work to all orders : "Factorized DIS"
- the basic statement is :

$$F_1^{\,n} \;\longrightarrow\; F_1^{\,n}\left(\frac{Q}{m}\right) \;=\; \hat{F}_1^{\,n}\left(\frac{Q}{\mu}\right)\; q^{\,n}\left(\frac{\mu}{m}\right) \;+\; \text{"small corrections"}$$

- we know on the other hand :

$$\mu\,\frac{dF_1^{\,n}\left(\frac{Q}{m}\right)}{d\mu} = 0$$

F_1 is physical !

- the consequence :

$$\frac{d\hat{F}_1^{\,n}\left(\frac{Q}{\mu}\right)}{d\log\mu}\; q^{\,n}\left(\frac{\mu}{m}\right) \;+\; \hat{F}_1^{\,n}\left(\frac{Q}{\mu}\right)\frac{dq^{\,n}\left(\frac{\mu}{m}\right)}{d\log\mu} \;=\; 0$$

- or,

$$- \frac{d \log \hat{F}_1^{\,n}\left(\frac{Q}{\mu}\right)}{d \log\left(\frac{Q}{\mu}\right)} \;=\; - \frac{d \log q^{\,n}\left(\frac{\mu}{m}\right)}{d \log\left(\frac{\mu}{m}\right)} \;\;\equiv\;\; \gamma^{\,n}$$

- solve it :

$$q^{\,n}\left(\frac{\mu}{m}\right) \;=\; q^{\,n}\left(\frac{\mu_0}{m}\right) \,\exp\left[-\,\gamma^{\,n}\,\log\left(\frac{\mu/m}{\mu_0/m}\right)\right]$$

- in short :

$$q^{\,n}(\mu) \;=\; q^{\,n}(\mu_0)\,\exp\left[-\,\gamma^{\,n}\,\log\left(\frac{\mu}{\mu_0}\right)\right]$$

- γ : "anomalous dimension"

- can be read off our example : had

$$\hat{F}_1^{\,n} \;=\; \left[\,1 \,+\, \frac{\alpha_s}{2\pi}\,\left(P_{qq}^{\,n}\,\log\frac{Q}{\mu} \,+\, r^{\,n}\right)\,\right]$$

therefore,

$$\gamma^{\,n} \;=\; -\frac{\alpha_s}{2\pi}\,P_{qq}^{\,n}$$

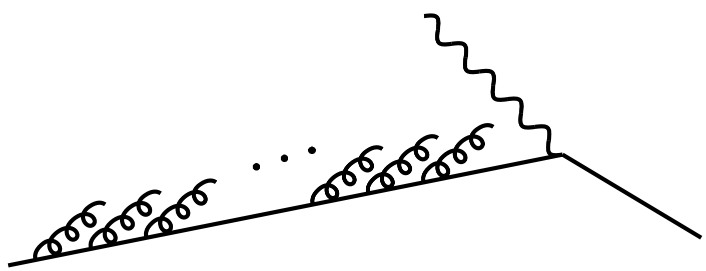
- let's put everything together. Let's choose $\mu = Q$ and get :

$$F_1^n(Q) = \left[1 + \frac{\alpha_s}{2\pi} r^n \right] \times q^n(\mu_0) \exp\left[\frac{\alpha_s}{2\pi} P_{qq}^n \log\left(\frac{Q}{\mu_0}\right) \right]$$

- key observations :
  - only "small" $\mathcal{O}(\alpha_s)$ corrections in $[\ldots]$ $\quad\longrightarrow$ neglect to a first approx.
  - expand exponent :

$$\exp[\ldots] = 1 + \frac{\alpha_s}{2\pi} P_{qq}^n \log\left(\frac{Q}{\mu_0}\right) + \frac{1}{2}\left[\frac{\alpha_s}{2\pi} P_{qq}^n \log\left(\frac{Q}{\mu_0}\right) \right]^2 + \ldots$$

  - we are getting all terms $\alpha_s^k \log^k(Q/\mu_0)$!

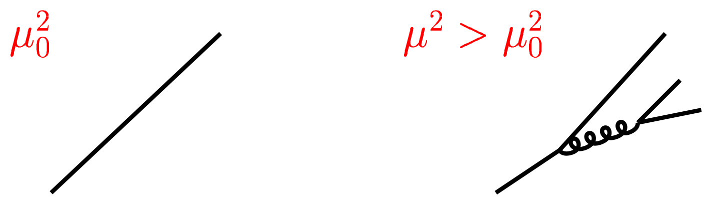


- this summation of all the large logarithms is called "evolution" of the parton distribution

- it is a direct consequence of factorization

- evolution is perturbative and a direct prediction of QCD

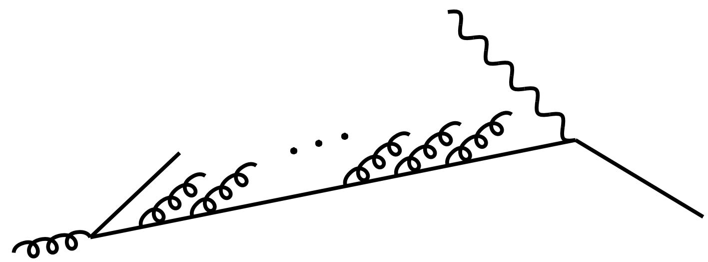
- physically, think of it as the effect of increasing the resolution scale :



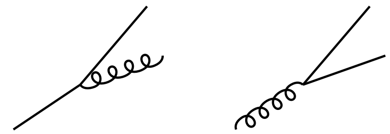
- the actual equations describing evolution are called DGLAP eqs :

  Dokshitzer,Gribov,Lipatov,Altarelli,Parisi

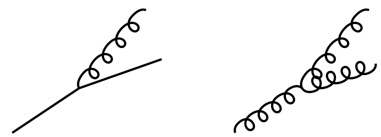
- they are in general somewhat more complicated than above :
  - renormalization of strong coupling $\alpha_s \to \alpha_s(\mu)$
  - the quark that eventually scatters may originate from a gluon :



• one finds :



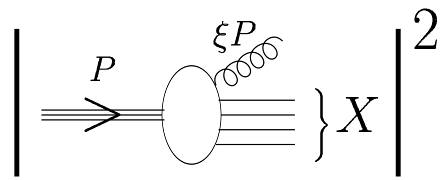
$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} \begin{pmatrix} q^n(\mu^2) \\ g^n(\mu^2) \end{pmatrix} = \begin{pmatrix} \mathcal{P}^n_{qq}(\alpha_s(\mu)) & \mathcal{P}^n_{qg}(\alpha_s(\mu)) \\ \mathcal{P}^n_{gq}(\alpha_s(\mu)) & \mathcal{P}^n_{gg}(\alpha_s(\mu)) \end{pmatrix} \cdot \begin{pmatrix} q^n(\mu^2) \\ g^n(\mu^2) \end{pmatrix}$$



$\mathcal{P}^n_{ij}(\alpha_s)$: ‘splitting’-functions, calculable in QCD perturbation theory,

$$\mathcal{P}^n_{ij}(\alpha_s) = \frac{\alpha_s}{2\pi} P^{(0),n}_{ij} + \left(\frac{\alpha_s}{2\pi}\right)^2 \underbrace{P^{(1),n}_{ij}}_{\text{NLO}} + \ldots$$

• involves gluon density $g(\xi, \mu^2)$:

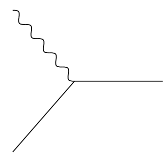
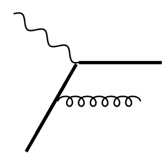


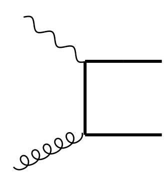
Let's take stock ! "QCD-improved Parton Model" for F_1

$$F_1(x,Q^2) = \sum_q \int_x^1 \frac{d\xi}{\xi}\left[q(\xi,Q^2) + \bar{q}(\xi,Q^2)\right] \cdot \hat{F}_1^{\,q}\left(\frac{x}{\xi},\alpha_s(Q)\right)$$

$$+ g(\xi,Q^2) \cdot \hat{F}_1^{\,g}\left(\frac{x}{\xi},\alpha_s(Q)\right) + \underbrace{\mathcal{O}\left(\frac{\lambda^2}{Q^2}\right)}_{\text{"small"}} \rightarrow$$

- where parton distributions evolve according to DGLAP . . .
- . . . and $\hat{F}_1^{\,q}$, $\hat{F}_1^{\,g}$ are calculated perturbatively from :

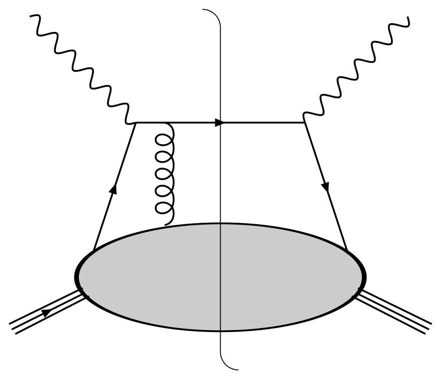
$$\hat{F}_1^{\,q} \sim \Big|$$  $$\Big|^2 + \Big|$$  $$+ \ldots \Big|^2 + \ldots$$

$$\hat{F}_1^{\,g} \sim \Big|$$  $$+ \ldots \Big|^2 + \ldots$$

LO NLO

Corrections to this ? $\mathcal{O}\left(\frac{\lambda^2}{Q^2}\right)$

- suppressed by powers of hard scale Q $\rightsquigarrow$ small
- typically,



g_1 follows exactly the same pattern :

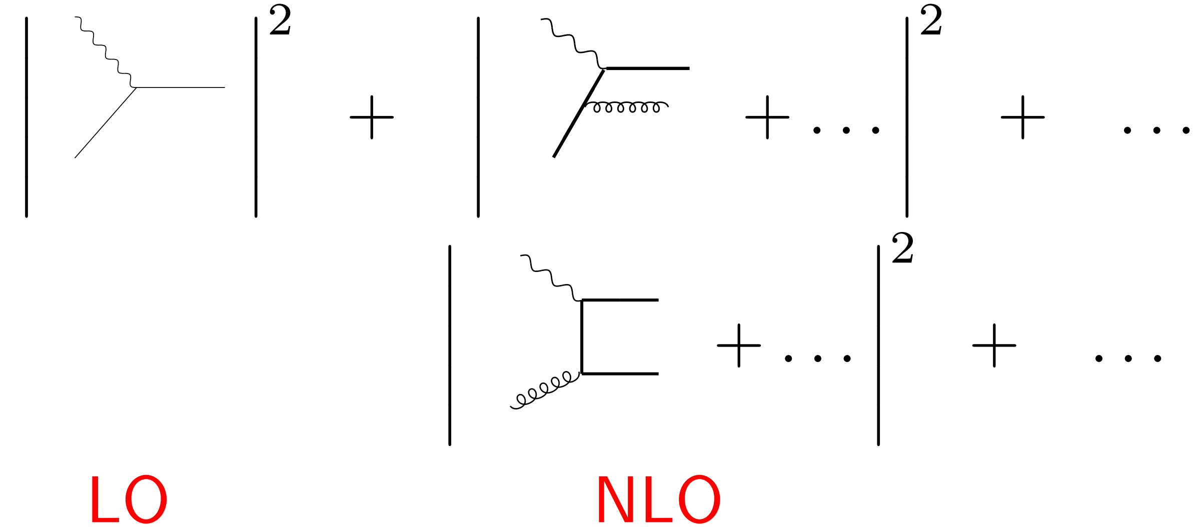
$$g_1(x, Q^2) = \sum_q \int_x^1 \frac{d\xi}{\xi} \left[\Delta q(\xi, Q^2) + \Delta \bar{q}(\xi, Q^2) \right] \cdot \hat{g}_1^{\,q} \left(\frac{x}{\xi}, \alpha_s(Q) \right)$$

$$+ \Delta g(\xi, Q^2) \cdot \hat{g}_1^{\,g} \left(\frac{x}{\xi}, \alpha_s(Q) \right) \qquad + \underbrace{\mathcal{O}\left(\frac{\lambda^2}{Q^2} \right)}_{\text{“small”}}$$

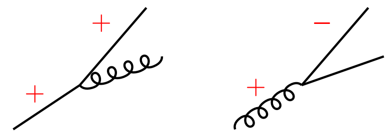
- $\hat{g}_1^{\,q}$, $\hat{g}_1^{\,g}$ perturbative – predicted by QCD :

$\hat{g}_1^{\,q} \sim$

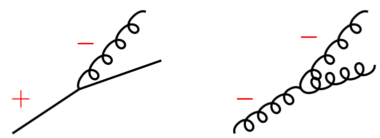
$\hat{g}_1^{\,g} \sim$



- spin-dependent DGLAP evolution equations :



$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} \begin{pmatrix} \Delta q(x,\mu^2) \\ \Delta g(x,\mu^2) \end{pmatrix} = \int_x^1 \frac{dz}{z} \begin{pmatrix} \Delta\mathcal{P}_{qq} & \Delta\mathcal{P}_{qg} \\ \Delta\mathcal{P}_{gq} & \Delta\mathcal{P}_{gg} \end{pmatrix}_{(z,\alpha_s(\mu))} \cdot \begin{pmatrix} \Delta q \\ \Delta g \end{pmatrix} \left(\frac{x}{z},\mu^2\right)$$

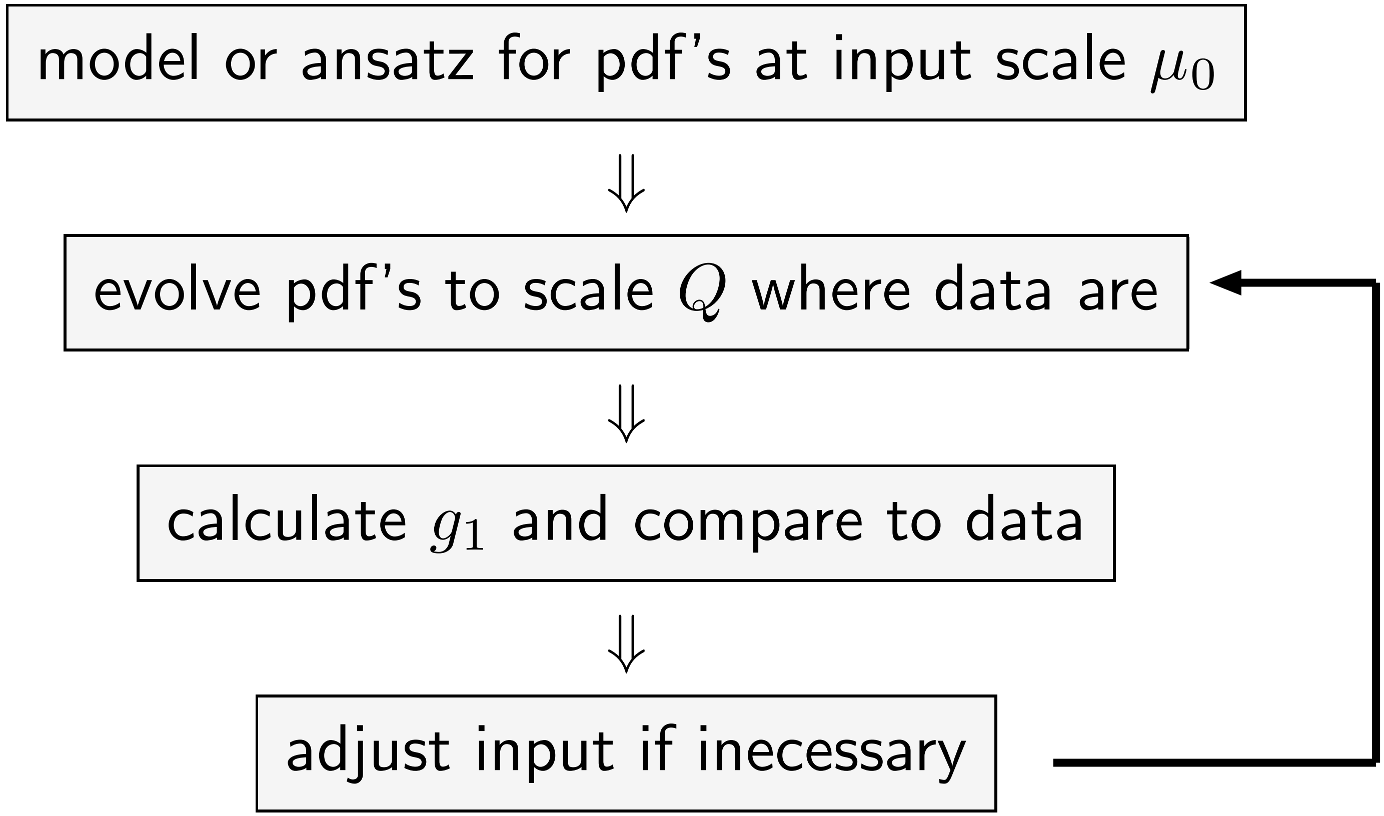


$\Delta\mathcal{P}_{ij}(z,\alpha_s(\mu))$: spin-dep. ‘splitting’-functions, calculable in QCD perturbation theory,

$$\Delta\mathcal{P}_{ij}(z,\alpha_s) \;=\; \frac{\alpha_s}{2\pi}\Delta P_{ij}^{(0)}(z) \;+\; \left(\frac{\alpha_s}{2\pi}\right)^2 \underbrace{\Delta P_{ij}^{(1)}(z)}_{\text{NLO}} + \ldots$$

(Ahmed,Ross; Altarelli,Parisi; Mertig,van Neerven; WV)

This leads the way to the analysis of DIS data :



in this way :

- get parton distribution functions
- test compatibility of Q^2 dependence of data with QCD

A few further important points :

- nothing forces us to choose exactly $\mu = Q$

  just $\mu \approx Q$ will do to avoid large logs in $\hat{F}_1$, $\hat{g}_1$

- the requirement was $dF_1^n / d\log(\mu) = 0$

  however, expect $\neq 0$ in truncated perturbation theory

- recall our "master formula" :

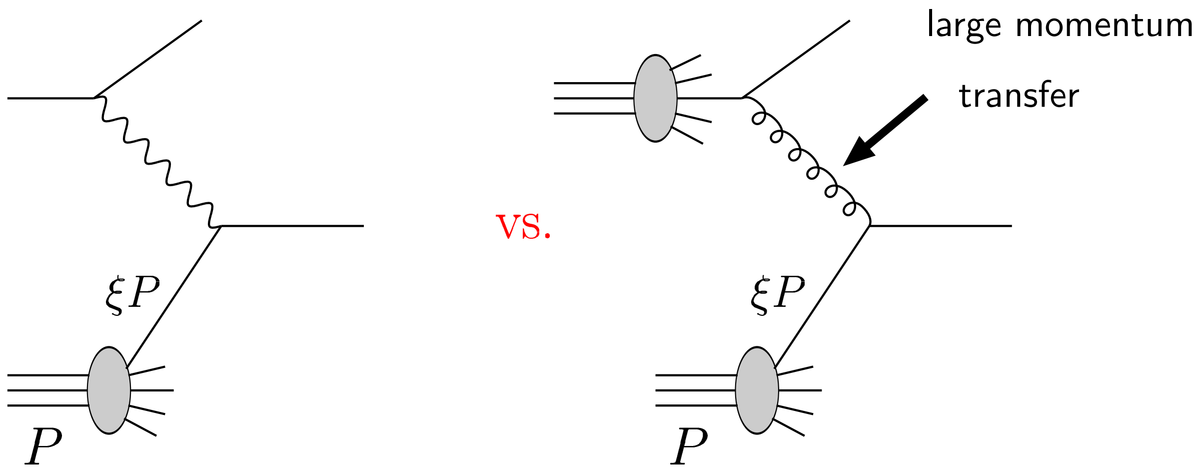
$$F_1^n = \left[1 + \frac{\alpha_s}{2\pi} \left(P_{qq}^n \log\frac{Q}{\mu} + r^n \right) \right] q^n \left(\frac{\mu}{m} \right)$$

  – could have absorbed the term r^n into parton distribution as well :

  $\leftrightarrow$ scheme (convention) dependence of pdfs

- parton distributions are universal

Parton distributions are universal :



- can be measured also in very inelastic hadron-hadron scattering
- formalized in terms of "factorization theorems"

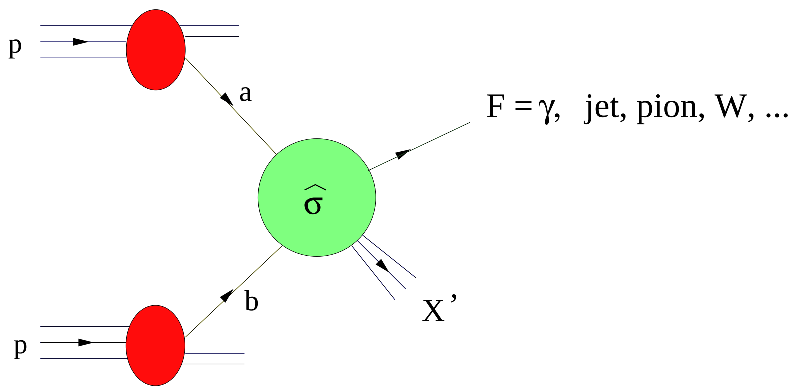
  (Sterman,Libby; Ellis et al.; Collins,Soper,Sterman; Collins)

- examples :
  - high-p_T reactions, $pp \to \mathrm{jet}X$, $pp \to \gamma X$, $pp \to \pi X$, ...
  - large produced masses, $pp \to (c\bar{c})X$, $pp \to WX$, ...
- makes notion of "nucleon structure" meaningful !

Factorized cross sections :

consider high-p_T final state : $\Rightarrow$ hard scale

$$p_T^3 \frac{d\sigma}{dp_T} = \left| \quad \right|^2 + \mathcal{O}\left(\frac{\lambda}{p_T}\right)^n$$



$$p_T^3 \frac{d\sigma^{pp\to \mathrm{F}X}}{dp_T} = \sum_{abc} \int dx_a\, dx_b\, dz_c\, f_a(x_a,\mu)\, f_b(x_b,\mu)$$

$$\times\, p_T^3 \frac{d\hat\sigma^{ab\to \mathrm{F}X'}}{dp_T}(x_a P_a, x_b P_b, P^{\mathrm{F}}/z_c, \mu) \;+\; \text{Power corr.}$$

IV. Lessons from Polarized DIS

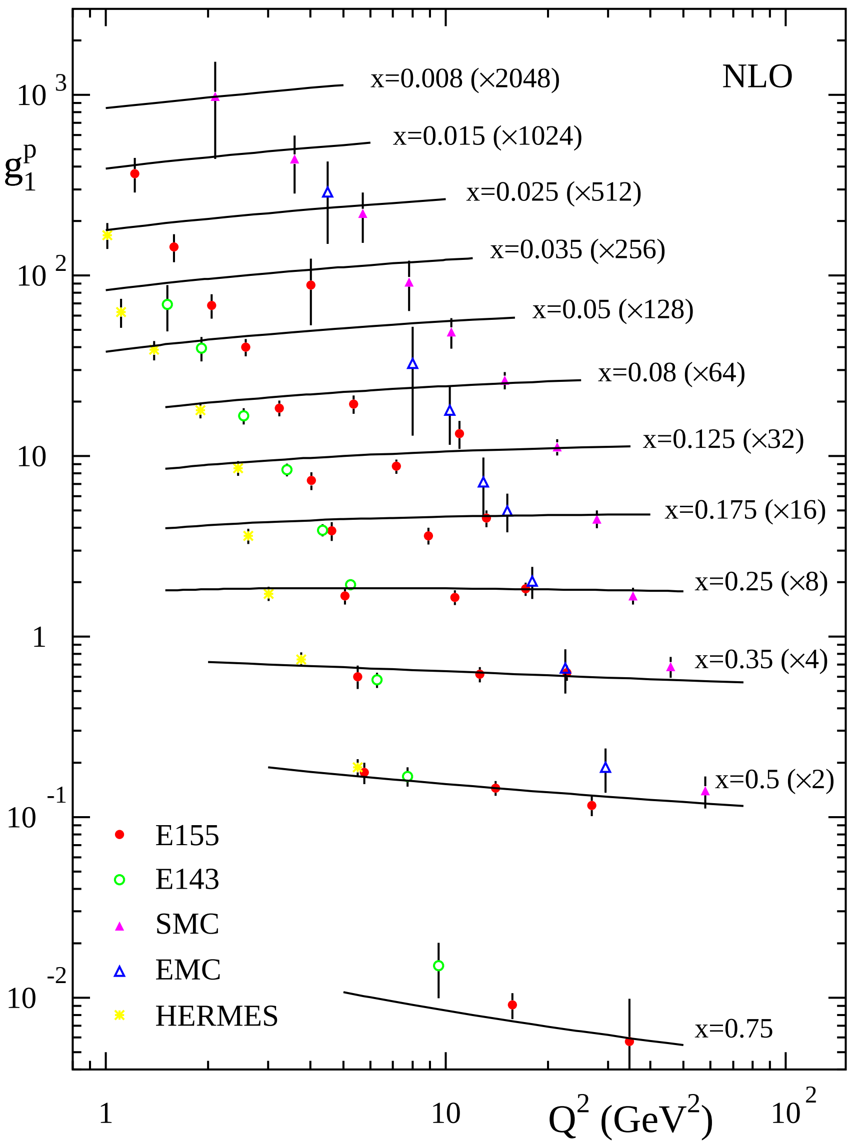
- NLO QCD fits describe data well
- Bjorken sum rule confirmed to $\sim 10\%$
- quarks do *not* carry the proton spin !

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g \qquad \frac{1}{2}\Delta\Sigma \approx 0$$

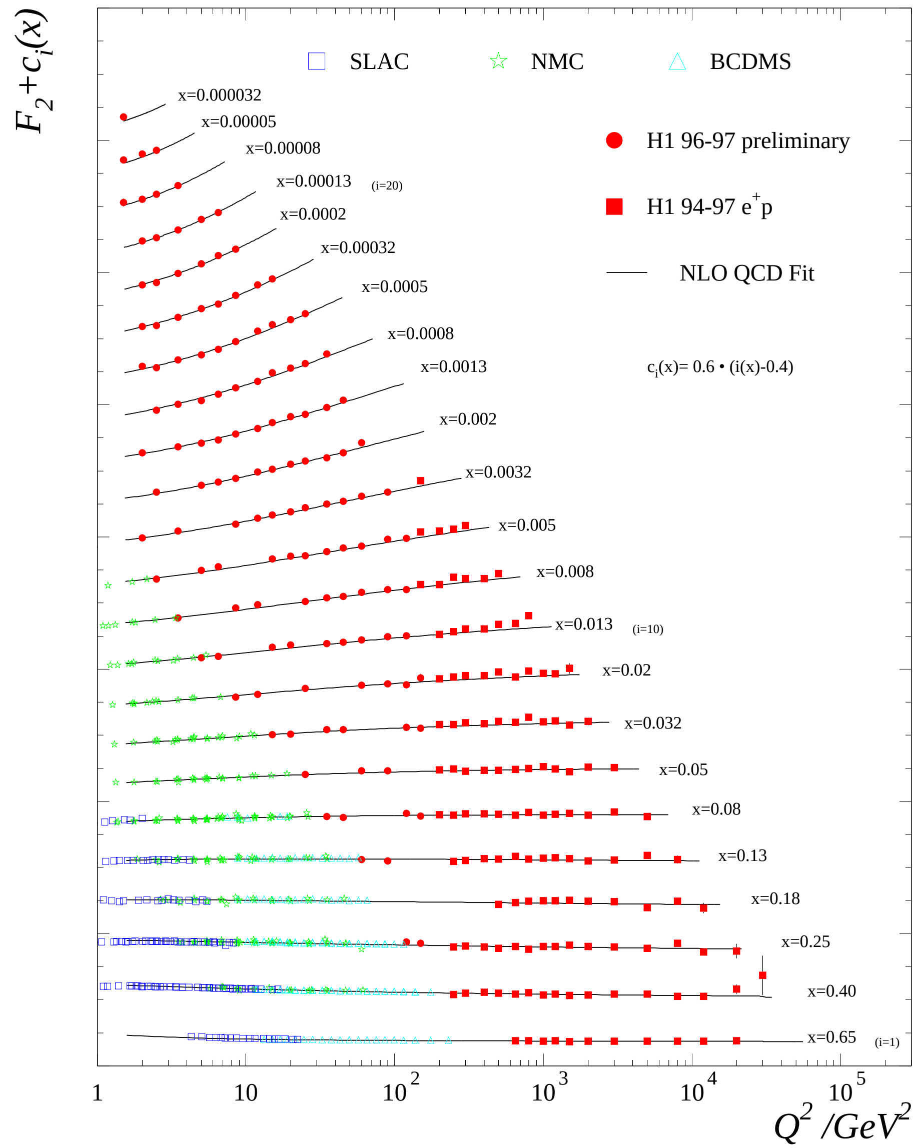
- still considerable lack of knowledge regarding
  - polarized gluon density $\Delta g(x, \mu^2)$
  - orbital angular momenta of quarks and gluons in nucleon
  - separation $u, \bar{u}, d, \bar{d}, s, \bar{s}, \ldots$

World data on pol. and unpol. deep-inelastic scattering

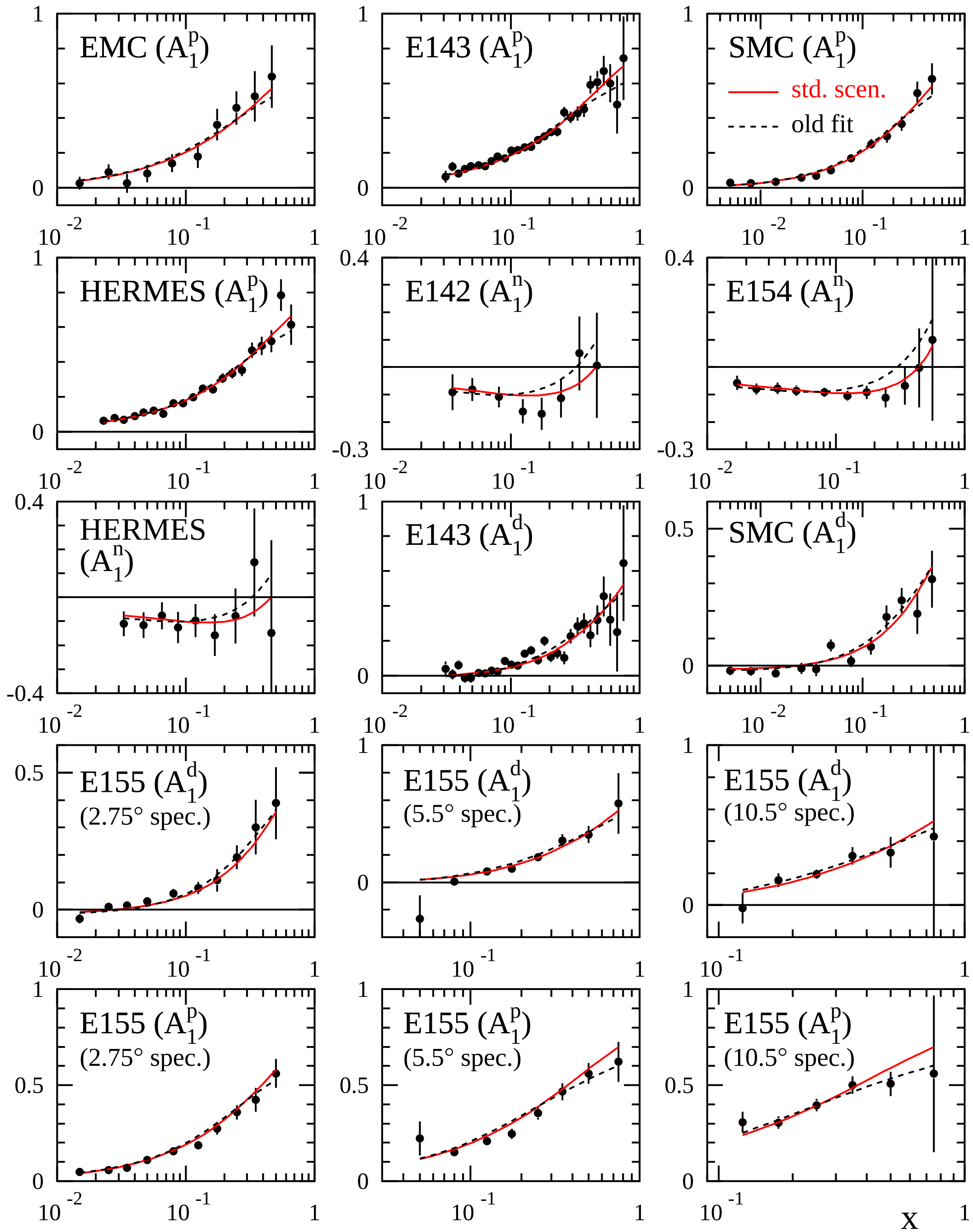
polarized



unpolarized



NLO QCD fit: Glück, Reya, Vogelsang, MS (2000 update)


EMC (A_1^p)
E143 (A_1^p)
SMC (A_1^p)
std. scen.
old fit
HERMES (A_1^p)
E142 (A_1^n)
E154 (A_1^n)
HERMES (A_1^n)
E143 (A_1^d)
SMC (A_1^d)
E155 (A_1^d) (2.75° spec.)
E155 (A_1^d) (5.5° spec.)
E155 (A_1^d) (10.5° spec.)
E155 (A_1^p) (2.75° spec.)
E155 (A_1^p) (5.5° spec.)
E155 (A_1^p) (10.5° spec.)
x

Things one can determine, and things one can't :

- assume : have proton and neutron data
- for DIS via photon exchange : only $\Delta q + \Delta\bar{q}$, not separately
- proton (lowest order) :

$$g_1^p = \frac{1}{2}\left[\frac{4}{9}\left(\Delta u + \Delta\bar{u}\right) + \frac{1}{9}\left(\Delta d + \Delta\bar{d} + \Delta s + \Delta\bar{s}\right)\right]$$

- for neutron assume isospin symmetry : $\Delta u^n = \Delta d^p$ etc.
- then,

$$g_1^n = \frac{1}{2}\left[\frac{4}{9}\left(\Delta d + \Delta\bar{d}\right) + \frac{1}{9}\left(\Delta u + \Delta\bar{u} + \Delta s + \Delta\bar{s}\right)\right]$$

rewrite jointly as

$$g_1^{p,n}(x,Q^2) \;=\; \pm\frac{1}{12}\Delta a_3(x,Q^2) \;+\; \frac{1}{36}\Delta a_8(x,Q^2) \;+\; \frac{1}{9}\Delta a_0(x,Q^2)$$

where

$$\Delta a_3 \;\equiv\; \Delta u + \Delta\bar{u} - \Delta d - \Delta\bar{d}$$

$$\Delta a_8 \;\equiv\; \Delta u + \Delta\bar{u} + \Delta d + \Delta\bar{d} - 2\left(\Delta s + \Delta\bar{s}\right)$$

$$\Delta a_0 \;\equiv\; \Delta u + \Delta\bar{u} + \Delta d + \Delta\bar{d} + \Delta s + \Delta\bar{s}$$

- if we had data at only one Q^2 :

  2 results for 4 functions Δa_3, Δa_8, Δa_0, Δg

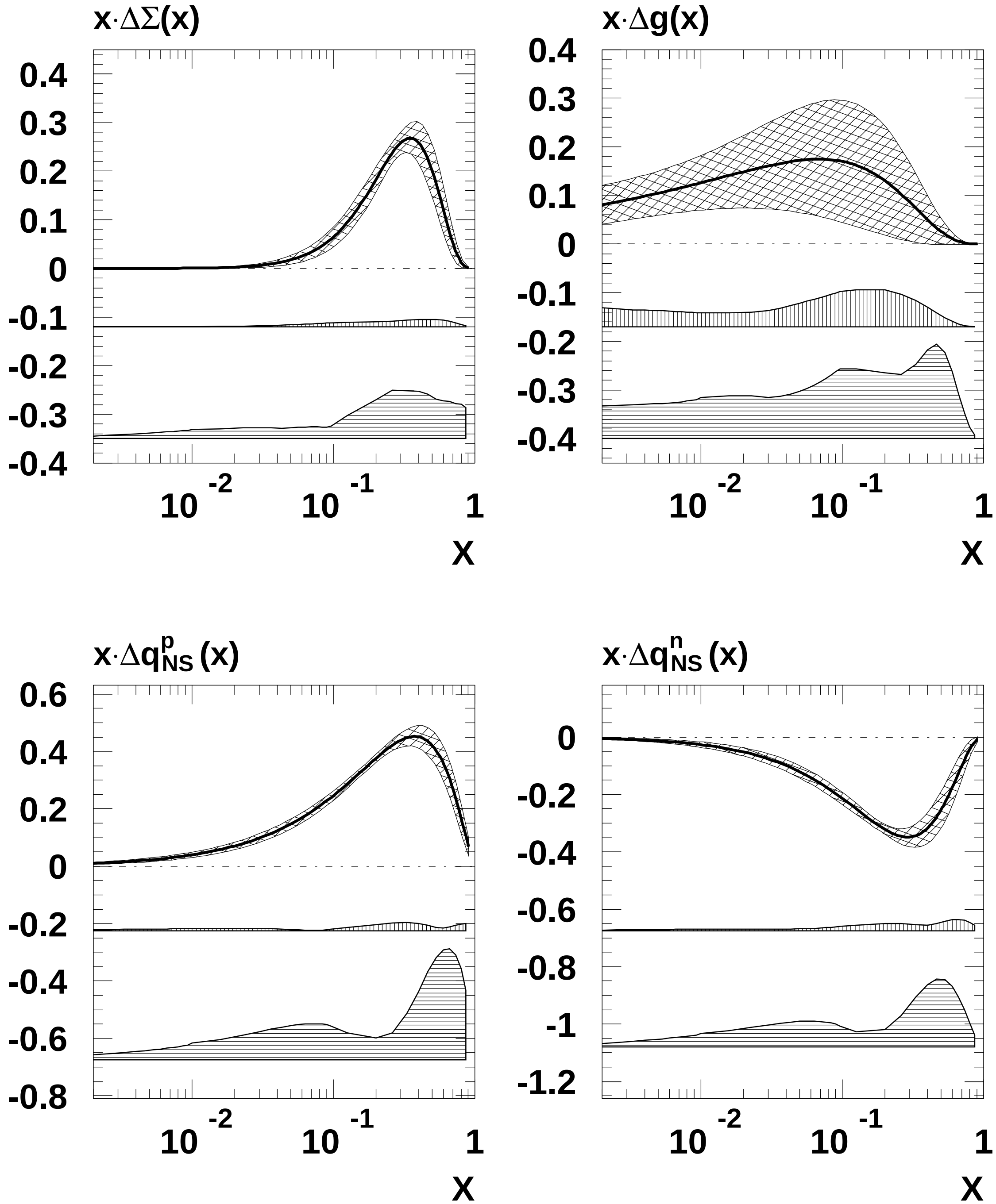
- with data at different Q^2 : evolution properties help
- full evolution equations :

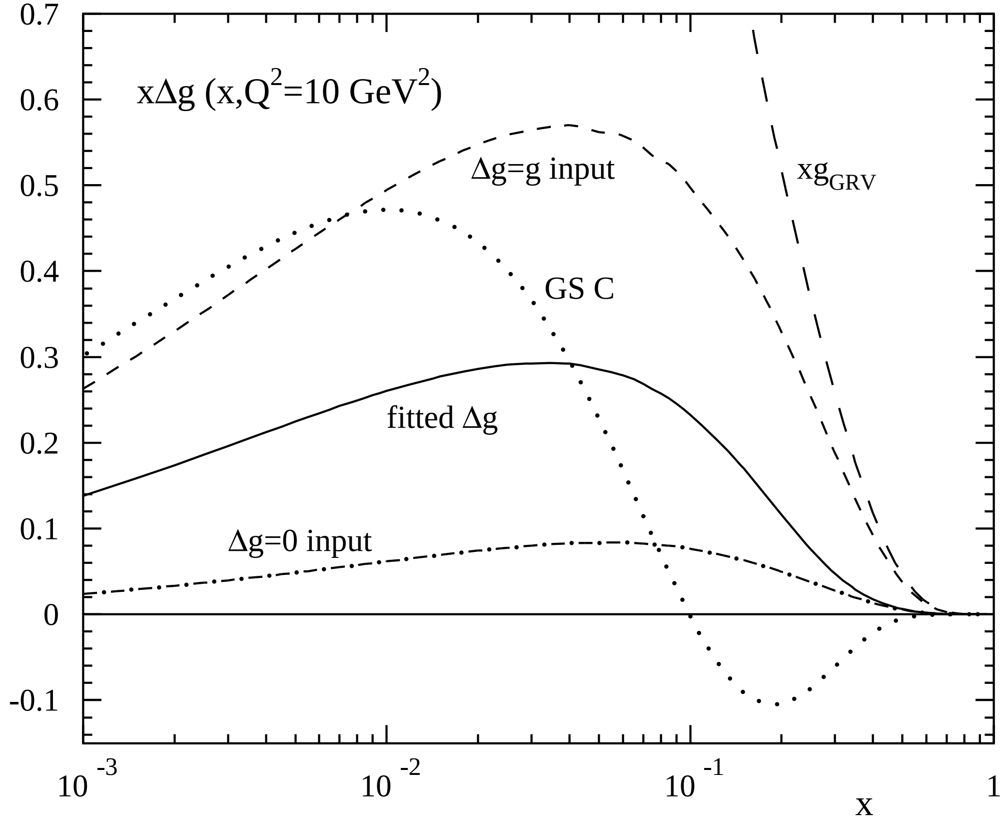
$$\mu\frac{\mathrm{d}}{\mathrm{d}\mu}\left(\begin{array}{c}\Delta a_0(x,\mu^2)\\ \Delta g(x,\mu^2)\end{array}\right)=\frac{\alpha_s}{2\pi}\int_x^1\frac{dz}{z}\left(\begin{array}{cc}\Delta P_{qq}(z) & \Delta P_{qg}(z)\\ \Delta P_{gq}(z) & \Delta P_{gg}(z)\end{array}\right)\cdot\left(\begin{array}{c}\Delta a_0\\ \Delta g\end{array}\right)\left(\frac{x}{z},\mu^2\right)$$

$$\mu\frac{\mathrm{d}}{\mathrm{d}\mu}\Delta a_{3,8}(x,\mu^2) = \frac{\alpha_s}{2\pi}\int_x^1\frac{dz}{z}\,\Delta P_{qq}(z)\,\Delta a_{3,8}\left(\frac{x}{z},\mu^2\right)$$

- $\Rightarrow$ can in principle extract Δa_3, Δa_8, Δa_0, Δg and hence all $\Delta q + \Delta\bar{q}$ and Δg
- however, in practice : fixed-target expt. $\leftrightarrow$ low lever arm in Q^2
- need "some extra help" . . .

SMC analysis :





Polarized gluon densities :

(Glück,Reya,Stratmann,WV; Gehrmann,Stirling)

Bjorken's sum rule

- originally, a main motivation for measuring g_1 !

$$\int_0^1 dx \left[g_1^p(x,Q^2) - g_1^n(x,Q^2) \right] = \frac{1}{6}\, g_A$$

$n \to p\, e^- \, \bar{\nu}_e$

- involves x-integral of the structure functions. Let's denote :

$$\int_0^1 \Delta f(\xi)\, d\xi = \int_0^1 \left[f^+(\xi) - f^-(\xi) \right] d\xi \equiv \Delta F \qquad \text{"first moment"}$$

$=$ net total parton polarization in nucleon

- when expressed in terms of parton densities :

$$\mathcal{G}_1^{p-n}(Q^2) = \frac{1}{6} \left[\Delta U(Q^2) + \Delta \bar{U}(Q^2) - \Delta D(Q^2) - \Delta \bar{D}(Q^2) \right]$$

$$\equiv \frac{1}{6} \Delta \mathcal{A}_3(Q^2)$$

- one can show :

$$s^\mu \, \Delta\mathcal{A}_3(Q^2) \;=\; \langle P,S \,|\; \underbrace{\bar\psi\,\gamma^\mu\,\gamma^5\,\frac{\lambda_3}{2}\,\psi}_{\text{non}-\text{sing. axial current } j_5^{\mu,3}} \;|\, P,S\rangle$$

- $\Delta\mathcal{A}_3$ doesn't evolve with Q^2 :

$$\Delta\mathcal{A}_3(Q^2) \;\equiv\; \Delta\mathcal{A}_3(\not{Q}^2)$$

- isospin rotation yields the sum rule

$$\langle\, p \,|\, j_5^{\mu,3} \,|\, p\rangle \;-\; \langle\, n \,|\, j_5^{\mu,3} \,|\, n\,\rangle \;=\; \langle\, p \,|\, j_5^{\mu,+} \,|\, n\,\rangle \;\propto\; g_A$$

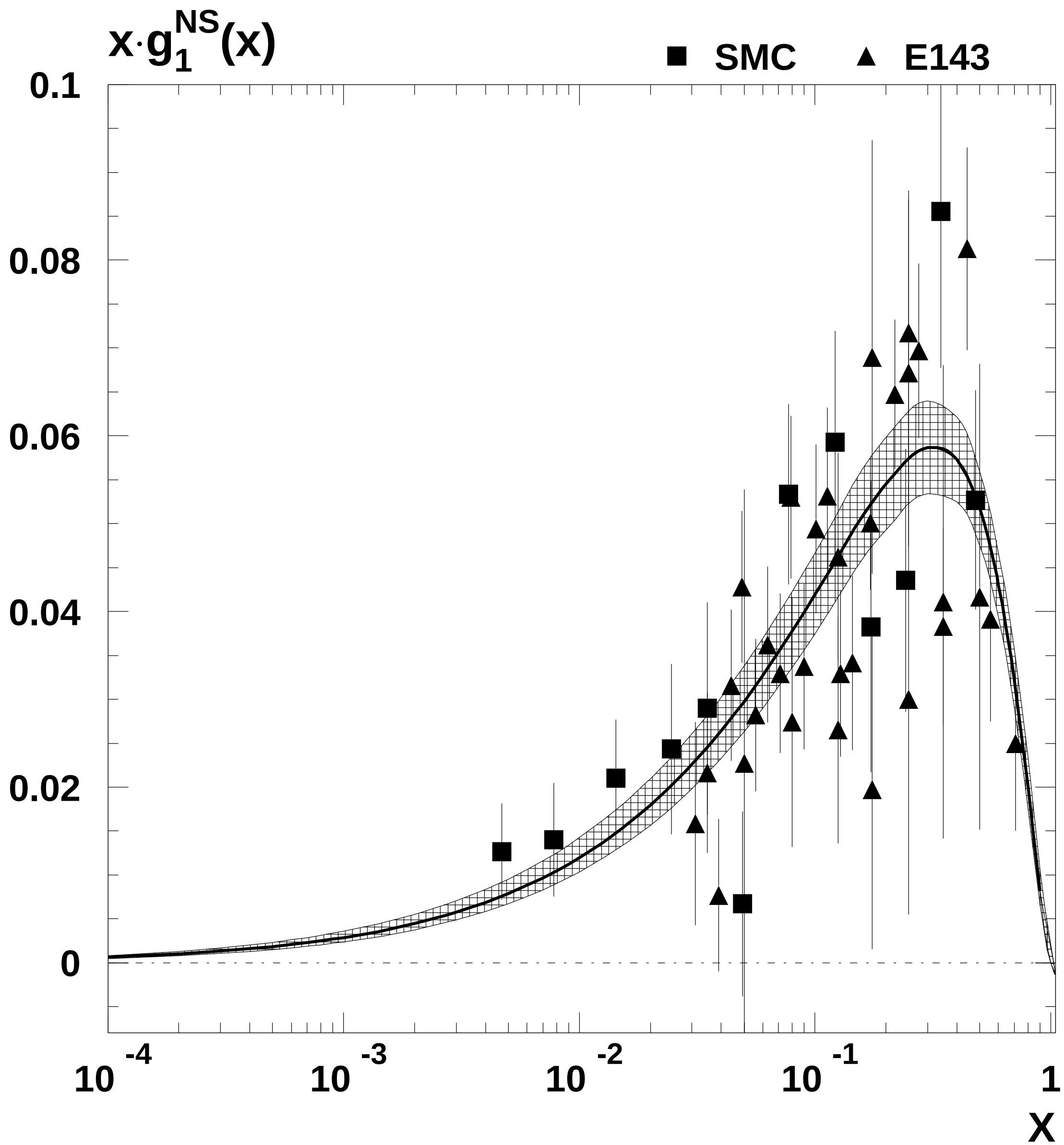
- Recall : "QCD formula" for g_1 contains $\mathcal{O}(\alpha_s), \mathcal{O}(\alpha_s^2), \ldots$ corrections to partonic cross sections

$$\Rightarrow \quad \text{get} \qquad \mathcal{G}_1^{p-n}(Q^2) \;=\; \frac{1}{6}\, \mathcal{C}_{\mathrm{Bj}}\Big(\alpha_s(Q)\Big)\; g_A$$

where

$$\begin{aligned}
\mathcal{C}_{\mathrm{Bj}}(\alpha_s) \;&=\; 1 - \frac{\alpha_s}{\pi} - \left[\frac{55}{12} - \frac{n_f}{3}\right]\left(\frac{\alpha_s}{\pi}\right)^2 \\
&\quad - \;\left[41.4 - 7.6\, n_f + \frac{115}{648}\, n_f^2\right]\left(\frac{\alpha_s}{\pi}\right)^3 \pm \ldots
\end{aligned}$$

(Kodaira et al.; Gorishny,Larin; Larin,Vermaseren)



$x \cdot g_1^{NS}(x)$
SMC
E143
0.1
0.08
0.06
0.04
0.02
0
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
X

$Q^2 = 5$ GeV^2 :

- Theory :

$$\mathcal{G}_1^{p-n}(Q^2) = 0.181 \pm 0.003$$

- SMC : (E155 similar)

$$\mathcal{G}_1^{p-n}(Q^2) = 0.174 \pm 0.005 \begin{array}{cc} +0.011 & +0.021 \\ -0.009 & -0.006 \end{array}$$

Ellis-Jaffe sum rule

- we had to lowest order :

$$g_1 = \frac{1}{2}\left[\frac{4}{9}\left(\Delta u + \Delta\bar{u}\right) + \frac{1}{9}\left(\Delta d + \Delta\bar{d} + \Delta s + \Delta\bar{s}\right)\right]$$

- reshuffle a little, and integrate over x,

$$\mathcal{G}_1^{p,n}(Q^2) = \int_0^1 dx\, g_1^{p,n}(x,Q^2) = \pm\frac{1}{12}\Delta\mathcal{A}_3 + \frac{1}{36}\Delta\mathcal{A}_8 + \frac{1}{9}\Delta\Sigma$$

where

$$\Delta\mathcal{A}_3 \equiv \Delta U + \Delta\bar{U} - \Delta D - \Delta\bar{D}$$

$$\Delta\mathcal{A}_8 \equiv \Delta U + \Delta\bar{U} + \Delta D + \Delta\bar{D} - 2\left(\Delta S + \Delta\bar{S}\right)$$

$$\Delta\Sigma \equiv \Delta U + \Delta\bar{U} + \Delta D + \Delta\bar{D} + \Delta S + \Delta\bar{S}$$

One finds :

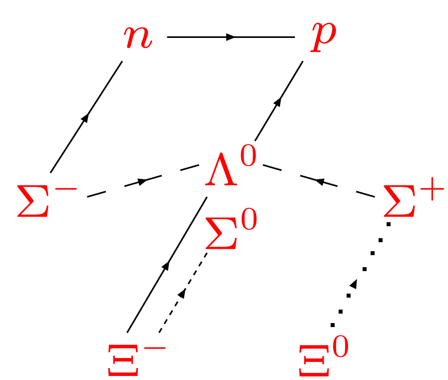
$$s^\mu \, \Delta\mathcal{A}_3 \;=\; \langle P,S \,|\, \bar\psi \,\gamma^\mu \,\gamma^5 \,\frac{\lambda_3}{2}\,\psi \,|\, P,S\rangle$$

$$s^\mu \, \Delta\mathcal{A}_8 \;=\; \langle P,S \,|\, \bar\psi \,\gamma^\mu \,\gamma^5 \,\frac{\lambda_8}{2}\,\psi \,|\, P,S\rangle$$

$$s^\mu \, \Delta\Sigma \;=\; \langle P,S \,|\, \bar\psi \,\gamma^\mu \,\gamma^5 \,\frac{1\!\!1}{2}\,\psi \,|\, P,S\rangle$$

- $\Delta\mathcal{A}_3 \;=\; g_A$ Bj. sum rule . . .

- SU(3) : $\Delta\mathcal{A}_8$ from baryonic β decays



decay	SU(3)	exp.
$n \longrightarrow pe^-\bar\nu_e$	$F + D$	1.2670 ± 0.0035
$\Lambda \longrightarrow pe^-\bar\nu_e$	$F + D/3$	0.718 ± 0.015
$\Sigma^- \longrightarrow ne^-\bar\nu_e$	$F - D$	-0.340 ± 0.017
$\Xi^- \longrightarrow \Lambda e^-\bar\nu_e$	$F - D/3$	0.25 ± 0.05
$\Xi^+ \longrightarrow \Sigma^+ e^-\bar\nu_e$	$F + D$	1.32 ± 0.22

$$\Delta\mathcal{A}_3 \;=\; F + D = g_A = 1.267 \pm 0.0035$$

$$\Delta\mathcal{A}_8 \;=\; 3F - D = 0.58 \pm 0.03$$

Ellis,Jaffe : assume $\Delta S + \Delta\bar{S} \approx 0$

$$\mathcal{G}_1^p(Q^2) \approx \frac{1}{2}F - \frac{1}{18}D = 0.186 \pm 0.004$$

$$\text{E155} \quad 0.118 \pm 0.004 \pm 0.007$$

$$\mathcal{G}_1^n(Q^2) \approx \frac{1}{3}F - \frac{2}{9}D = -0.025 \pm 0.004$$

$$\text{E155} \quad -0.058 \pm 0.005 \pm 0.008$$

Hence, we find that $\boxed{\Delta S + \Delta\bar{S} \approx -0.14}$ or

$\boxed{\frac{1}{2}\Delta\Sigma \approx 0.08 \pm 0.04 \quad \ll \frac{1}{2}}$ "spin crisis"

recall : $$\frac{1}{2}\Delta\Sigma \equiv \frac{1}{2}\left[\Delta U + \Delta\bar{U} + \Delta D + \Delta\bar{D} + \Delta S + \Delta\bar{S}\right]$$

= quark spin contribution to nucleon spin

What could it be due to ?

Let's look a little more at $\Delta\Sigma$:

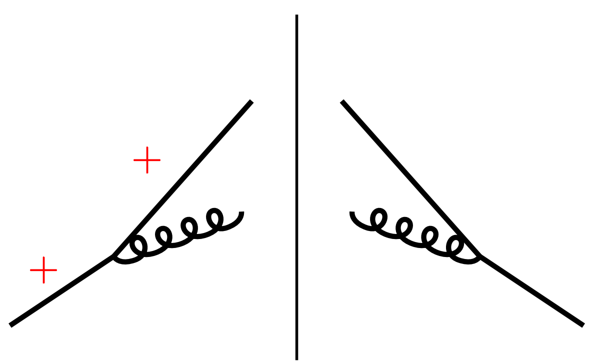
- recall $\mathcal{A}_{3,8}(\cancel{Q^2})$

$$\Delta\mathcal{A}_{3,8}(Q) \;\sim\; \langle P,S\,|\,\bar{\psi}\,\gamma^\mu\,\gamma^5\,\frac{\lambda_{3,8}}{2}\,\psi\,|\,P,S\rangle \qquad\qquad \partial_\mu\, j_5^\mu \;=\; 0$$

- However, $\Delta\Sigma$ independent of Q^2 only at lowest order in evolution :

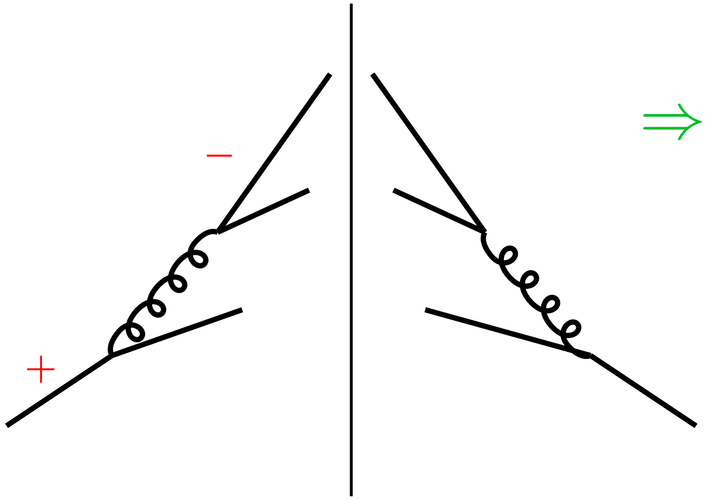
$$\frac{d\,\Delta\Sigma_{\overline{\rm MS}}(Q^2)}{d\ln Q^2} = \Bigg[\; \underbrace{\frac{\alpha_s}{2\pi}\cdot 0}_{\rm LO} \;+\; \underbrace{\left(\frac{\alpha_s}{2\pi}\right)^2 \cdot (\neq 0)}_{\rm NLO} \;\Bigg]\;\Delta\Sigma_{\overline{\rm MS}}(Q^2)$$

- why ? At LO :



$\Rightarrow$ no net helicity change

- at NLO : this parton diagram is responsible (in $\overline{\text{MS}}$ scheme) :

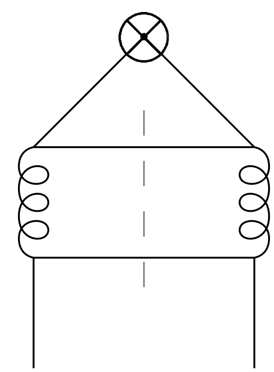


$\Rightarrow$ net helicity change

- deeper reason : $s^\mu \, \Delta\Sigma \; = \; \langle P, S \, | \; \underbrace{\bar{\psi} \, \gamma^\mu \, \gamma^5 \, \frac{1}{2} \, \psi}_{\text{singlet axial current } j_5^{\mu,0}} \; | \, P, S \rangle$

- *not conserved* due to axial anomaly :

$$\partial_\mu \, j_5^{\mu,0} \equiv \partial_\mu \left[\bar{\psi} \, \gamma^\mu \, \gamma^5 \, \psi \right] = n_f \, \frac{\alpha_s}{2\pi} \, \epsilon^{\mu\nu\rho\sigma} \, \mathrm{Tr} \left[G_{\mu\nu} \, \tilde{G}^{\mu\nu} \right]$$



- find

$$\Delta\Sigma(Q^2) \;=\; \left(1 \;+\; \frac{6n_f}{(33-2n_f)\,\pi}\,\left[\,\alpha_s(Q^2) \;-\; \alpha_s(\mu_0^2)\,\right] \right) \,\Delta\Sigma(\mu_0^2)$$

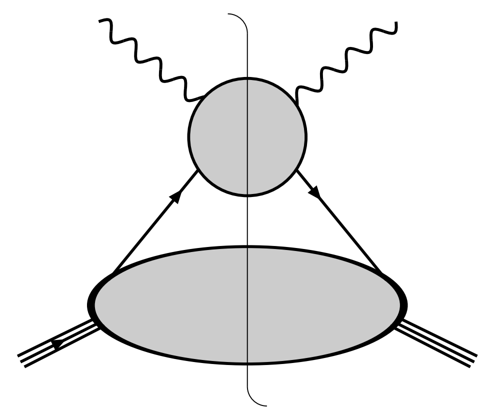
- $\Rightarrow$ moderate decrease in perturbative region (Jaffe)

- proposals for alternative definitions of $\Delta\Sigma$: (Altarelli, Ross; Carlitz, Collins, Mueller)

  – scheme dependence at NLO : may redefine parton distributions without affecting the physics

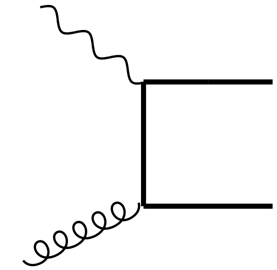
  – affects NLO evolution of pdfs and NLO partonic scattering cross sections

  – normally rather irrelevant :
  $\text{pdf}_{\text{sch. I}} \;=\; \text{pdf}_{\text{sch. II}} + \mathcal{O}(\alpha_s)$



- a particular scheme transformation of the first moment of $\Delta\Sigma$:

(Altarelli, Ross; Carlitz, Collins, Mueller)



$$\Delta\Sigma(Q^2) = \Delta\Sigma'(\cancel{Q}^2) - n_f \frac{\alpha_s(Q)}{2\pi} \underbrace{\int_0^1 dx \Delta g(x, Q^2)}_{\equiv \Delta G(Q^2)}$$

- LO evolution of first moment of gluon density :

$$\frac{d\,\Delta G(Q^2)}{d\ln Q^2} = \frac{\alpha_s(Q)}{2\pi}\left[2\,\Delta\Sigma + \frac{\beta_0}{2}\,\Delta G(Q^2)\right]$$

- gives $\alpha_s(Q)\,\Delta G(Q^2) \rightarrow \mathrm{const.}$ as $Q \rightarrow \infty$!

- therefore, not a "genuine" $\mathcal{O}(\alpha_s)$ correction

- however, separation in terms of underlying operators not gauge inv.

- no compelling reason to identify $\Delta\Sigma'$ with quark spin

- however, perfectly allowed in context of high-energy factorization

If quarks don't – who does ?

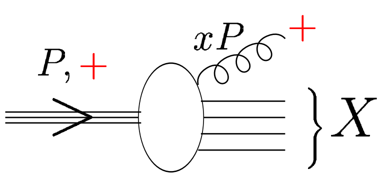
$$\frac{1}{2} = \langle P, \frac{1}{2} | \int d^3x \left[x^1 T^{02}(x) - x^2 T^{01}(x) \right] | P, \frac{1}{2} \rangle$$

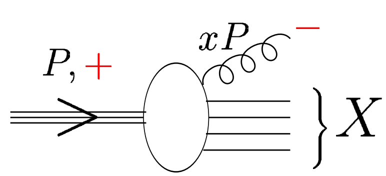
- one finds (Jaffe,Manohar; Ji)

$$M_{012} = \underbrace{\bar{\psi}\,\gamma^3 \gamma_5\, \psi}_{\sim \text{ quark spin}} + \underbrace{\psi^\dagger \left(\vec{x} \times \left(-i\vec{\mathcal{D}} \right) \right)_3 \psi}_{\sim \text{ quark OAM}} + \underbrace{\left(\vec{E} \times \vec{A} \right)_3}_{\text{gluon spin}} + \underbrace{E_j \left(\vec{x} \times i\vec{\mathcal{D}} \right)_3 A_j}_{\text{gluon OAM}}$$

leads to : $\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g$

- perhaps gluons ? $\Delta G(Q^2) = \int_0^1 dx\, \Delta g(x, Q^2)$ where

$\Delta g(x) = \Big|$ 

$\Big|^2 - \Big|$ 

$\Big|^2$

- a promising candidate

Good reasons for wanting to measure $\Delta g(x, \mu^2)$:

- a fundamental property of the nucleon !
- quark spin contribution small $\Rightarrow$
  $\int \Delta g$ possible candidate for major contributor to proton spin
- $\int \Delta g$ has a striking evolution with Q^2
  (fits to DIS data see tendency to sizeable gluon polarization)
- involved in relating QCD hard-scattering cross sections

V. Learning about Δg, Δq from hadronic collisions

- in inclusive DIS :
  - only $\Delta q + \Delta\bar{q}$
  - Δg contributes to evolution

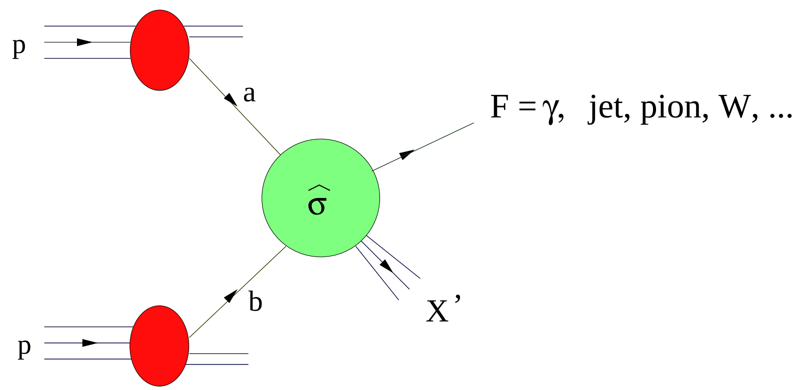
$$\mu\frac{\mathrm{d}}{\mathrm{d}\mu}\begin{pmatrix}\Delta q(x,\mu^2)\\ \Delta g(x,\mu^2)\end{pmatrix} = \int_x^1 \frac{dz}{z}\begin{pmatrix}\Delta\mathcal{P}_{qq} & \Delta\mathcal{P}_{qg}\\ \Delta\mathcal{P}_{gq} & \Delta\mathcal{P}_{gg}\end{pmatrix}_{(z,\alpha_s(\mu))} \cdot \begin{pmatrix}\Delta q\\ \Delta g\end{pmatrix}\left(\frac{x}{z},\mu^2\right)$$

$\Rightarrow$ difficult to get precise information if Q^2 lever arm is small

- one may look for special final states in DIS : $\gamma p \to (c\bar{c})X$ COMPASS
- at RHIC : polarized proton-proton scattering
- recall, parton distributions are universal !
- high-p_T / mass reactions, $pp \to \mathrm{jet}X$, γX , πX , $(c\bar{c})X$, WX ...

- spin asymmetry $\quad A_{LL} \equiv \frac{\sigma_{++} - \sigma_{+-}}{\sigma_{++} + \sigma_{+-}} \equiv \frac{\Delta\sigma}{\sigma}$

$$p_T^3 \frac{d\Delta\sigma}{dp_T} = \left| \quad \right|^2 + \mathcal{O}\left(\frac{\lambda}{p_T}\right)^n$$

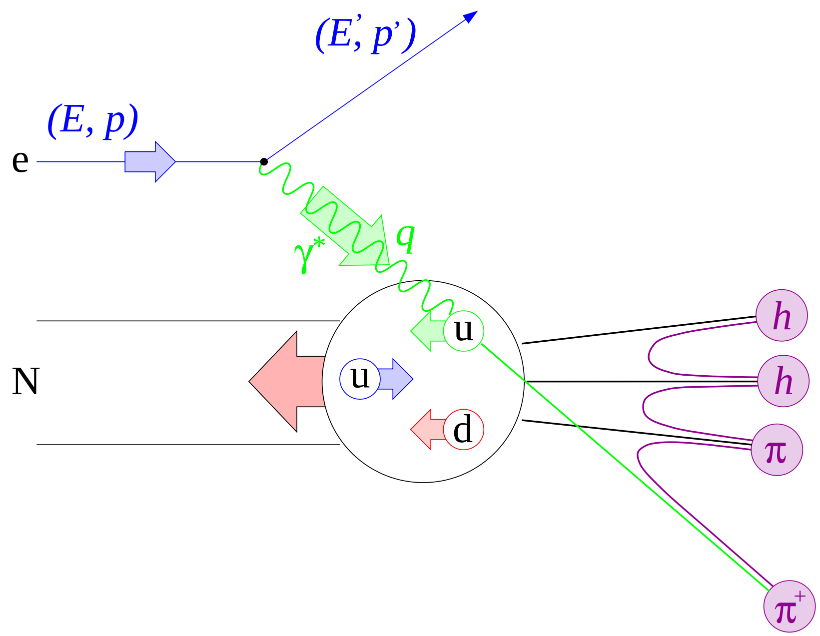


$$p_T^3 \frac{d\Delta\sigma^{pp\to \mathbf{F}X}}{dp_T} = \sum_{abc} \int dx_a\, dx_b\, dz_c\, \Delta f_a(x_a, \mu)\, \Delta f_b(x_b, \mu)$$

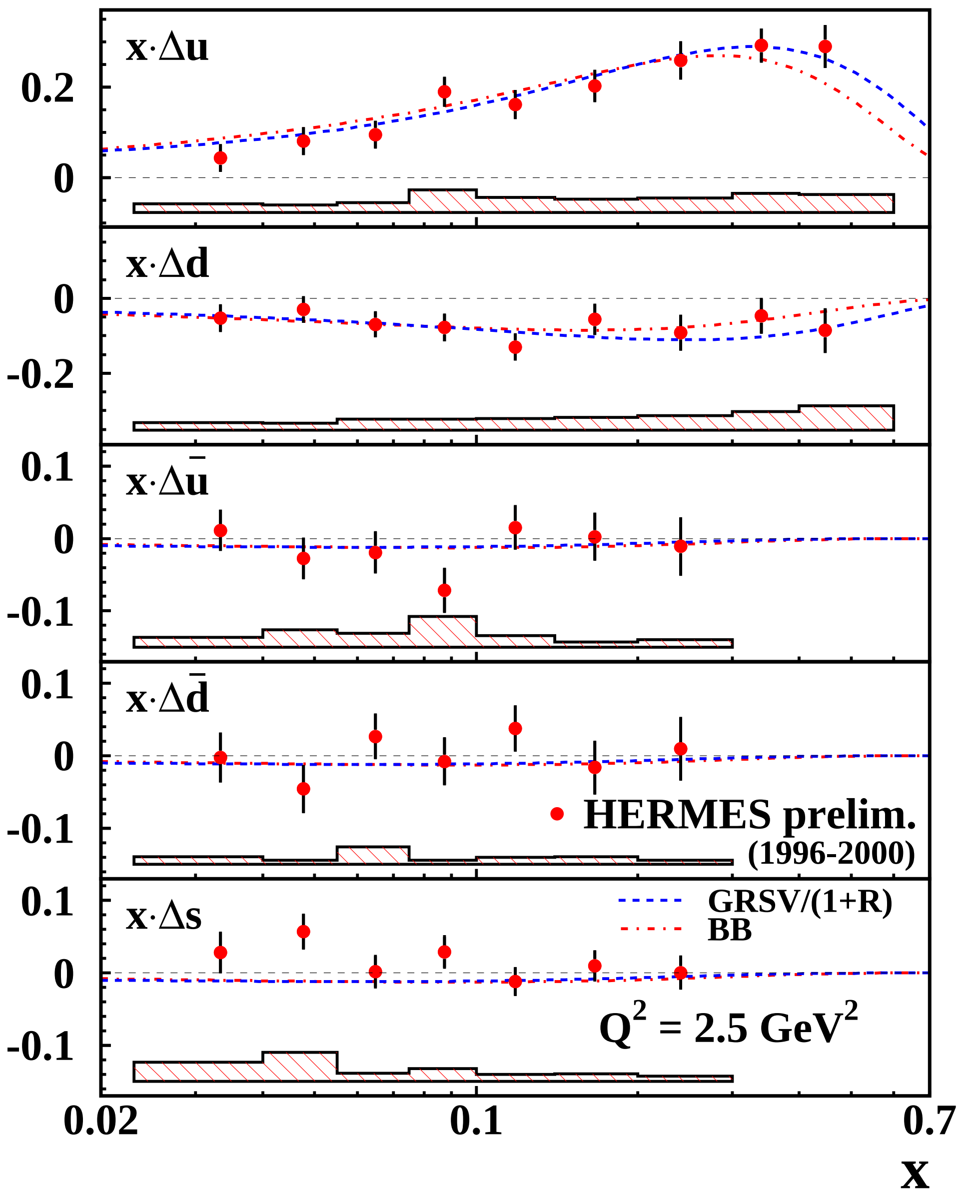
$$\times\; p_T^3 \frac{d\Delta\hat{\sigma}^{ab\to \mathbf{F}X'}}{dp_T}(x_a P_a, x_b P_b, P^{\mathbf{F}}/z_c, \mu) \;+\; \text{P.C.}$$

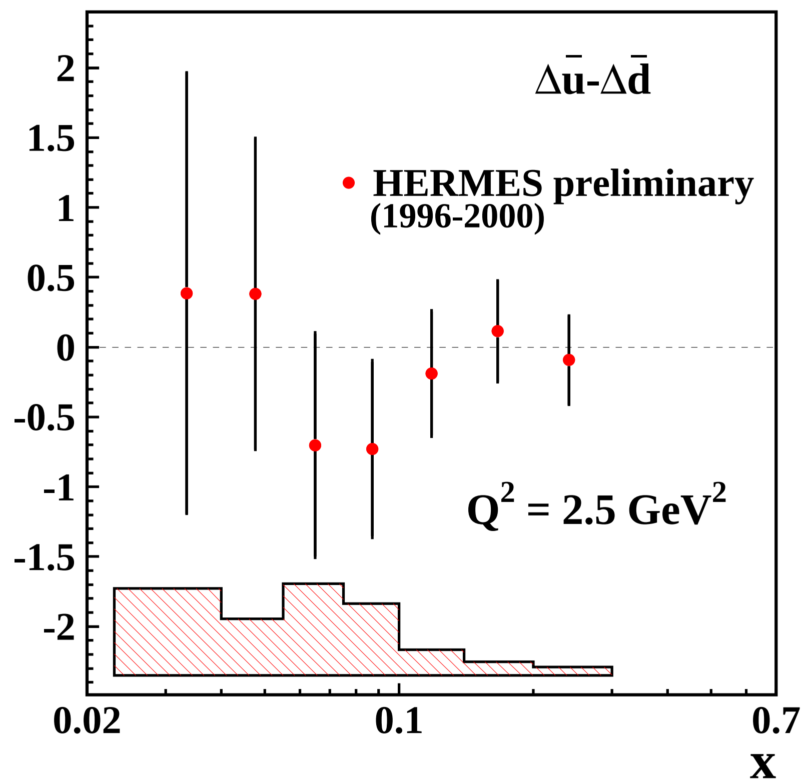
5.1 Further information on quark distributions

- inclusive DIS cannot distinguish between q and $\bar{q}$
- considerable interest :
  SU(2) breaking in sea (pion cloud models, Pauli exclusion, . . .)
- one option : semi-inclusive DIS. Detect a hadron $h = \pi^+, \pi^-, K^\pm, \ldots$



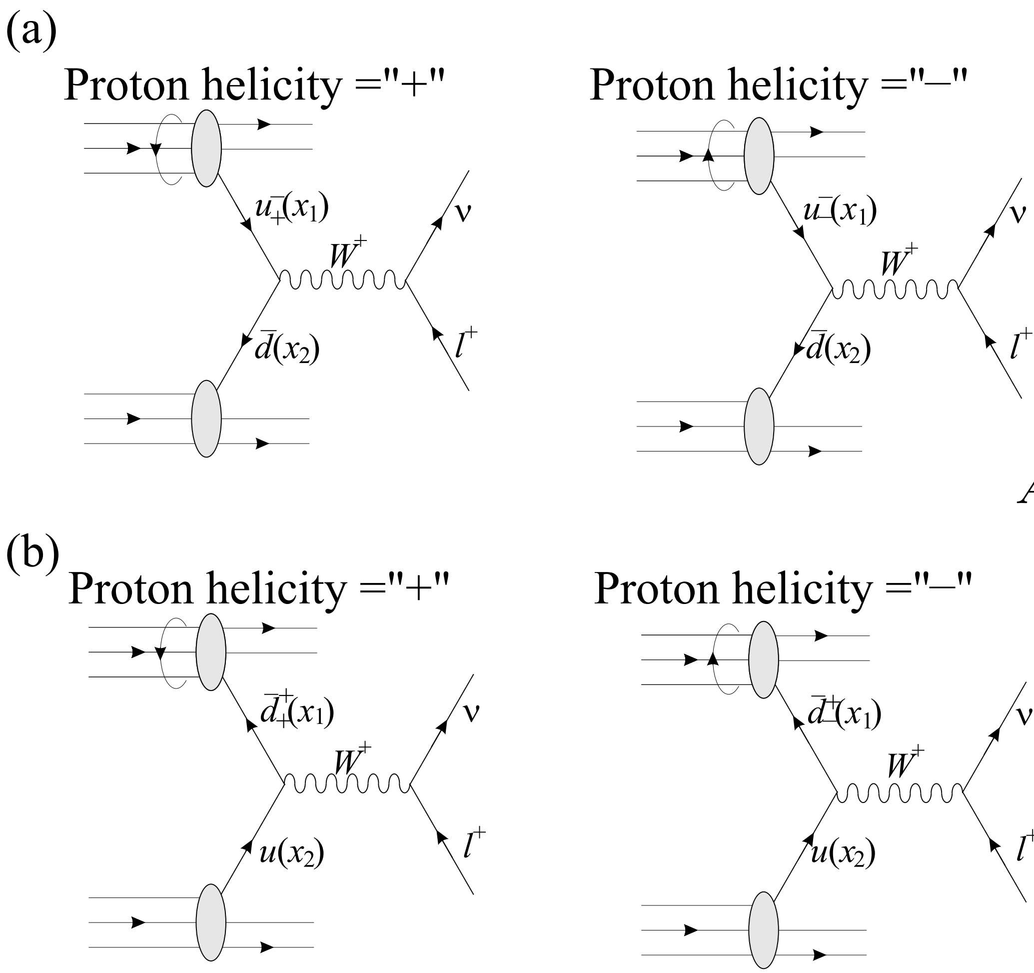
New HERMES results (Spin-2002) :





The way it is done at RHIC : W production

Parity violation $\leftrightarrow$ W selects parton helicity !

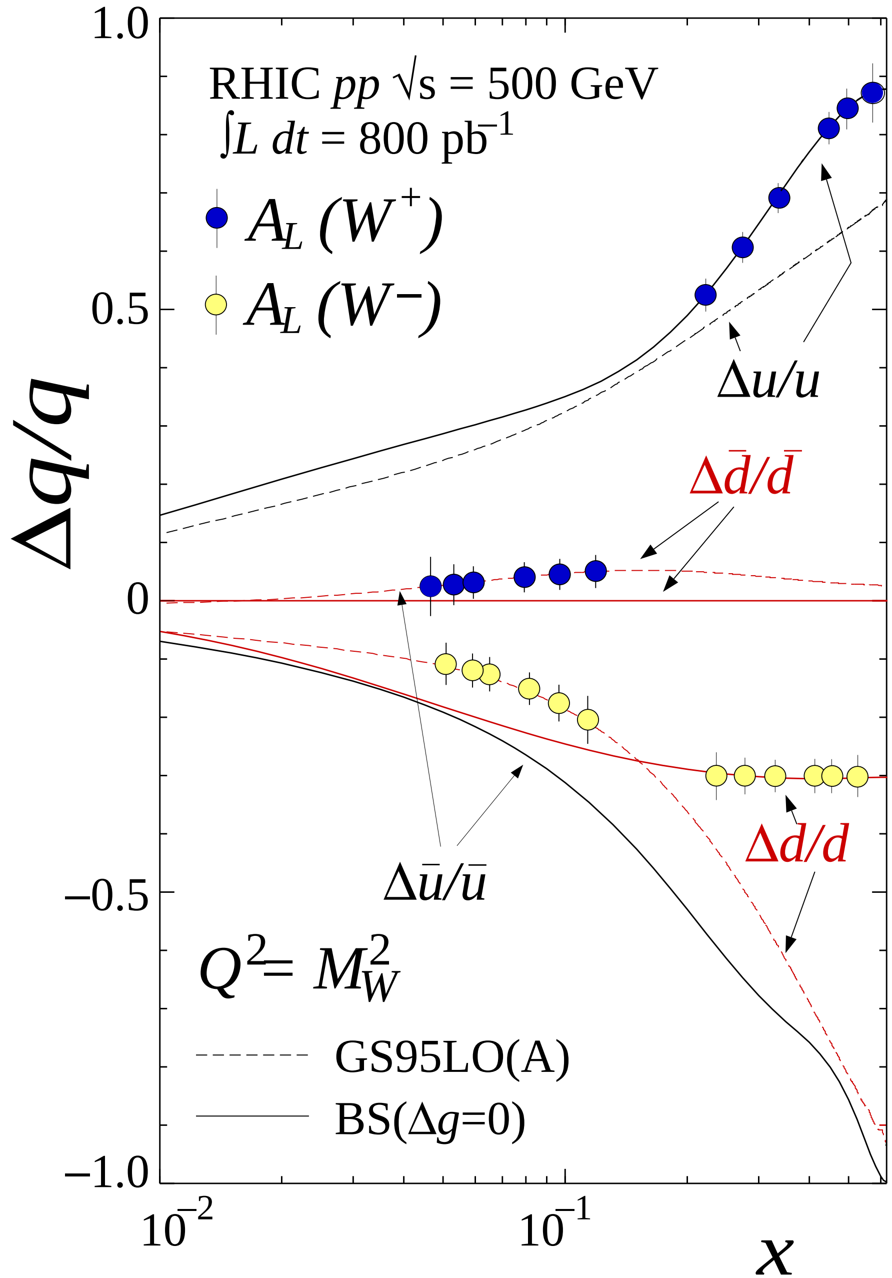


$$A_L^W \equiv \frac{\sigma_-(W) - \sigma_+(W)}{\sigma_-(W) + \sigma_+(W)}$$

$$A_L^{W^+} \approx \frac{\Delta u(x_1)\,\bar{d}(x_2) - \Delta\bar{d}(x_1)\,u(x_2)}{u(x_1)\,\bar{d}(x_2) + \bar{d}(x_1)\,u(x_2)}$$

(Bourrely,Soffer)

What you get :



NLO corrections, acceptance issues :

Weber; Kamal; Gehrmann; Nadolsky,Yuan

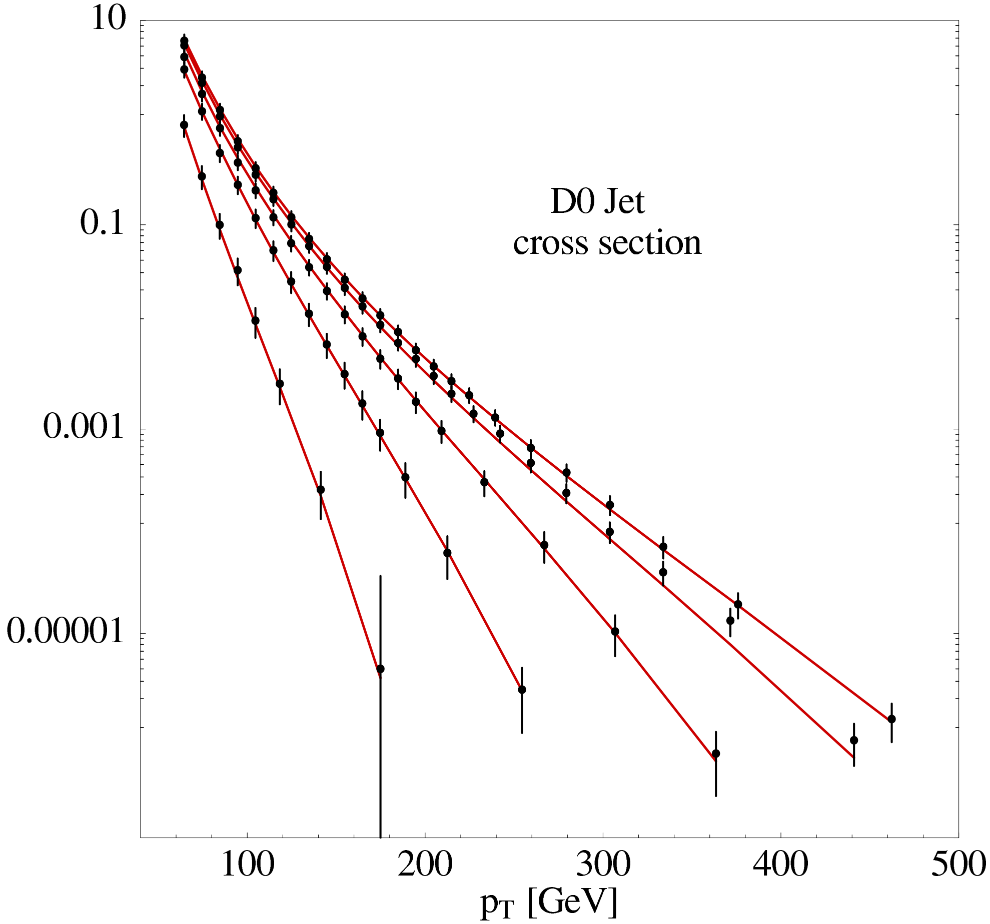
5.2 Measuring Δg

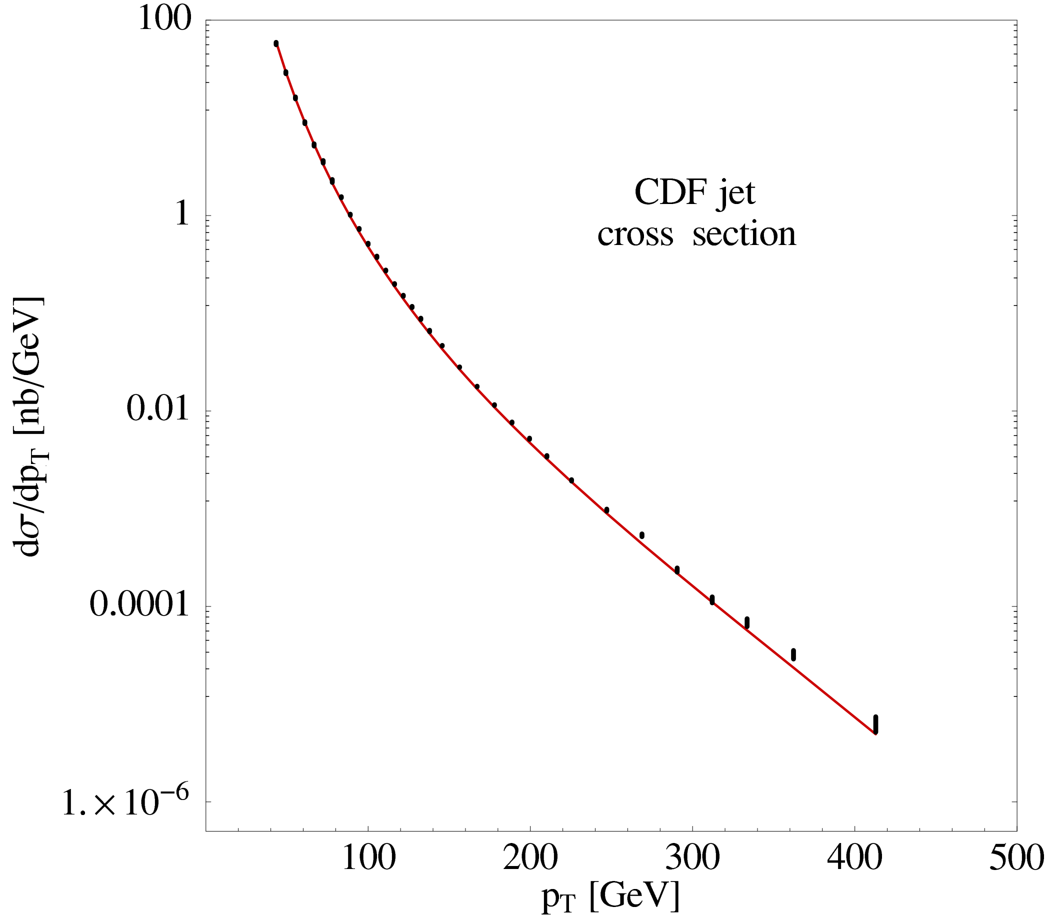
- several different reactions with sensitivity to Δg can be studied

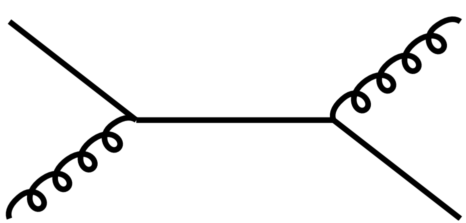
  for example, $pp \to \gamma X$, $pp \to \mathrm{jet} X$, $pp \to \pi X$, $pp \to (c\bar{c})X$, $\ldots$

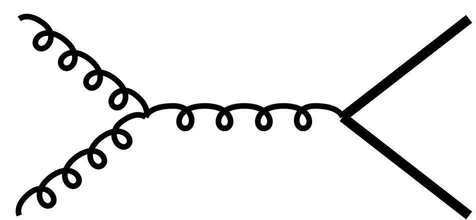
- for the first time, we can test universality properties of polarized pdfs
- emphasize : we are not only measuring nucleon structure
  - we also test QCD spin interactions !
- all studied extensively in the unpolarized case

Example : High-p_T jets at the Tevatron :

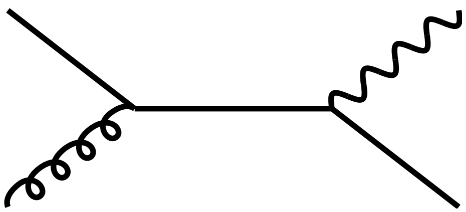


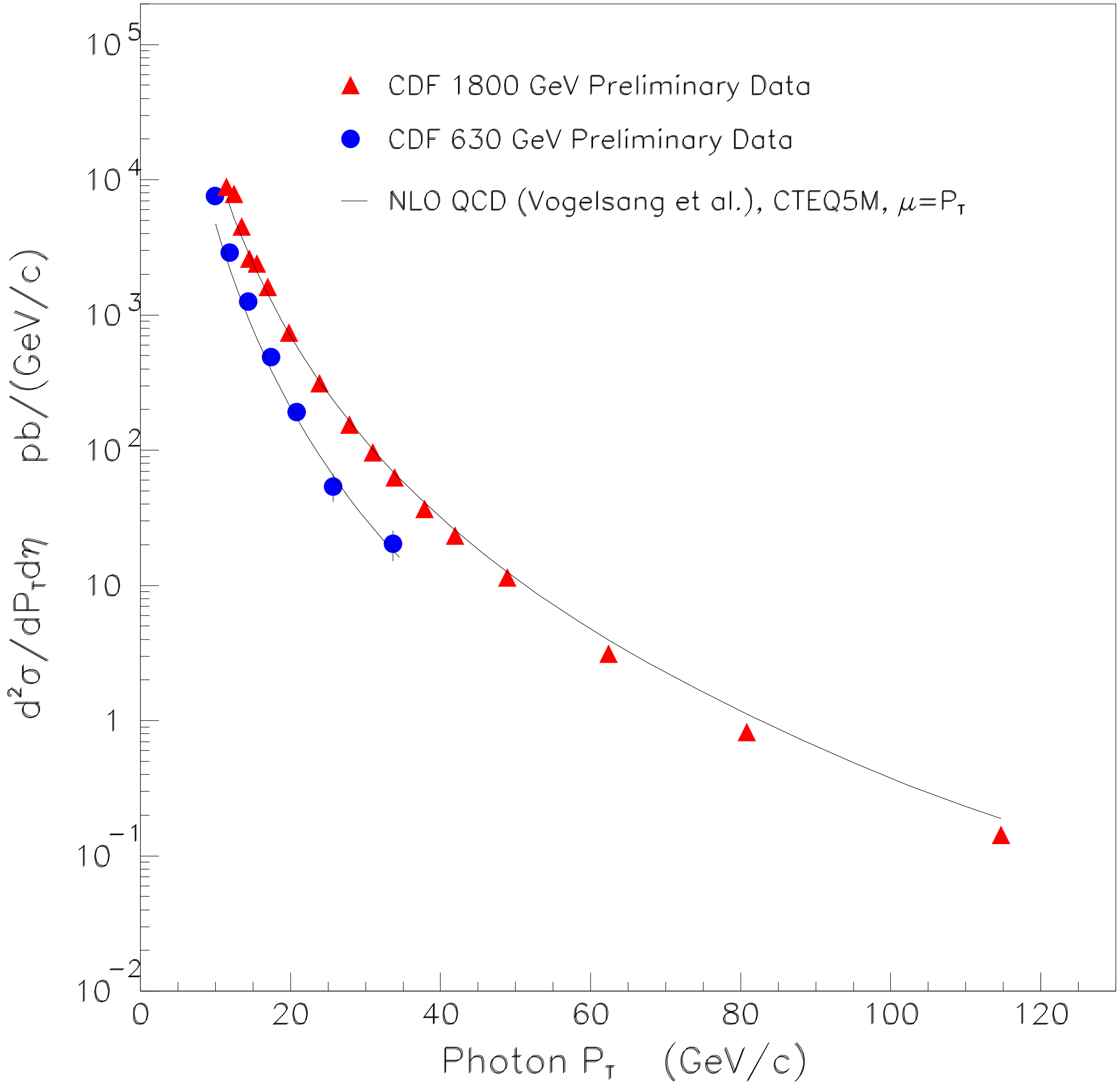






Example : High-p_T photons at the Tevatron :

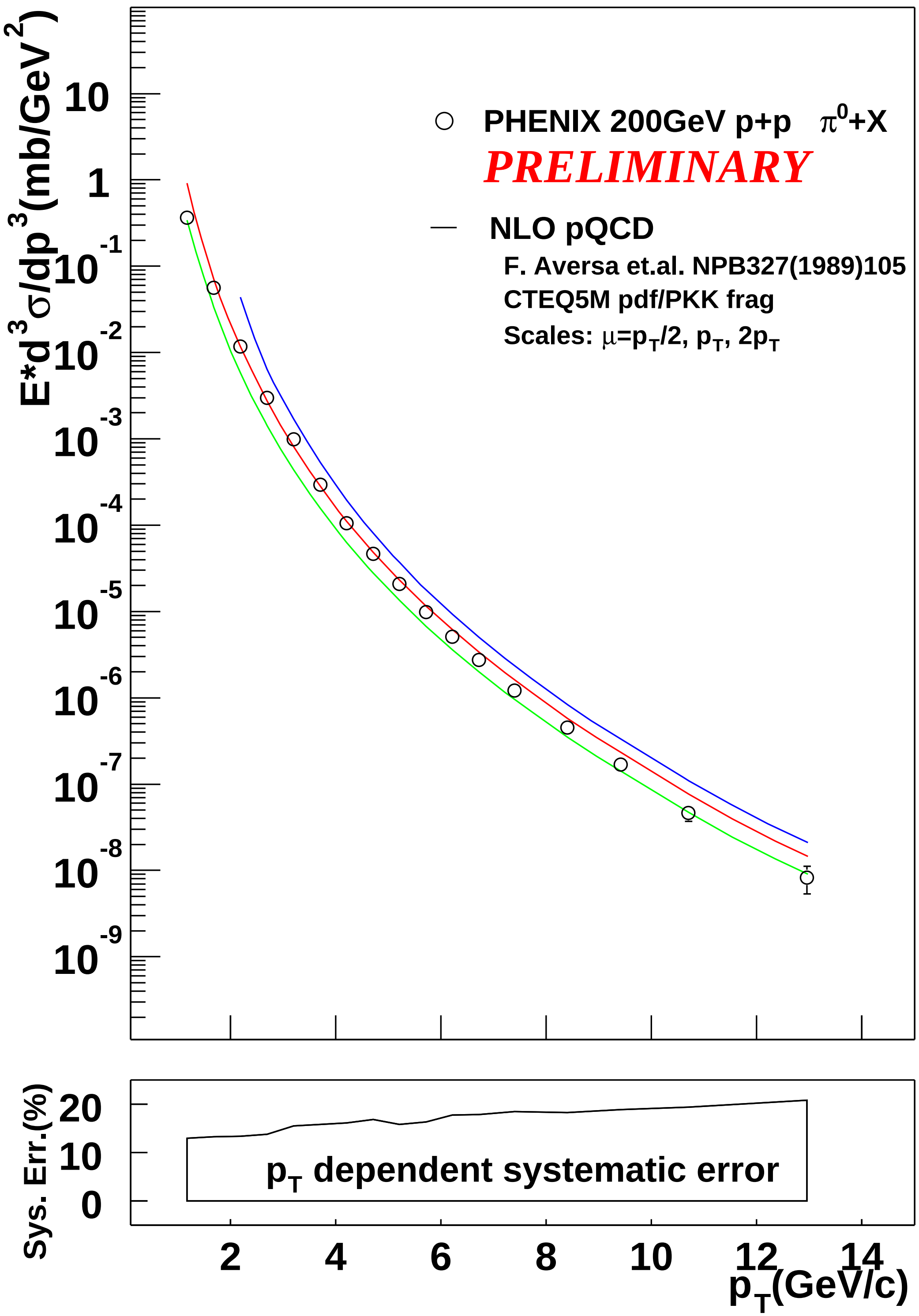




AND :

$pp \rightarrow \pi^0 X$ **by**

PHENIX



(±30% normalization unc.)

pQCD hard scattering works very well at colliders !

- in all cases, it is crucial to know QCD corrections to hard scattering :

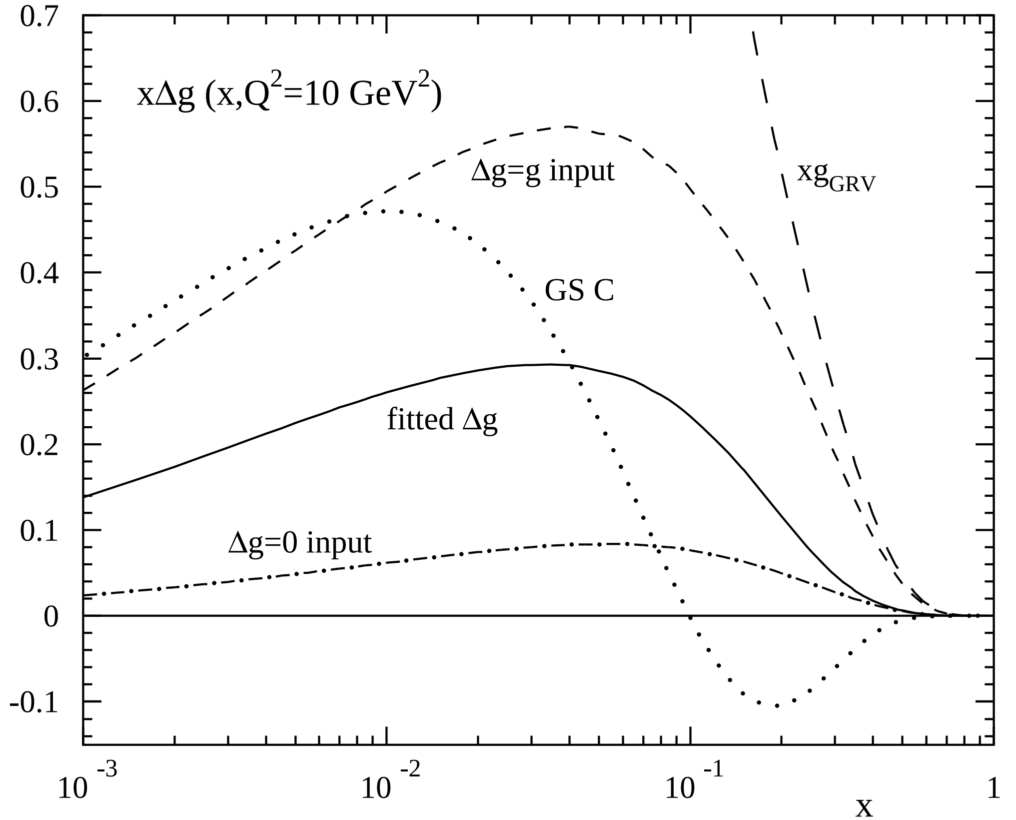
$$\hat{\sigma} = \underbrace{\hat{\sigma}^0}_{\text{LO}} + \underbrace{\alpha_s \hat{\sigma}^1}_{\text{NLO}} + \ldots$$

- lowest order : good for qualitative descriptions
  "catches the most important effects"

- next-to-leading order corrections :

  · often sizable

  · reduce dependence on unphysical scale μ

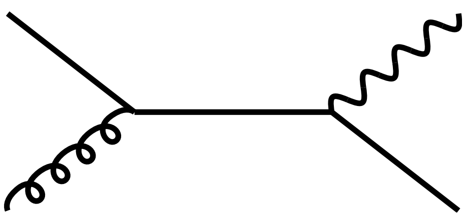


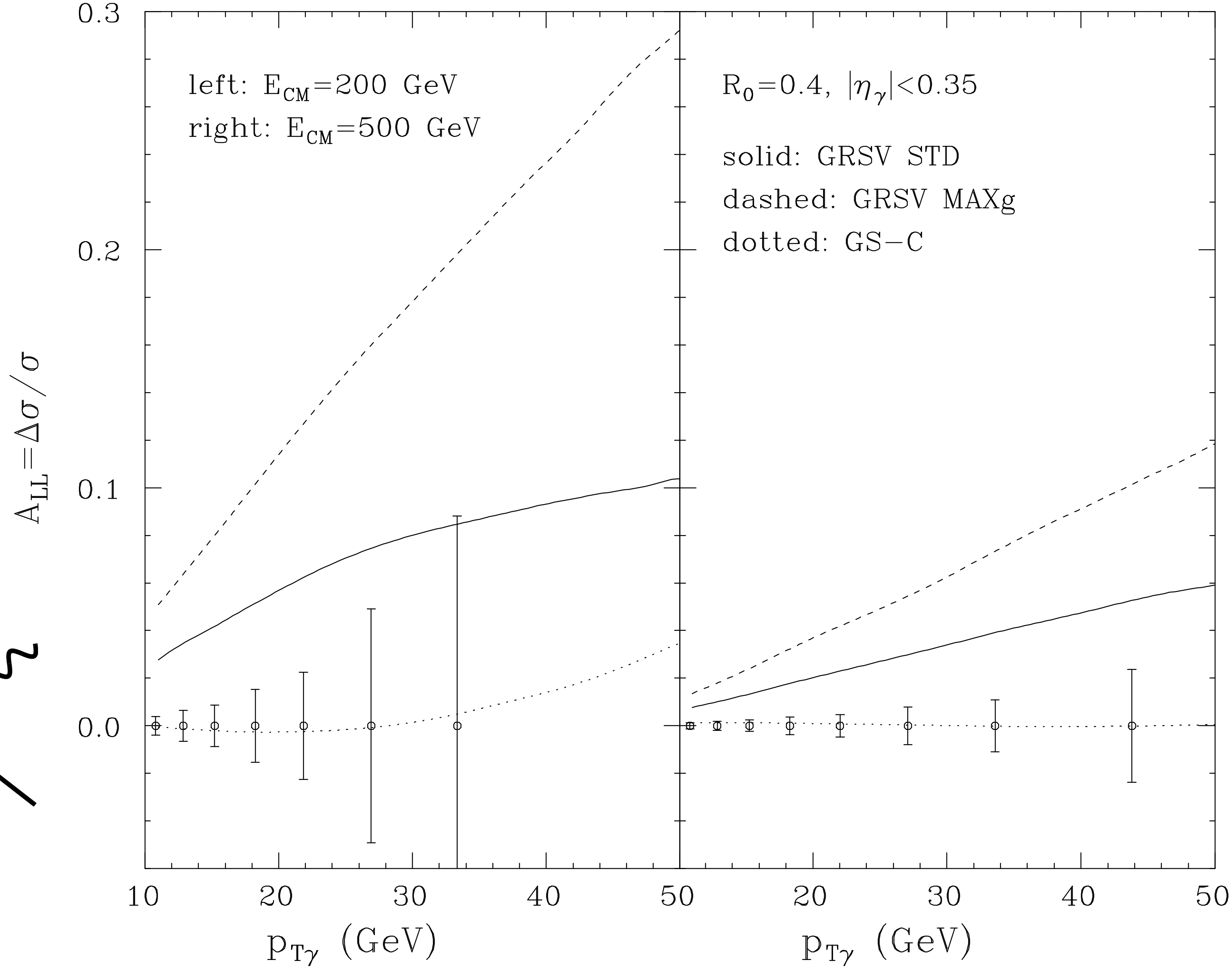
Polarized gluon densities :

(Glück,Reya,Stratmann,WV; Gehrmann,Stirling)

Some predictions : $\vec{p}\vec{p} \to \gamma X$

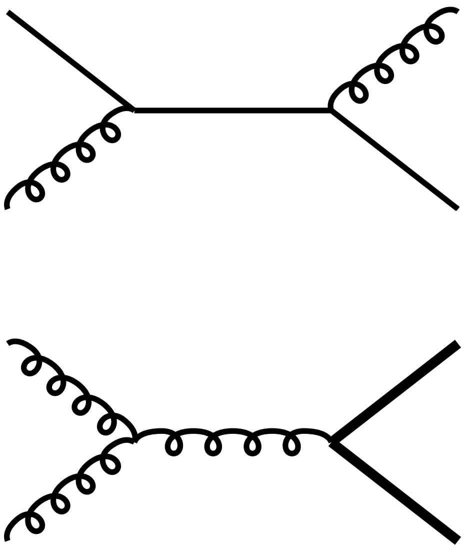
Frixione,WV

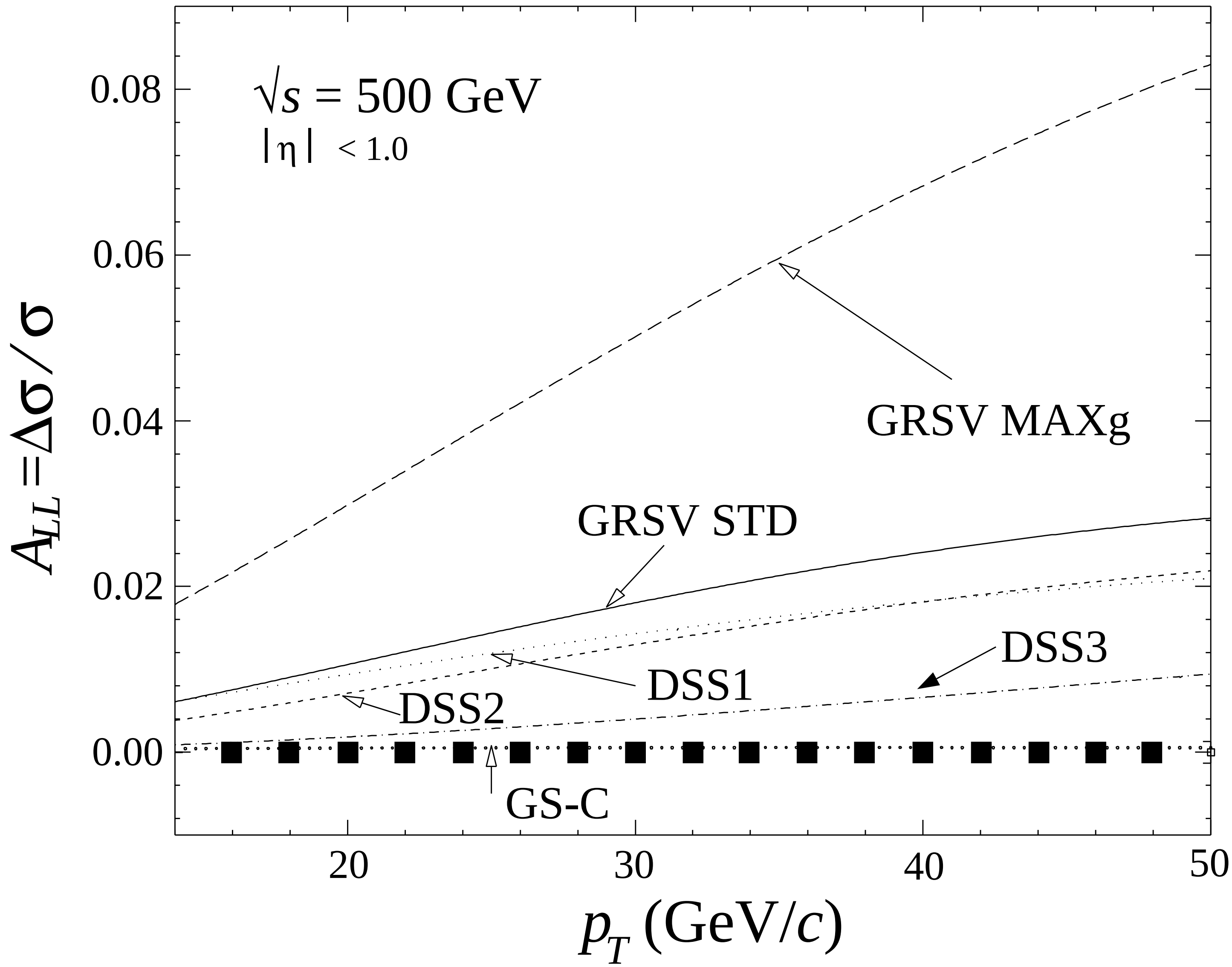




Some predictions : $\vec{p}\vec{p} \longrightarrow$ jet$X$

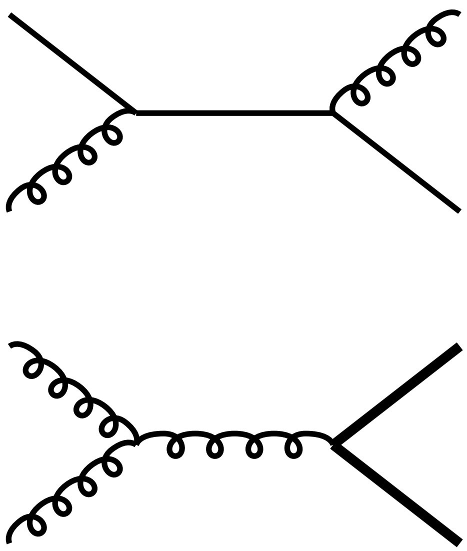
de Florian,Frixione,Signer,WV

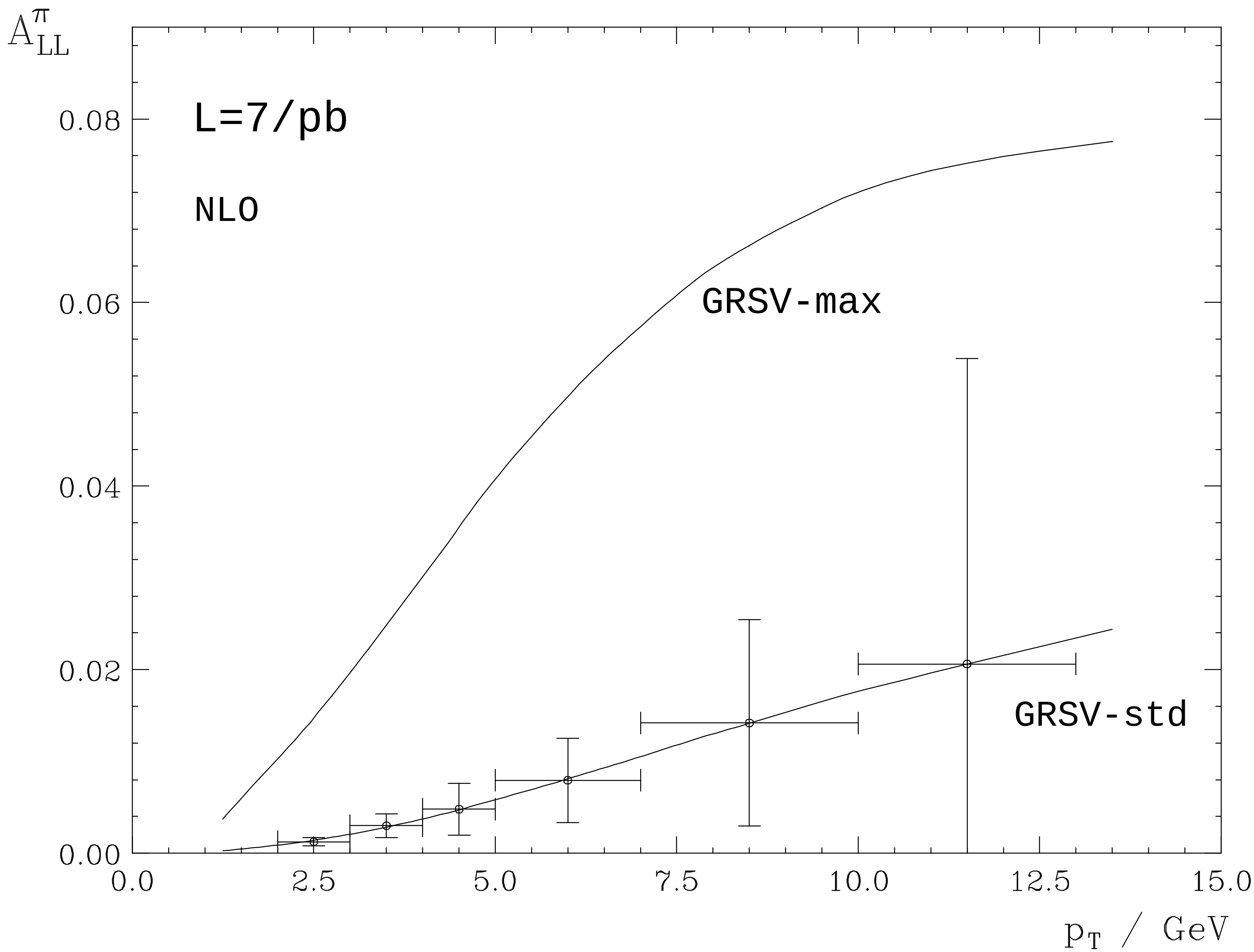




Already in coming run : $\vec{p}\vec{p} \to \pi^0 X$

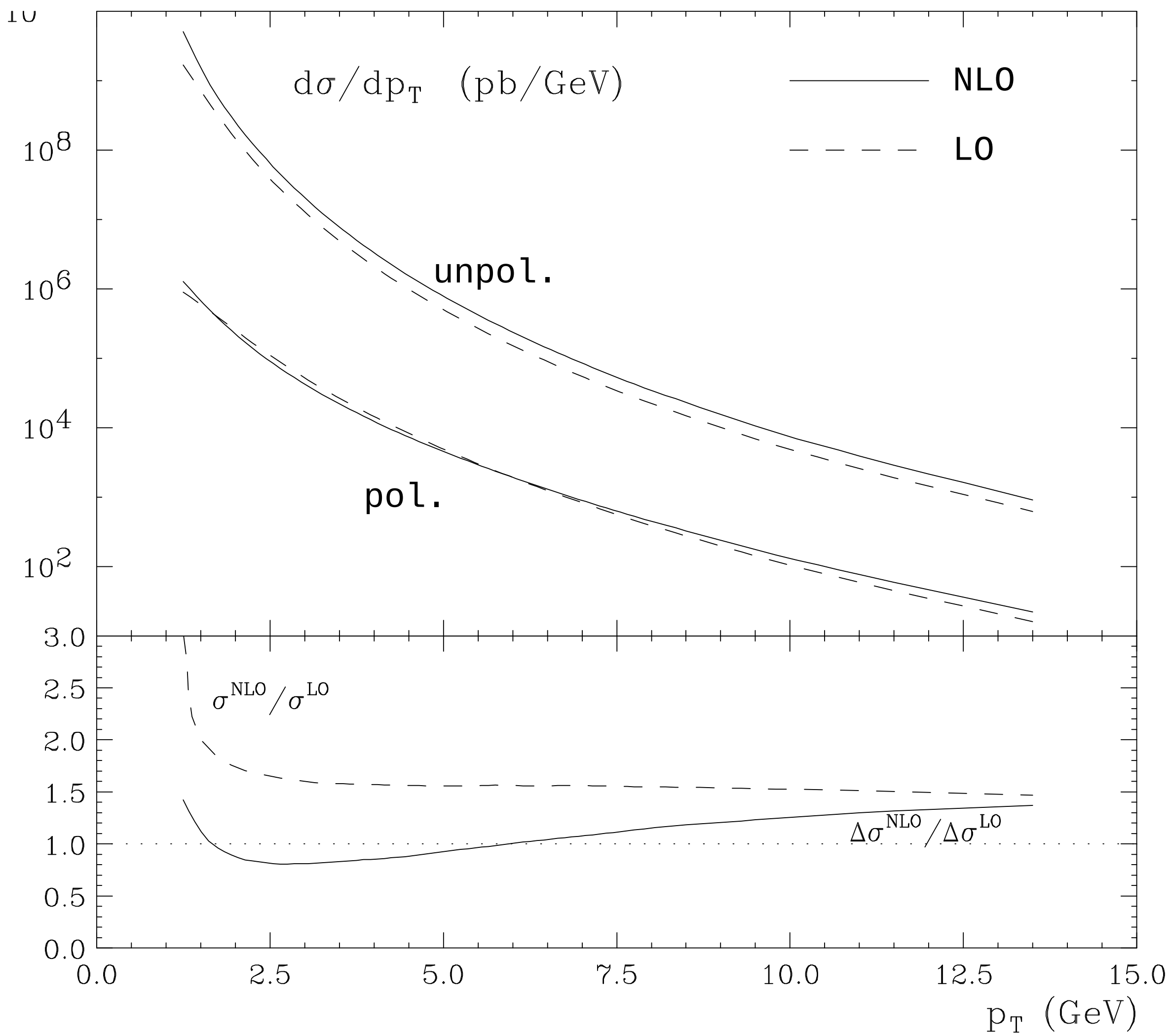
Jäger,Stratmann,WV





importance of NLO corrections:

$\sqrt{S} = 200\,\mathrm{GeV}$

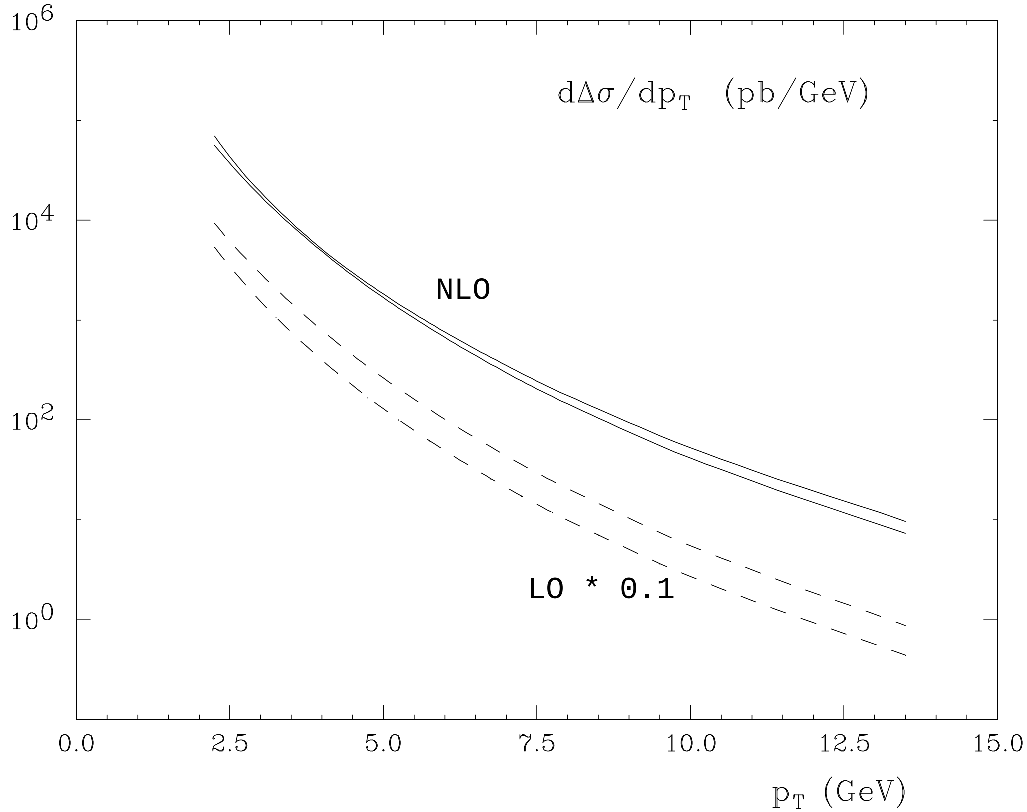


pdfs: CTEQ 5M (unpol.),

GRSV std. (pol.)

frag. fcts: Kniehl et al.

improvement in scale dependence:



variation of scales: $\mu_f = \mu_r = p_T \ldots 2p_T$

$\Downarrow$

NLO results much more reliable

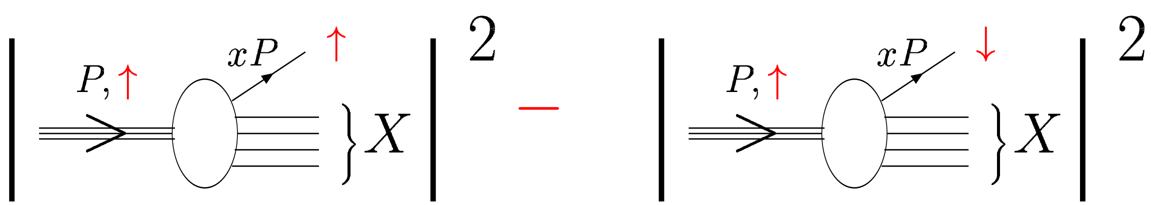
VI. Transverse-Spin Effects

(1) How to measure transversity ?

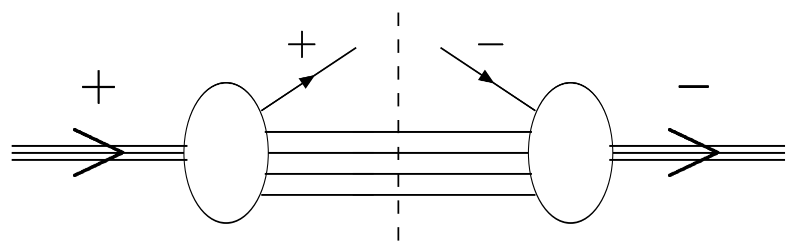
Some things we know about transversity :

- its definition and its structure in helicity basis :

$\delta q(x) =$



$\propto$



helicity-flip

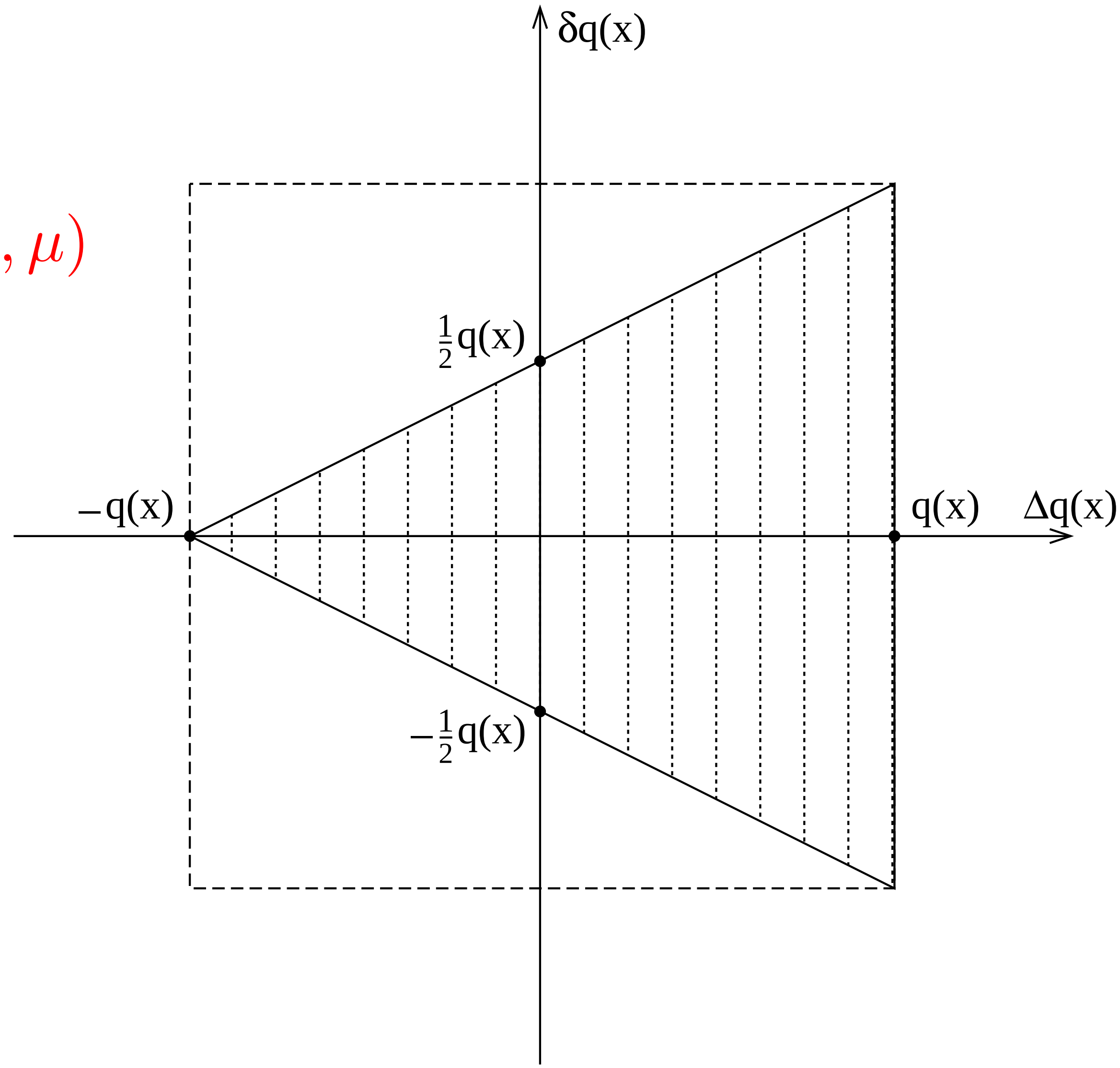
- therefore, no transversity gluon density !

- simple DGLAP evolution :

$$\frac{d\,\delta q(x,\mu)}{d\,\ln\mu} = \int_x^1 \frac{dz}{z}\,\delta\mathcal{P}_{qq}(z,\alpha_s(\mu))\,\delta q\left(\frac{x}{z},\mu\right)$$

- Soffer's inequality :

$$2\,|\delta q(x,\mu)| \le q(x,\mu) + \Delta q(x,\mu)$$

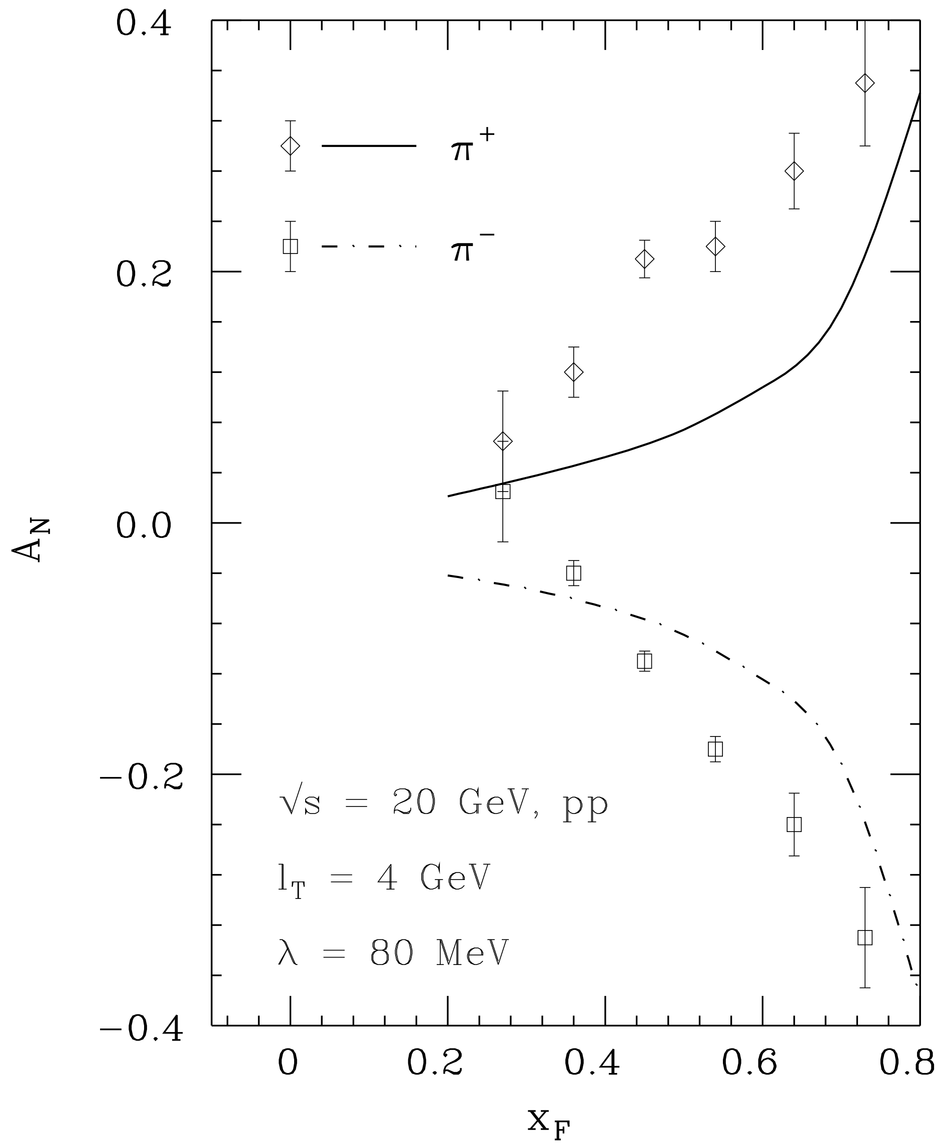


Measurement ?

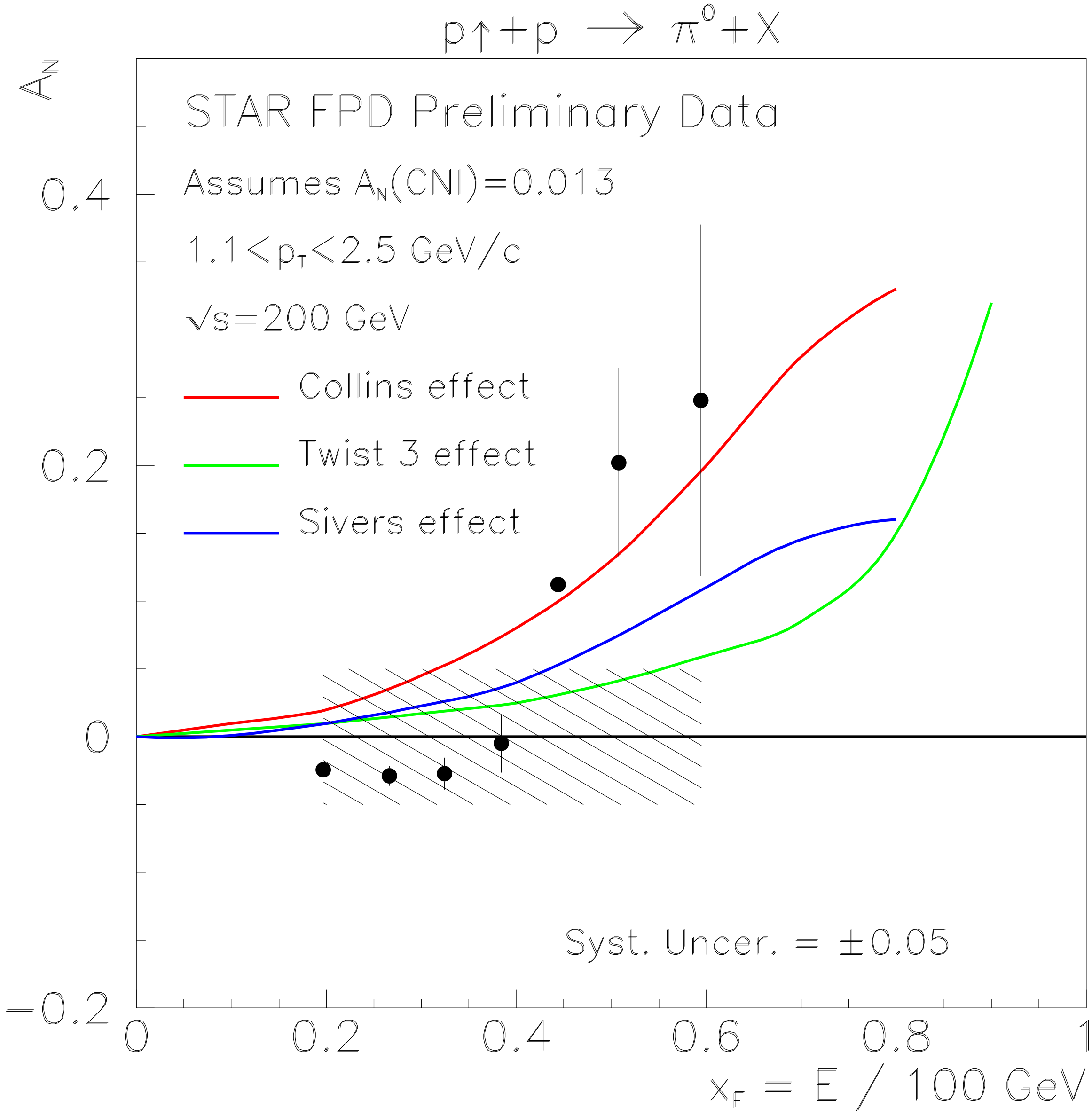
- not in inclusive DIS
- a first candidate : single-transverse spin asymmetry, say, $p^{\uparrow} p \to \pi X$

$$A_{\rm N} \equiv \frac{\sigma_{\uparrow} - \sigma_{\downarrow}}{\sigma_{\uparrow} + \sigma_{\downarrow}}$$

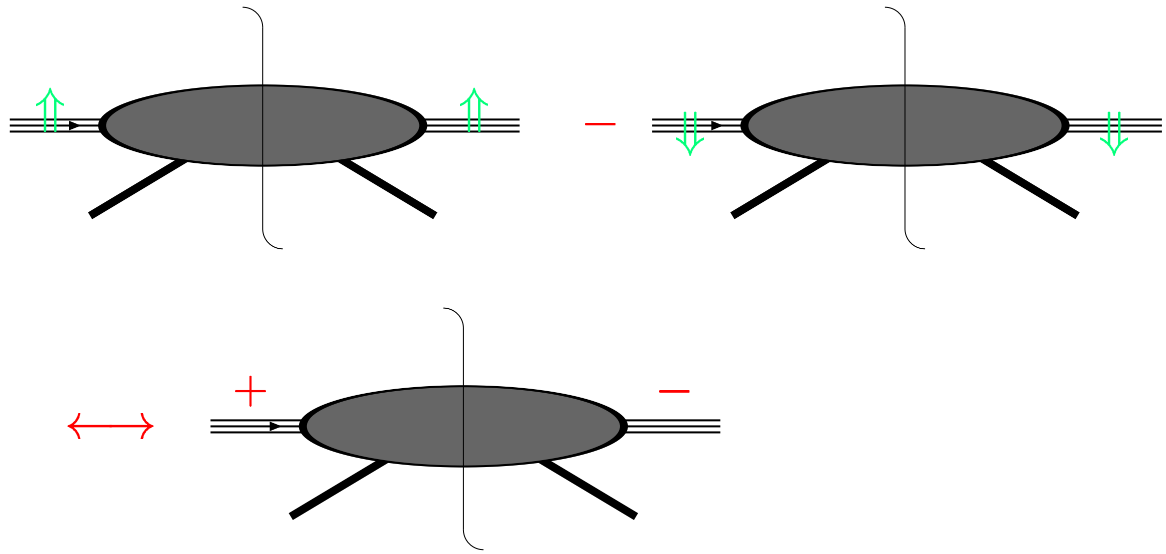
- after all, it's large experimentally !



E704

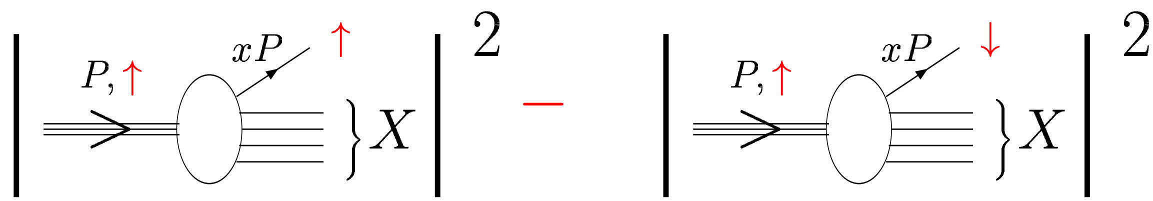

p↑+p → π⁰+X
A_N
STAR FPD Preliminary Data
Assumes A_N(CNI)=0.013
1.1<p_T<2.5 GeV/c
√s=200 GeV
Collins effect
Twist 3 effect
Sivers effect
Syst. Uncer. = ±0.05
0.4
0.2
0
−0.2
0
0.2
0.4
0.6
0.8
1
x_F = E / 100 GeV

- recall, contribution to A_{N} enters via

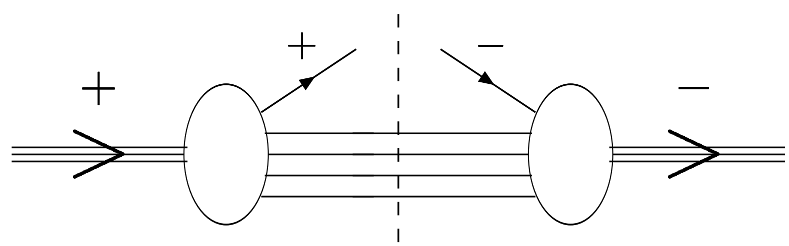


- for transversity to contribute :

$\delta q(x) =$

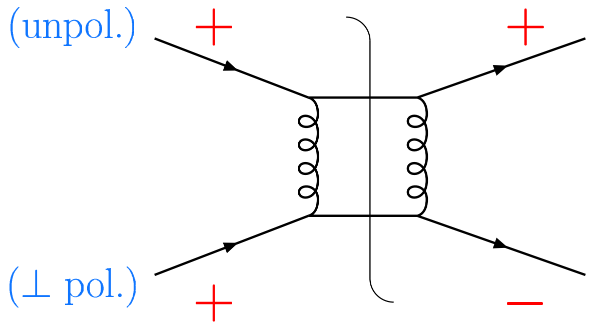


$\propto$



helicity-flip

- recall, we found for our $e\,\mu \longrightarrow e\,\mu$ example

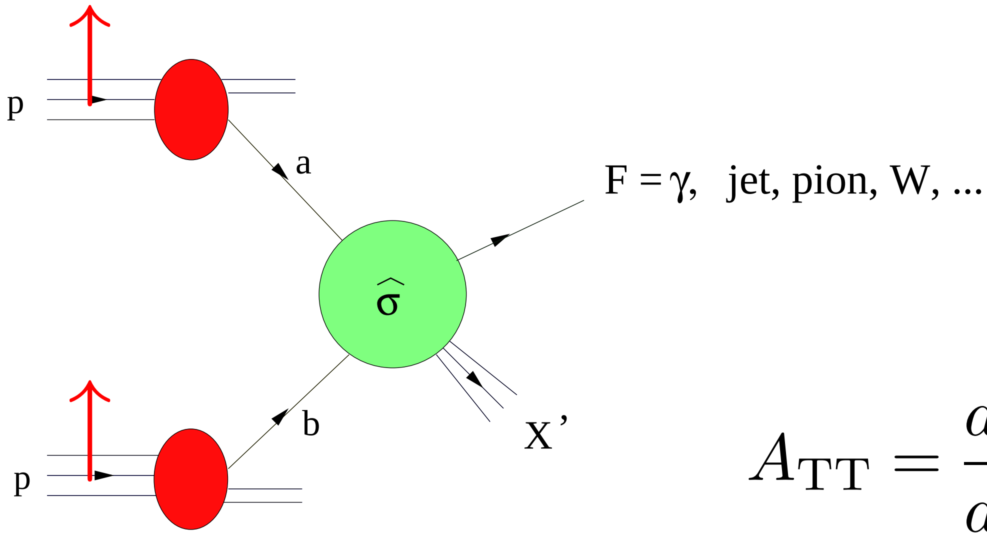


$$A_{\mathrm{N}} \;\propto\; \sin(\varphi) \sum_{\Lambda,\Lambda'} \mathrm{Im}\left[\, h_{++;\Lambda\Lambda'}\, h^{*}_{+-;\Lambda\Lambda'} \,\right]$$

$$\propto\; \mathrm{Im}\{(\mathrm{helicity\ non-flip})^{*}\,(\mathrm{single\ flip})\}$$

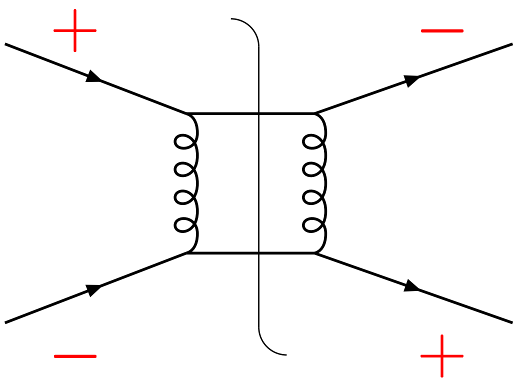
- chirality conservation at qqg vertices permits only helicity flip $\propto m_q$
- tree-amplitudes are real. Need imaginary part $\longrightarrow$ loop diagrams
- net effect is $A_{\mathrm{N}} \propto \frac{m_q}{p_T}\alpha_s \ll 1$
- so, no transversity this way !
- (explains why it is so hard to get description of single-spin asymm.)

• a viable way (among others) : double-transverse spin asymmetries



$$A_{\mathrm{TT}} = \frac{d\sigma^{p^\uparrow p^\uparrow} - d\sigma^{p^\uparrow p^\downarrow}}{d\sigma^{p^\uparrow p^\uparrow} + d\sigma^{p^\uparrow p^\downarrow}}$$

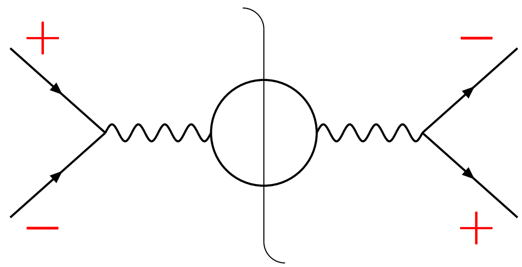
• recall, we found for our $e\,\mu \longrightarrow e\,\mu$ example

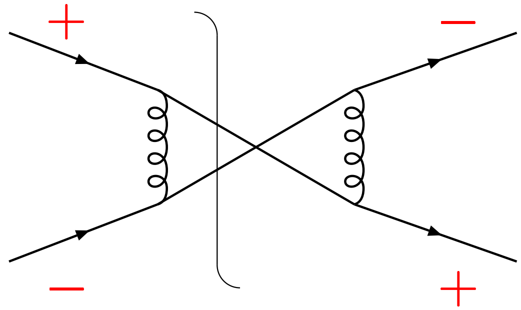


$$A_{\mathrm{TT}} \;\propto\; \cos(2\varphi) \sum_{\Lambda,\Lambda'} \mathrm{Re}\left[\, h_{+-;\Lambda\Lambda'}\, h^*_{-+;\Lambda\Lambda'} \,\right]$$

• still not possible ?

- however, there are two possibilities :





annihilation — identical quarks

Drell-Yan, direct photons — jet production

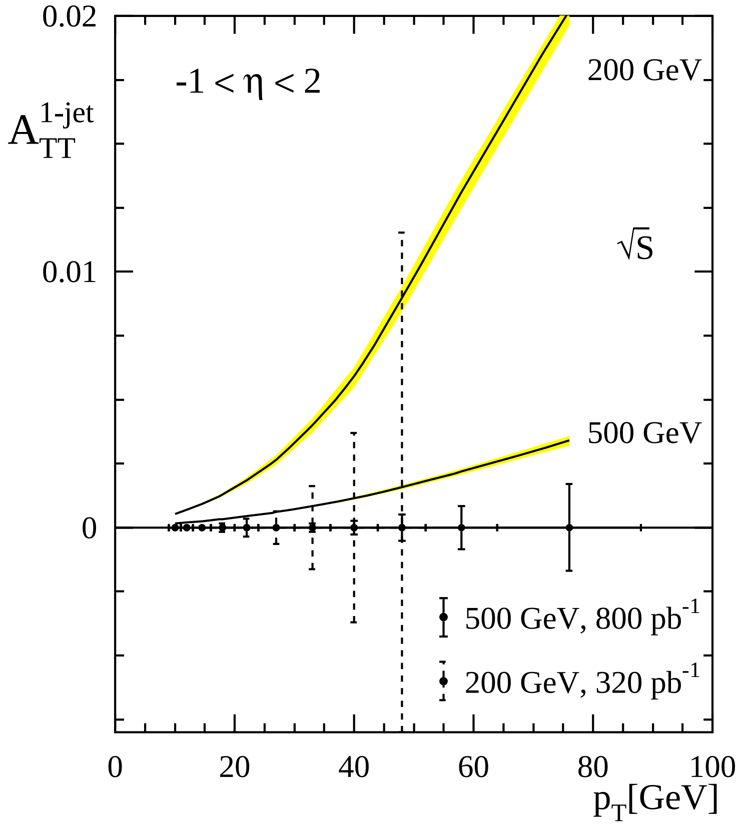
- caveats :
  - Drell-Yan : relies on presence of antiquark transversity
    no gluon density $\Rightarrow$ few antiquarks ? not necessarily
  - jets etc. : suffer from small asymmetries due to
    unpolarized gluon distribution in the denominator
    (also, some cross sections color suppressed) $A_{\rm LL} \ll A_{\rm TT}$

- nonetheless . . .

Some predictions :

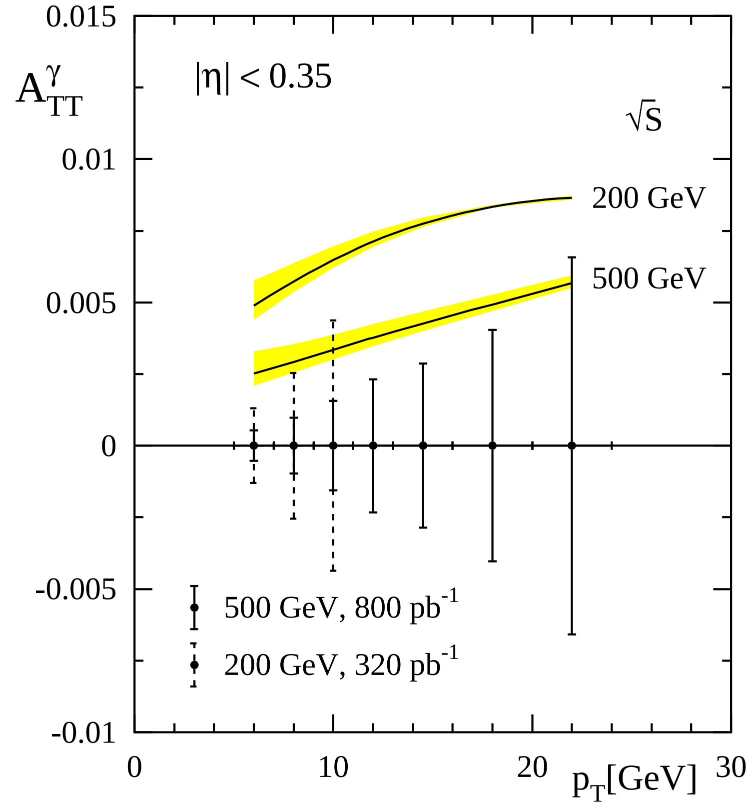
1-jet

STAR



high-p_T photon

PHENIX

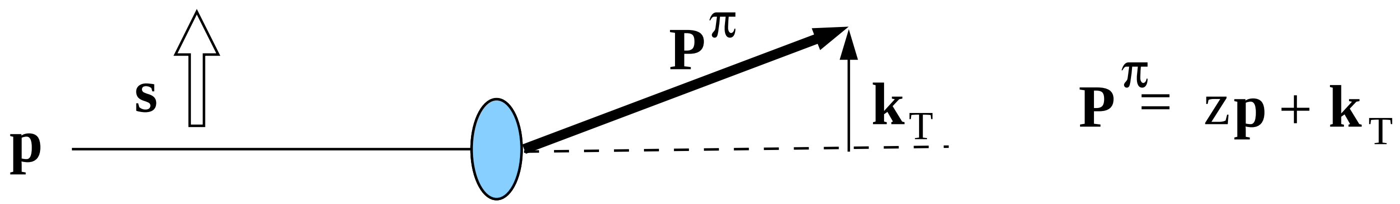


(Soffer,Stratmann,WV)

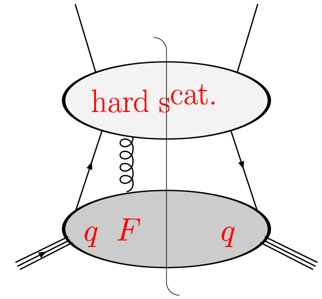
(2) Back to single-transverse-spin asymmetries

- saw that A_N is a *power-suppressed* contribution – but a sizeable one !
- in pQCD, it has to behave as $1/p_T$ at large p_T
- there is more dynamics in QCD that can lead to power-suppressed contributions
- $k_\perp$-dependence of distribution/fragmentation fcts.

  Sivers; Collins; Boer; Anselmino et al.; Leader et al.; Brodsky et al. . . .



- "Twist-3 quark-gluon correlation functions"



Qiu,Sterman; Teryaev et al.; Koike et al.; . . .

$\Rightarrow$ new universal nucleon matrix elements such as $\langle p|\,\bar{q}\,F\,q\,|p\rangle$

- RHIC will be the ideal testing ground :

enter the regime where pQCD should really work !

. . . to be continued . . .